The Split Delivery Vehicle Routing Problem with Minimum Delivery Amounts

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CVRP

The capacitated vehicle routing problem (CVRP) is a classical problem in operations research

In the CVRP a fleet of vehicles with the same capacity, leaving from and returning to a depot, must satisfy the demands of all customers in such a way that the total cost (distance or time) across all routes is minimized

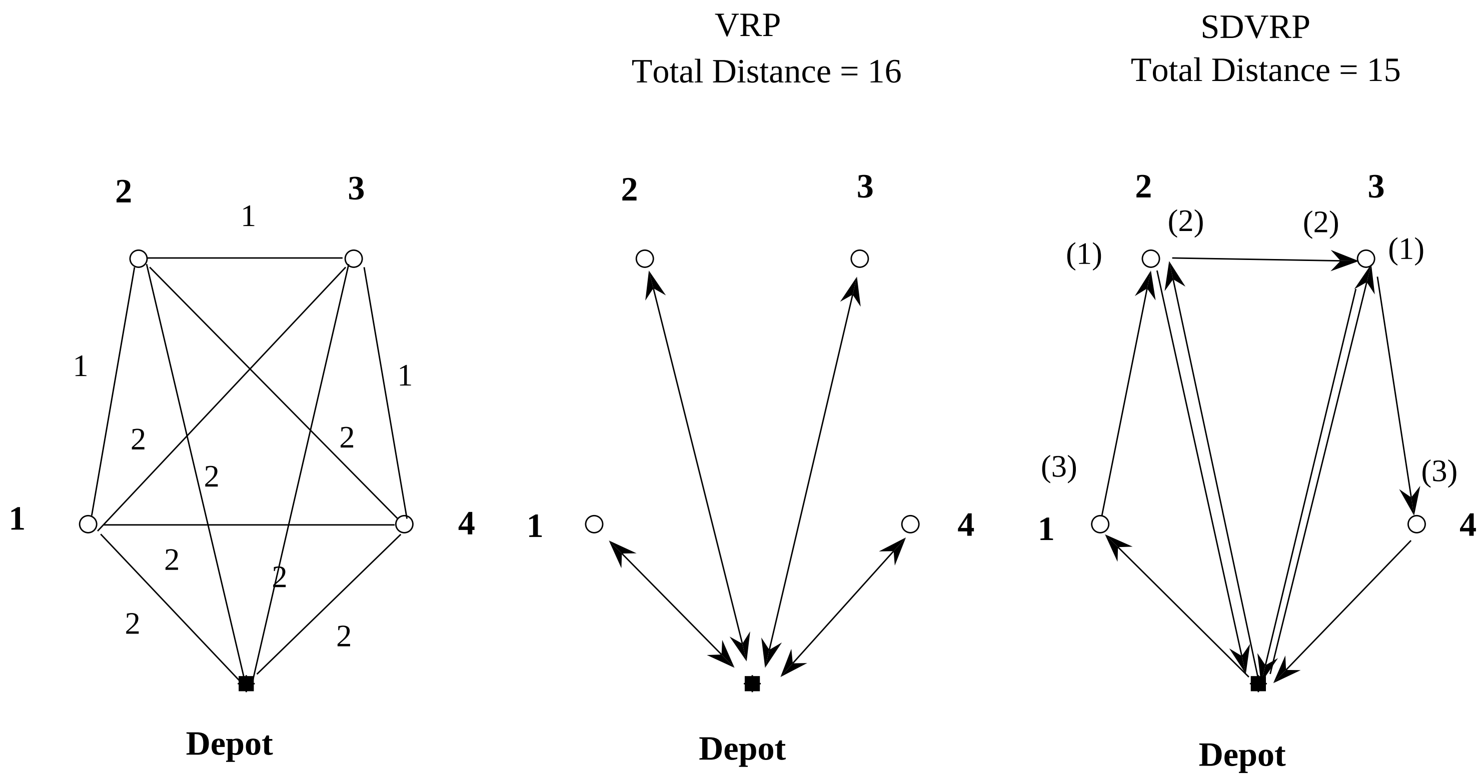
SDVRP

The split delivery vehicle routing problem (SDVRP) is a variant of the CVRP in which a customer can be serviced by more than one vehicle (i.e., the demand at a customer can be split among vehicles)

