Solving the Close-Enough Traveling Salesman Problem: New Algorithms and Computational Results

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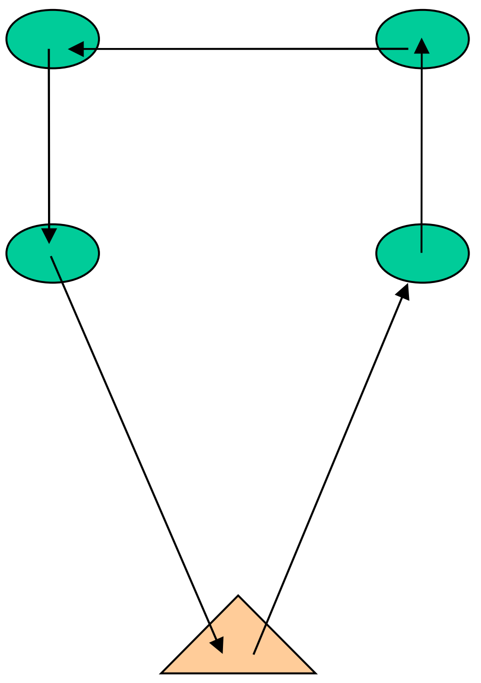
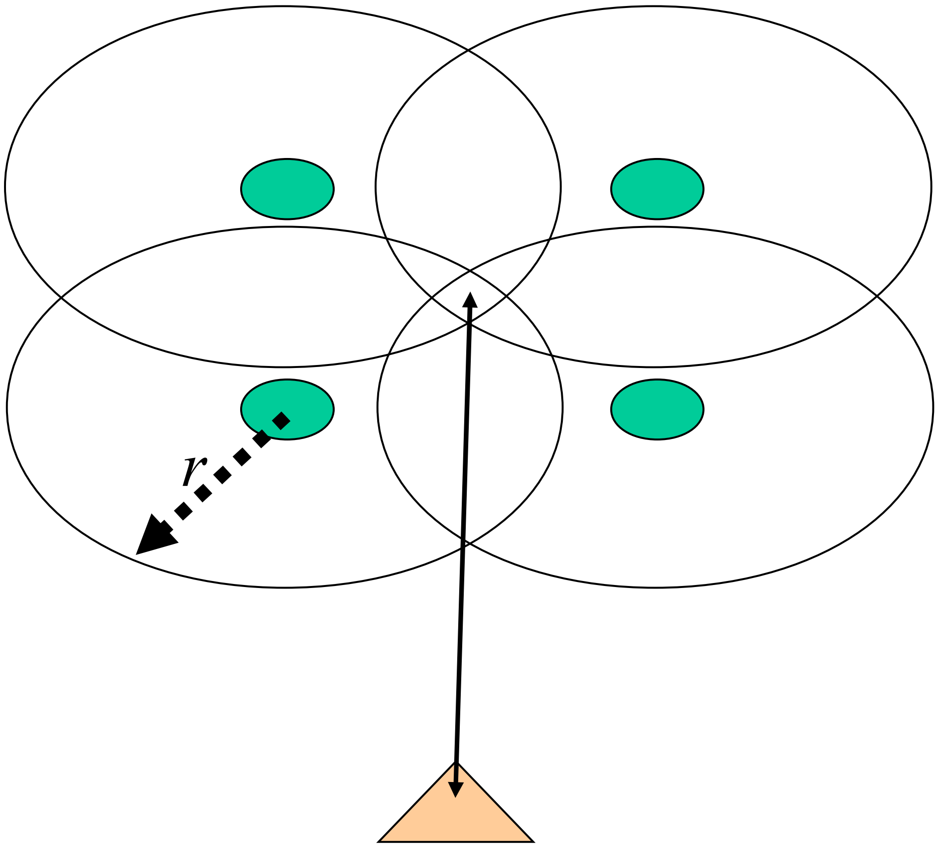
Outline

- Problem Illustration
- Review of Heuristic and Previous Results
- Observations
- Minimum Angle of Intersection
- Phase IV
- Results