The SDVRP was introduced by Dror and Trudeau (1990)

SDVRP

Customer demand is 3 and vehicle capacity is 4



SDVRP

Observations from Archetti, Savelsberg, and Speranza (2006):

By allowing split deliveries, cost can potentially be reduced by as much as 50%

When demands are small or large with respect to vehicle capacity (<25%, >90%), splitting does little to improve a solution

Potential for improvement is greatest when demand is between 50%-75% of vehicle capacity

Applications

Mullaseril, Dror, and Leung (1997)
    Distribution of livestock feed on a large ranch

Sierksma and Tijssen (1998)
    Routing helicopters to offshore platforms for crew exchanges

Archetti and Speranza (2004)
    Waste collection

Solution Procedures

Belenguer, Martinez, and Mota (2000)
Cutting plane

Archetti, Speranza, and Hertz (2006)
Tabu search algorithm

Chen, Golden, and Wasil (2007)
Endpoint mixed integer program with record-to-record travel algorithm

Jin, Liu, and Eksioglu (2008)
Column generation

SDVRP-MDA

The split delivery vehicle routing problem with minimum delivery amounts (SDVRP-MDA) is a new variant of the SDVRP

In the SDVRP-MDA, a customer can be serviced by more than one vehicle only if each vehicle visiting the customer satisfies a minimum amount of its demand

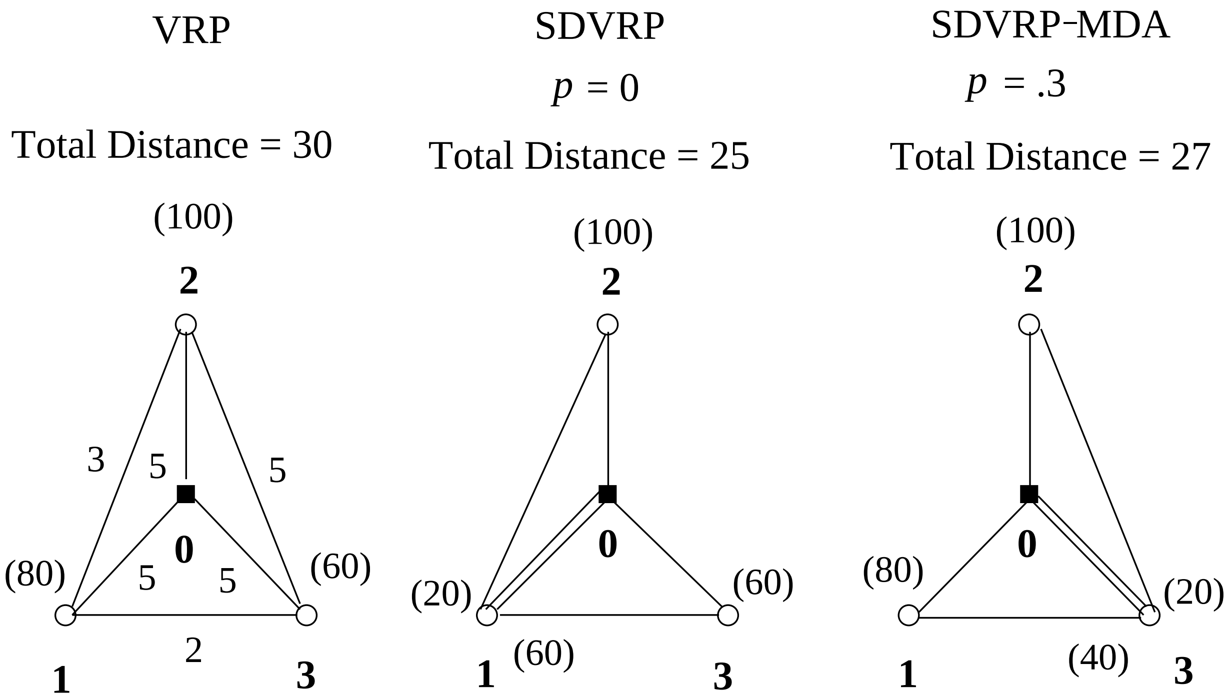
Reduces to the SDVRP when there is no minimum delivery amount, and to the CVRP when customer demand is satisfied by a single vehicle, so the SDVRP-MDA is NP-hard

SDVRP-MDA

Although a customer would prefer to have his demand delivered at one time, the customer would be willing to be serviced by more than one vehicle provided that each vehicle delivers a minimum amount

SDVRP-MDA

Demands are in parentheses, the vehicle capacity is 120, and p is the minimum fraction of a customer's demand that must be satisfied by a vehicle



Solution Procedure for SDVRP-MDA

Two-stage procedure

Stage 1. Find splits using the endpoint mixed integer program with minimum delivery amount constraints (EMIP-MDA)

Stage 2. Clean-up routes using the enhanced record-to-record travel algorithm (ERTR)

EMIP-MDA

We start with an initial VRP solution (no splits) generated by the Clarke-Wright savings procedure

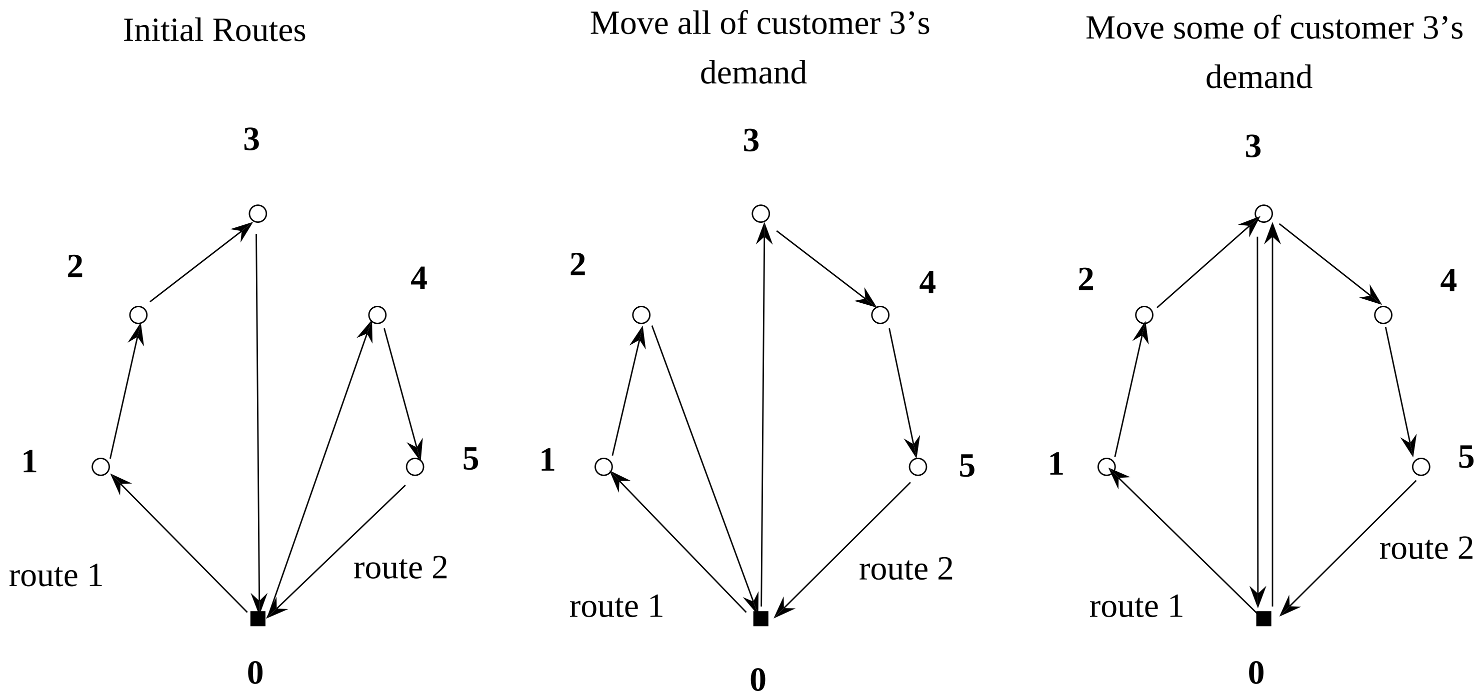
Since all routes meet at the depot, we look for splits near the depot