CETSP



Classical TSP



CETSP

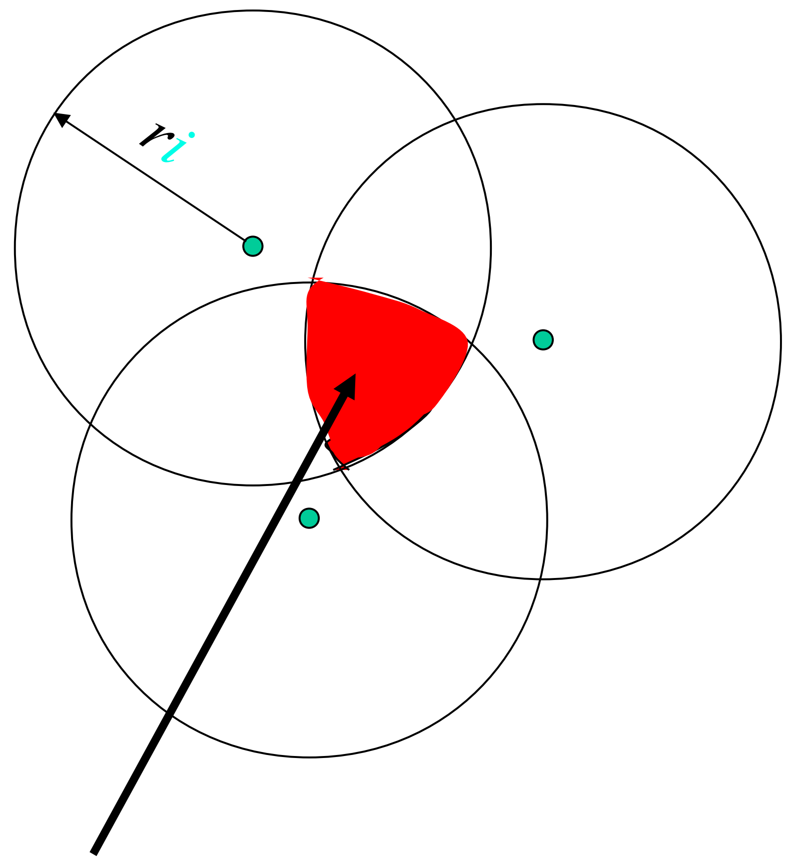
- CETSP can be used to model aerial reconnaissance planning

Steiner Zone

- Every node i is surrounded by a disc of radius r_i

- *Steiner zone of degree k:* SZ(*k*)

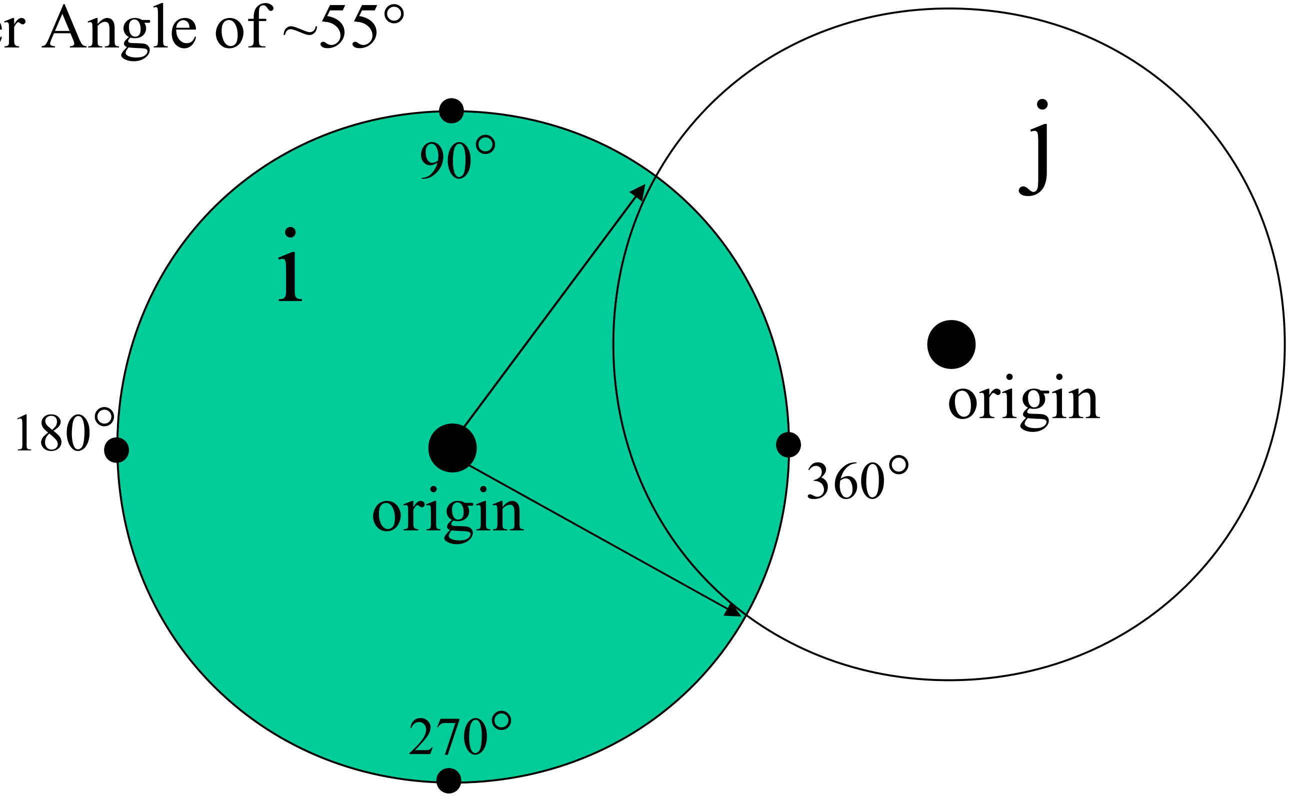
  -a region in which all points are close enough to all *k* member nodes



SZ of degree 3

Geometry

- SZij is characterized by:
  - Origin of i
  - Lower Angle of ~330°
  - Upper Angle of ~55°



Steiner Zone Heuristic

- Three key phases
  - Phase I: Graph reduction

    A graph of n nodes is transformed into a graph of m Steiner zones
  - Phase II: Solve the underlying TSP

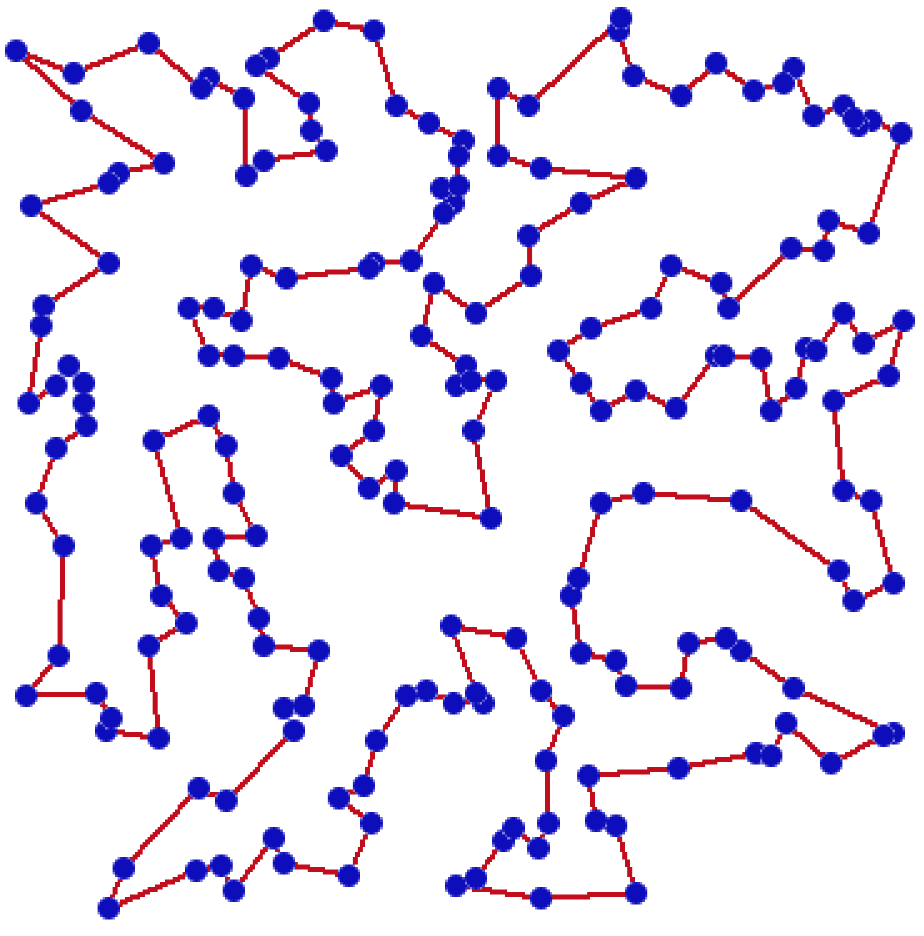
    A single point from each SZ is chosen to represent it

    A TSP is solved on this set of points
  - Phase III: Optimize the TSP tour with respect to the Steiner zones

    The point representing each SZ is moved around inside the zone to minimize tour distance