Each endpoint (node adjacent to the depot) is allowed to reallocate some of its demand according to three possibilities:

EMIP-MDA

1. No change is made
2. The endpoint is removed from its current route(s) and all of its demand is reallocated to other routes
3. Some of an endpoint's demand is removed from its current route(s) and reallocated to other routes

Endpoint moves



MIP Formulation

Objective Function

Maximize total savings from endpoint reallocation

Constraints

Demand reallocated to a route minus the demand reallocated from the route $\leq$ residual capacity of the route

Demand reallocated from an endpoint $\leq$ total demand of the endpoint

MIP Formulation

If an endpoint is removed from a route, then all of its demand is reallocated

If we reallocate some demand from endpoint i prior to endpoint j, then we insert i before j

If we remove an endpoint from a route, we do not insert an endpoint before it or its successor

If a route has only two customers, then we remove at most one of them

MIP Formulation

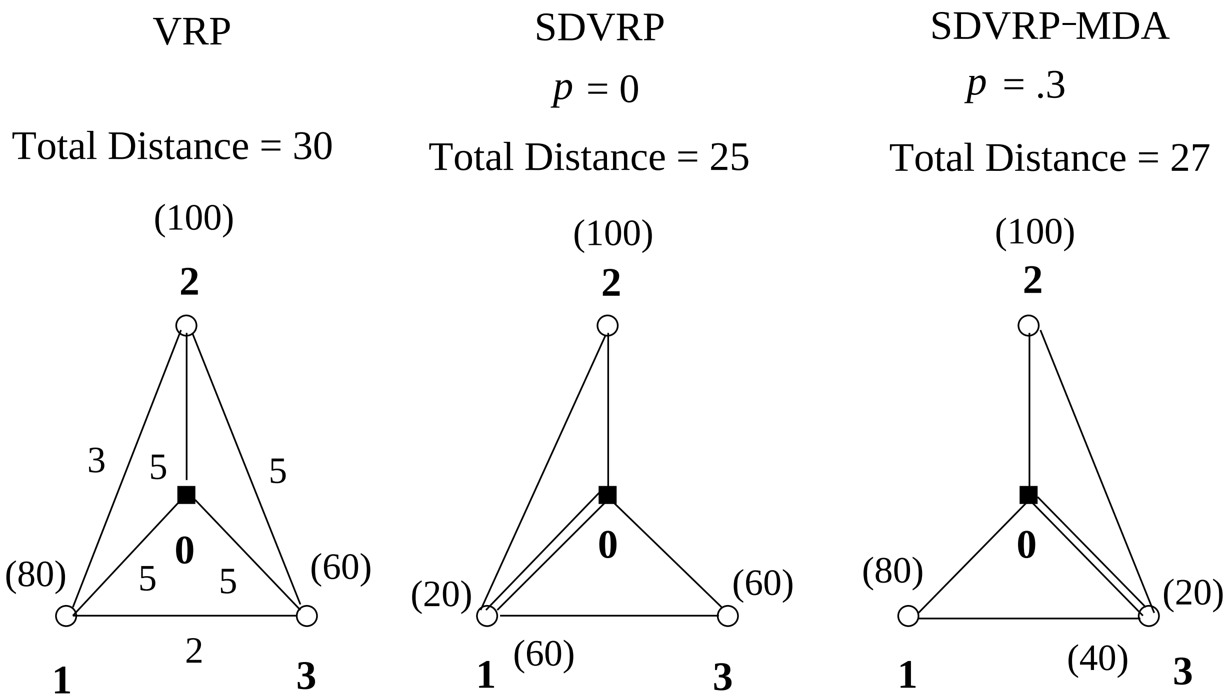
If we reallocate some demand from an endpoint, then the total reallocated amount is nonzero

If we reallocate any demand, then we must reallocate at least the minimum delivery amount

If we reallocate some, but not all, of a customer's demand, then the amount remaining must be at least the minimum delivery amount

EMIP-MDA Example

Initial solution is three direct routes. EMIP-MDA optimal objective function value is 3.



ERTR

Groër, Golden, Wasil (2008)

- VRP heuristic (no new splits)
- One-point move, two-point exchange, two-opt move
- Three-point move and OR-opt enhancement
- Route clean-up procedure

Test Problems

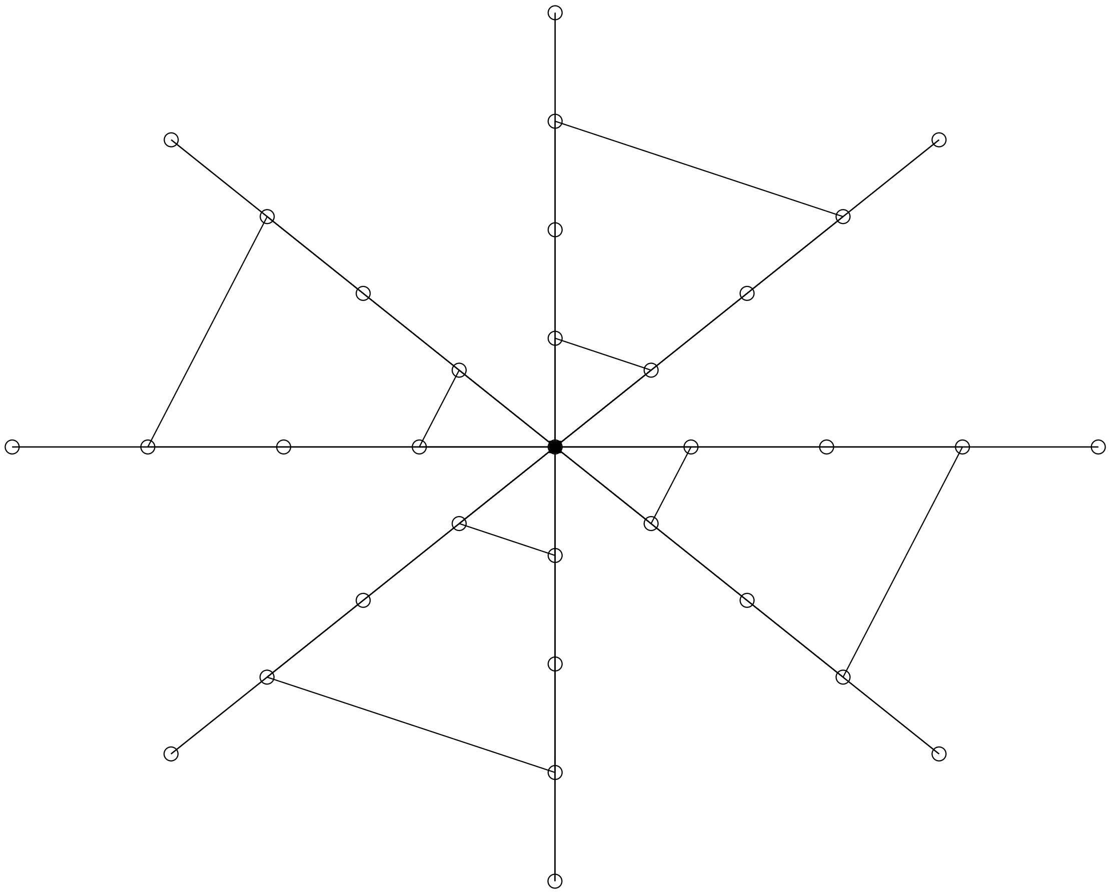
Because the SDVRP-MDA is a new variant of the SDVRP, we generated test problems with good solutions that can be estimated visually