Before Phase 1

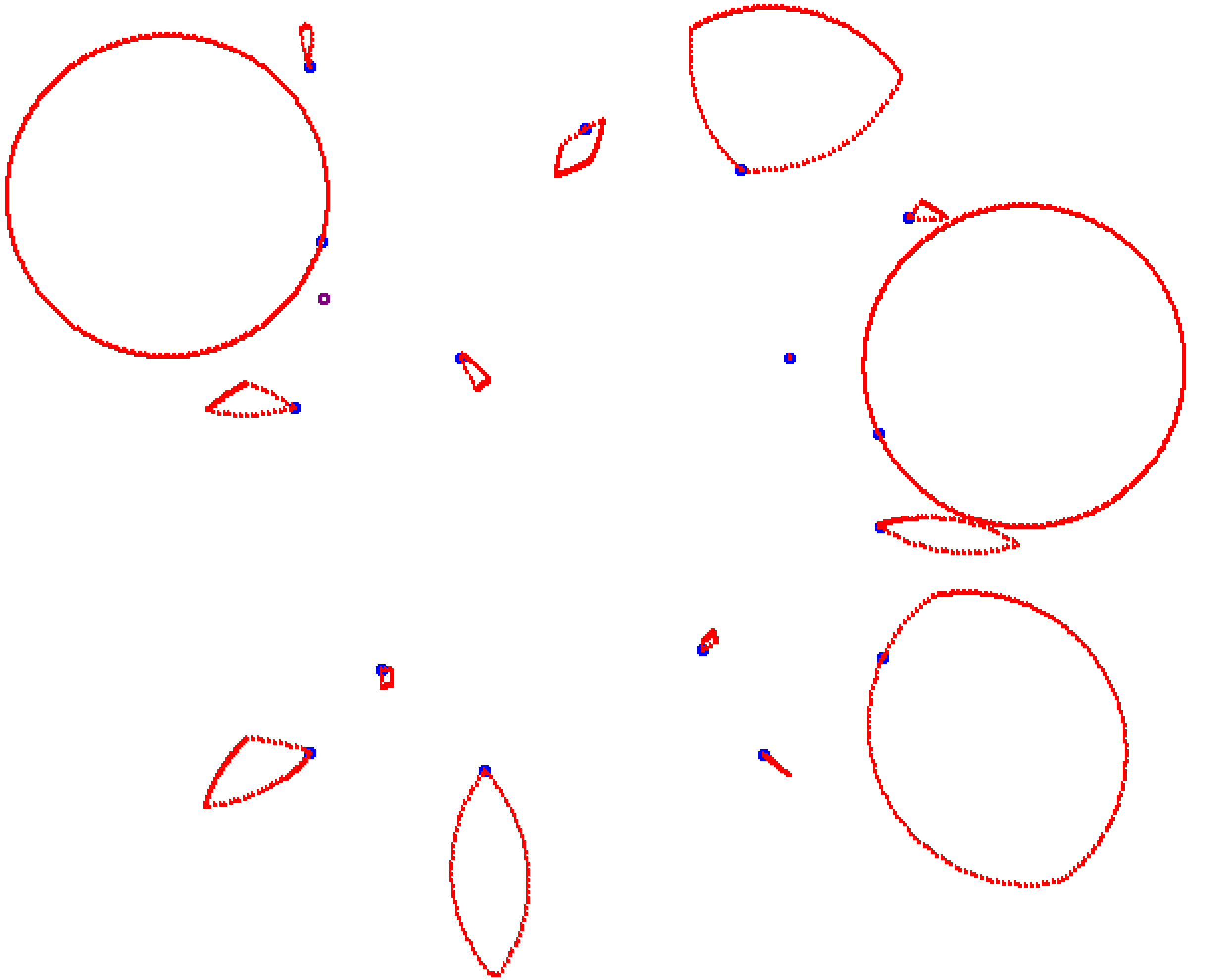
200-node randomly generated problem



Optimal TSP solution from Concorde: 1074.4

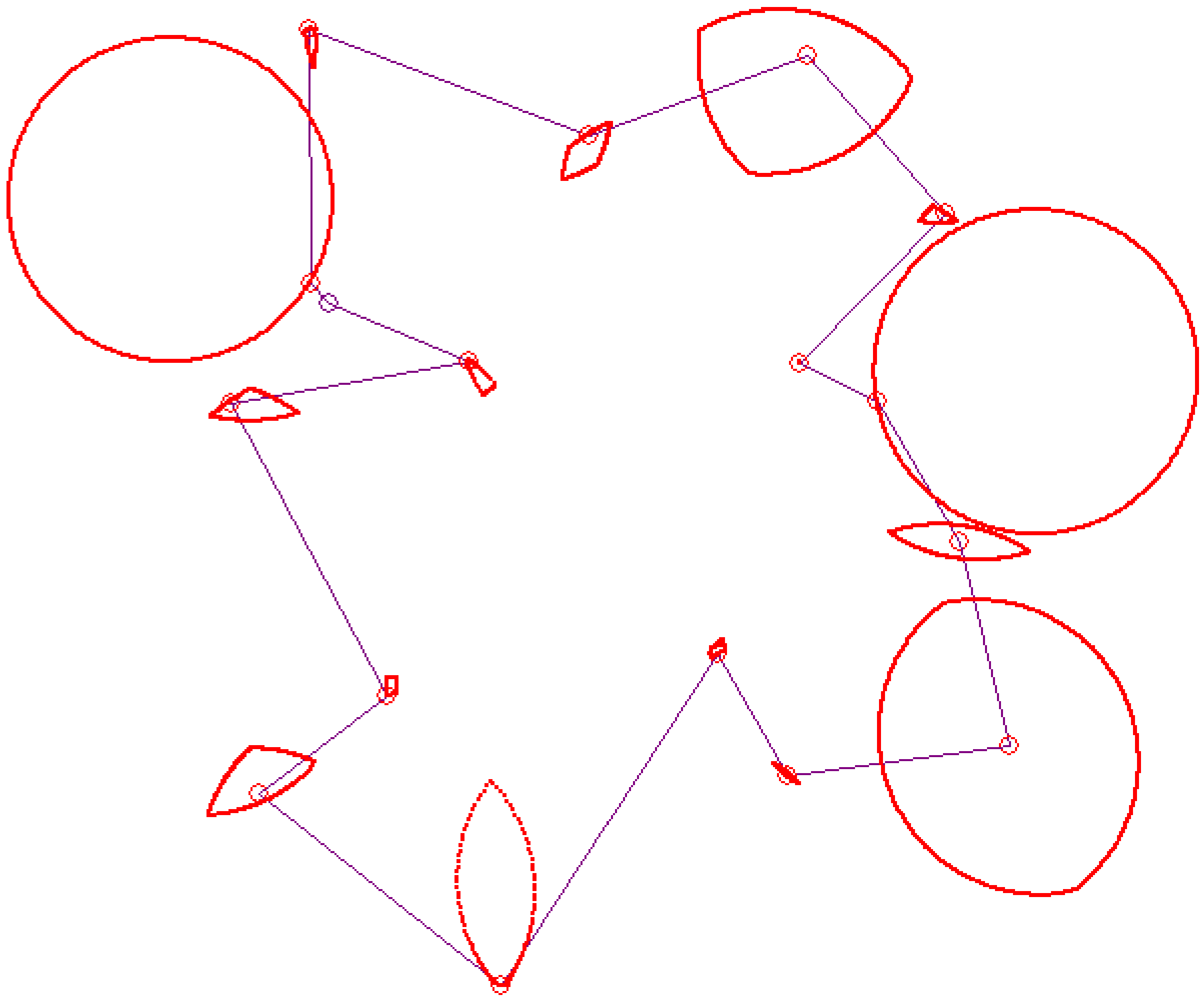
End of Phase 1

200 nodes reduced to 16 SZs



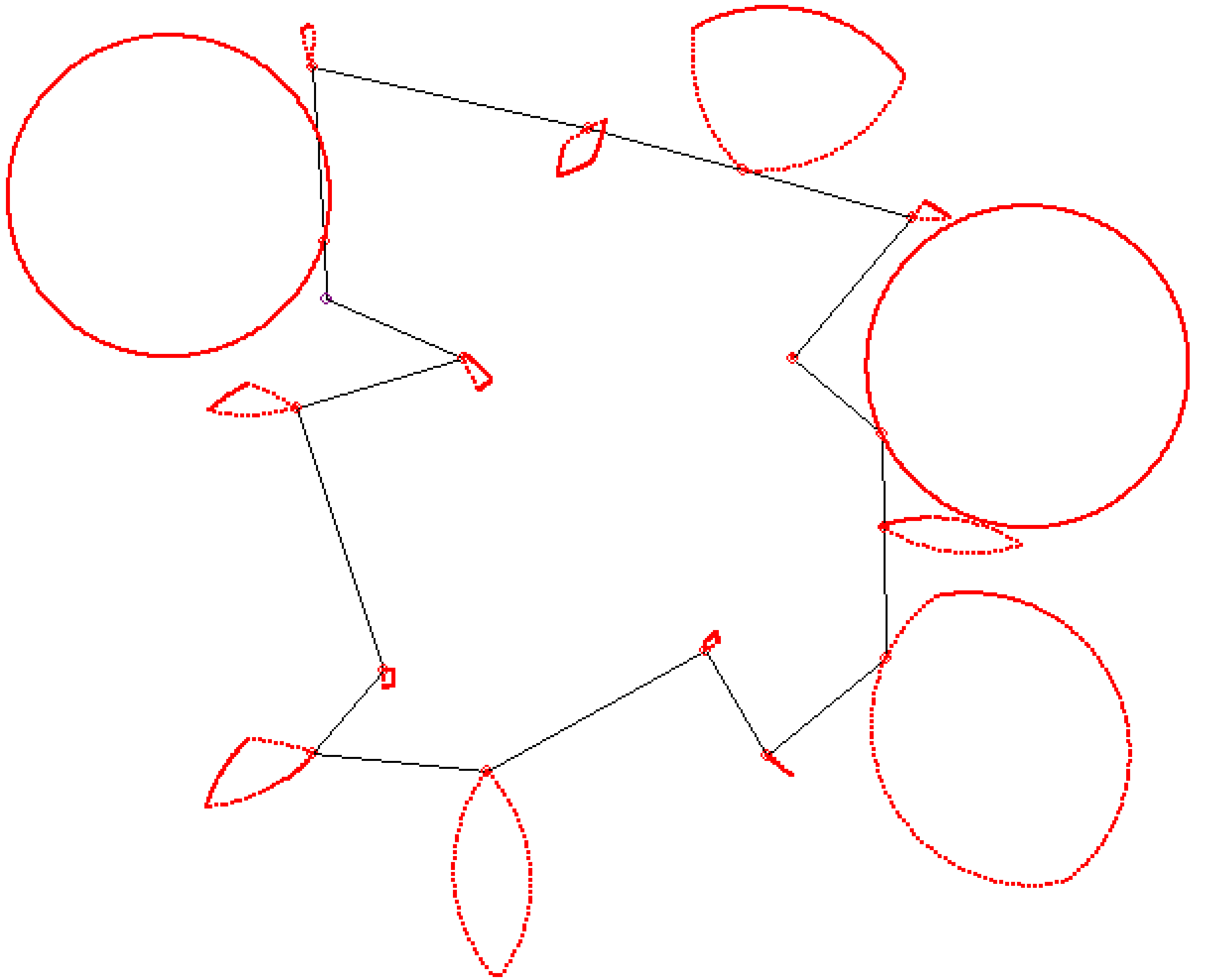
End of Phase 2

200-node problem with tour on representative points



Distance: 404.8

Final Solution



Distance: 312.3

Summary of Previous Results

- Steiner Zone Heuristic (SZH)
  - Very fast: few seconds to solve a problem
  - Consistent performer with very good solution quality

- State-of-the-art genetic algorithm (GA)
  - Silberholz and Golden (2007)
  - Very slow: hours, days, even months to solve a problem
  - On problems with low or medium amount of disk overlap, GA usually outperforms SZH by 2% to 10% in tour length
  - On problems with high amount of disk overlap, SZH outperforms GA by 50% to 300% in tour length