We varied the customer demands in these problems in order to test EMIP-MDA + ERTR using different minimum delivery fractions

We generated 21 test problems, ranging from 8 customers to 288 customers, with four different minimum delivery fractions ($.1$, $.2$, $.3$, and $.4$)

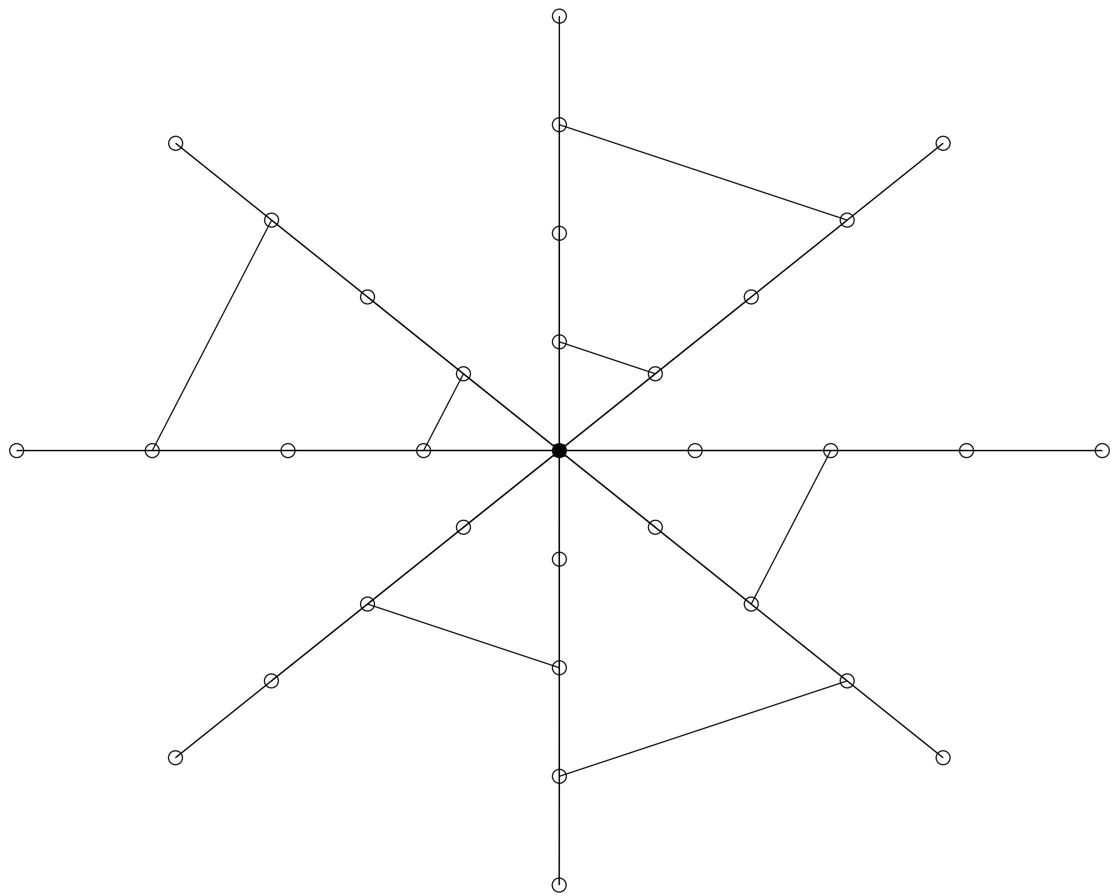
Computational Testing

A problem with 32 customers; the estimated solution cost is 831.21



Computational Testing

EMIP-MDA + ERTR generates a solution with a cost of 839.62; this is an increase of 1.01% from the estimated solution; $p = .2$