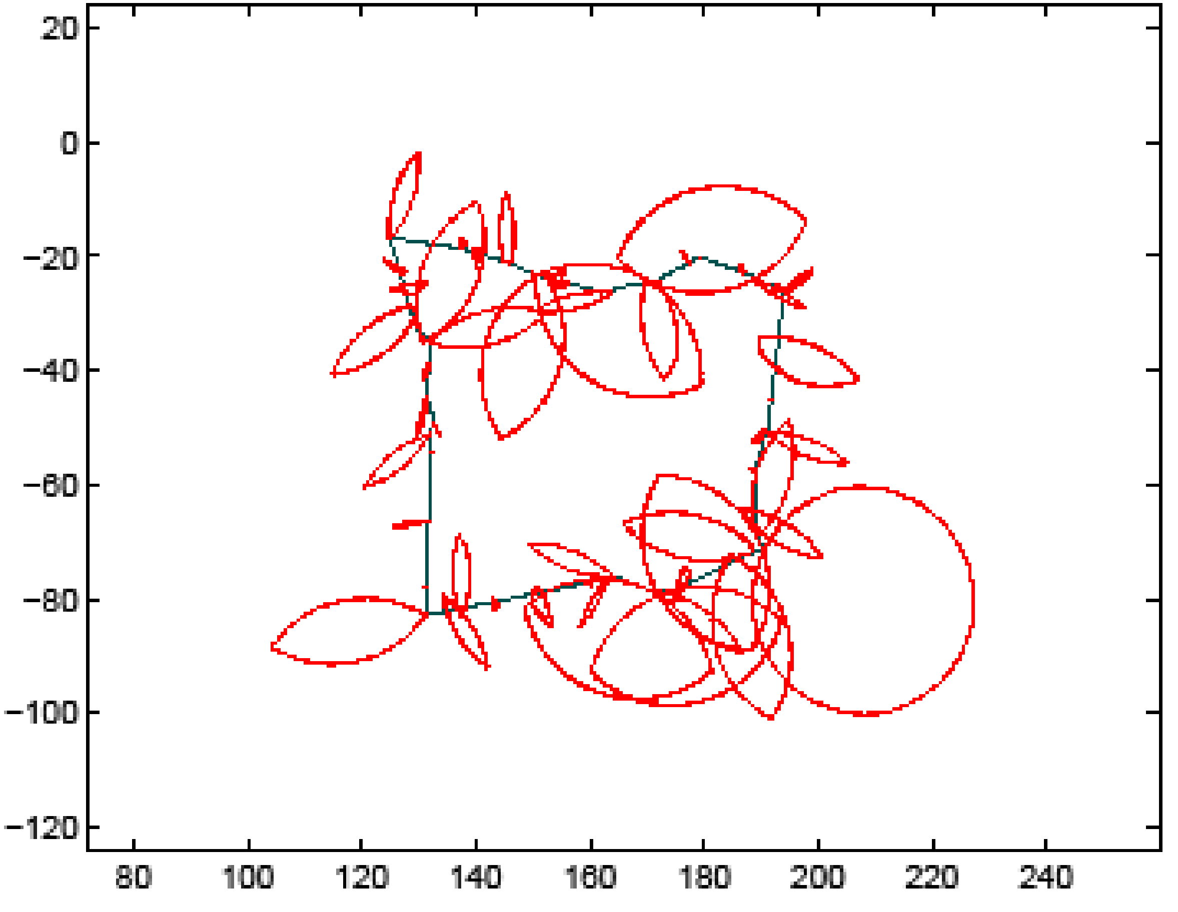
Motivation for Current Work

How do we alter SZH so that it finds tours for difficult problem classes that are comparable in quality to the tours produced by GA without increasing computation time significantly?