Computational Testing

N $=$ number of customers

Problem	N	$p = .4$	$p = .3$	$p = .2$	$p = .1$	Estimated Solution
SD1	8	**228.28**	**228.28**	**228.28**	**228.28**	**228.28**
SD2	16	**708.28**	714.40	**708.28**	734.79	**708.28**
SD3	16	**430.58**	**430.58**	**430.58**	**430.58**	**430.58**
SD4	24	**631.06**	**631.06**	640.02	**631.06**	**631.06**
SD5	32	**1390.57**	1408.12	**1390.57**	**1390.57**	**1390.57**
SD6	32	**831.21**	**831.21**	839.62	852.88	**831.21**
SD7	40	**3640.00**	3714.40	**3640.00**	**3640.00**	**3640.00**
SD8	48	5100.00	5200.00	**5068.28**	5094.79	**5068.28**
SD9	48	**2044.20**	2059.84	2071.05	2137.94	**2044.20**
SD10	64	2704.69	2749.11	2785.01	2772.91	**2684.85**
SD11	80	13363.90	13612.12	**13280.00**	**13280.00**	**13280.00**

Computational Testing

Problem	N	$p = .4$	$p = .3$	$p = .2$	$p = .1$	Estimated Solution
SD12	80	**7258.92**	7399.06	7279.97	7279.97	7280.00
SD13	96	10171.60	10367.06	**10110.57**	**10110.57**	**10110.57**
SD14	120	**10780.00**	11023.00	10819.29	10920.01	10920.00
SD15	144	15216.30	15271.77	15160.04	15223.42	**15151.10**
SD16	144	3382.16	3449.05	3497.97	3755.42	**3381.32**
SD17	160	26651.70	26665.76	**26559.91**	26559.93	26560.00
SD18	160	14357.80	14546.58	**14302.22**	14560.00	14380.30
SD19	192	20349.2	20559.21	20231.15	20212.84	**20191.20**
SD20	240	40022.70	40408.22	**39739.27**	39840.00	39840.00
SD21	288	11436.50	11491.67	11598.60	12445.52	**11271.10**

Computational Testing

We tested the EMIP-MDA + ERTR on three benchmark problem sets from the literature

1. Belenguer, Martinez, and Mota (2000)
2. Archetti, Speranza, and Hertz (2006)
3. Chen, Golden, Wasil (2007)

Customer demands fall into different ranges:
$[aQ, bQ], \quad 0 < a < b < 1, \quad Q =$ vehicle capacity

Computational Testing

Percent deterioration from $p = 0$ case

Demand Range	Number of Problems	$p = .4$	$p = .3$	$p = .2$	$p = .1$
$.01 - .3$	15	0.1	0.1	-0.1	0.0
$.1 - .5$	9	0.6	0.3	0.2	0.0
$.1 - .9$	8	2.0	1.1	0.4	0.4
$.3 - .7$	8	2.4	1.7	1.1	0.8
$.6 - .9$	28	8.0	4.6	3.7	0.7

Observations

When $p > 0$, we consider how solution cost is affected by increasing the value of p

Solution cost increases as p increases (what we expect)

When demand is small, EMIP-MDA + ERTR finds few splits, so cost varies little as p increases

When demand is large, the solution cost increases significantly with p because it becomes harder to fill vehicles with small splits

Conclusions

We defined a new problem: the split delivery vehicle routing problem with minimum delivery amounts (SDVRP-MDA)

We developed a two-stage heuristic for solving the SDVRP-MDA

Conclusions

In the first stage, we solved an endpoint mixed integer program with minimum delivery constraints (EMIP-MDA) that finds splits

In the second stage, we cleaned up the routes using an enhanced record-to-to record travel algorithm (ERTR)

Computational results showed that EMIP-MDA + ERTR is effective on a wide range of problems

Future Research Questions

How do we revise our procedures to solve the multi-depot SDVRP? Another layer of difficulty is added to the problem – which depots will service which customers?

How do we solve the multi-depot SDVRP with minimum delivery amounts?