Observations

200-node problem: Best GA result is 249 with 52 SZs

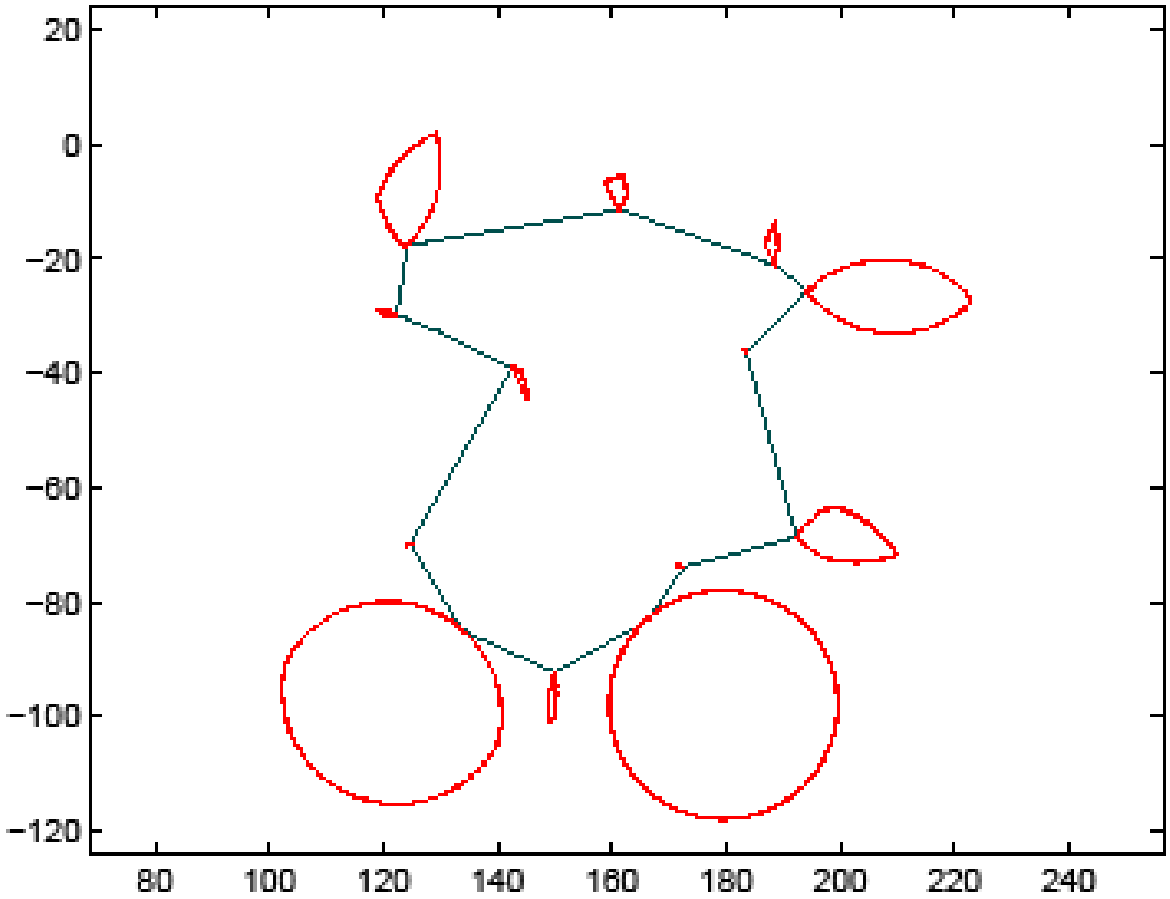
Tour is almost a square



Observations

200-node problem: Best SZ result is 277 with 13 SZs

Tour has sharp twists and turns



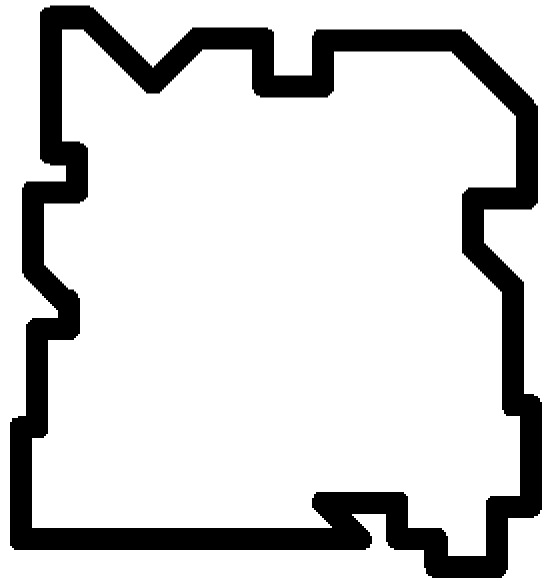
Turn Angle

- Given consecutive edges e_{ij} and e_{jk} in a tour, the turn angle for e_{jk} is the angle of deflection (modulo 90º) between e_{jk} and e_{ik}
  - With a turn angle of 90º, e_{ij} and e_{jk} form a right-angle
  - With a turn angle of 0º, e_{ij} and e_{jk} form a straight line

- If the sum of all edge turn angles in a tour is small, then the tour has a simple geometric shape
  - More pleasing to the eye
  - Likely to be shorter

- If the sum of all edge turn angles in a tour is large, then the tour has a more complex geometric shape
  - Likely to be longer

Good Tour or Bad Tour?



- By our current definition, a tour containing many sharp turns would have a very large sum of turn angles
- However, this tour is approximately a square – a simple shape ideal for minimum total distance
- Thus, the current definition of turn angle is inadequate

Normalized turn angle

- Divide e_{ij}'s turn angle by the percentage of route distance contributed by e_{ij}
- Each edge in a tour contributes an amount of turn angle proportionate to its length relative to the total tour length

Observations

- Typical tours produced by GA
  - Many Steiner zones of low degree
  - Simple geometric shape (low sum of normalized turn angles)

- Typical tours produced by SZH
  - Few Steiner zones, mostly very high degree
    - Result of the Phase I reduction from nodes to SZs
  - Tour shape is highly constrained by the location of the Steiner zones
    - Tour has sharp twists and turns

Improving SZH

- GA produces better tours with a simple shape using Steiner zones of much lower degree. We try to guide SZH in the same way.

  - Two approaches to improving SZH

    1. Force SZH to construct Steiner zones of lower degree

    1. Post-process a tour produced by SZH to reduce the degree of the Steiner zones

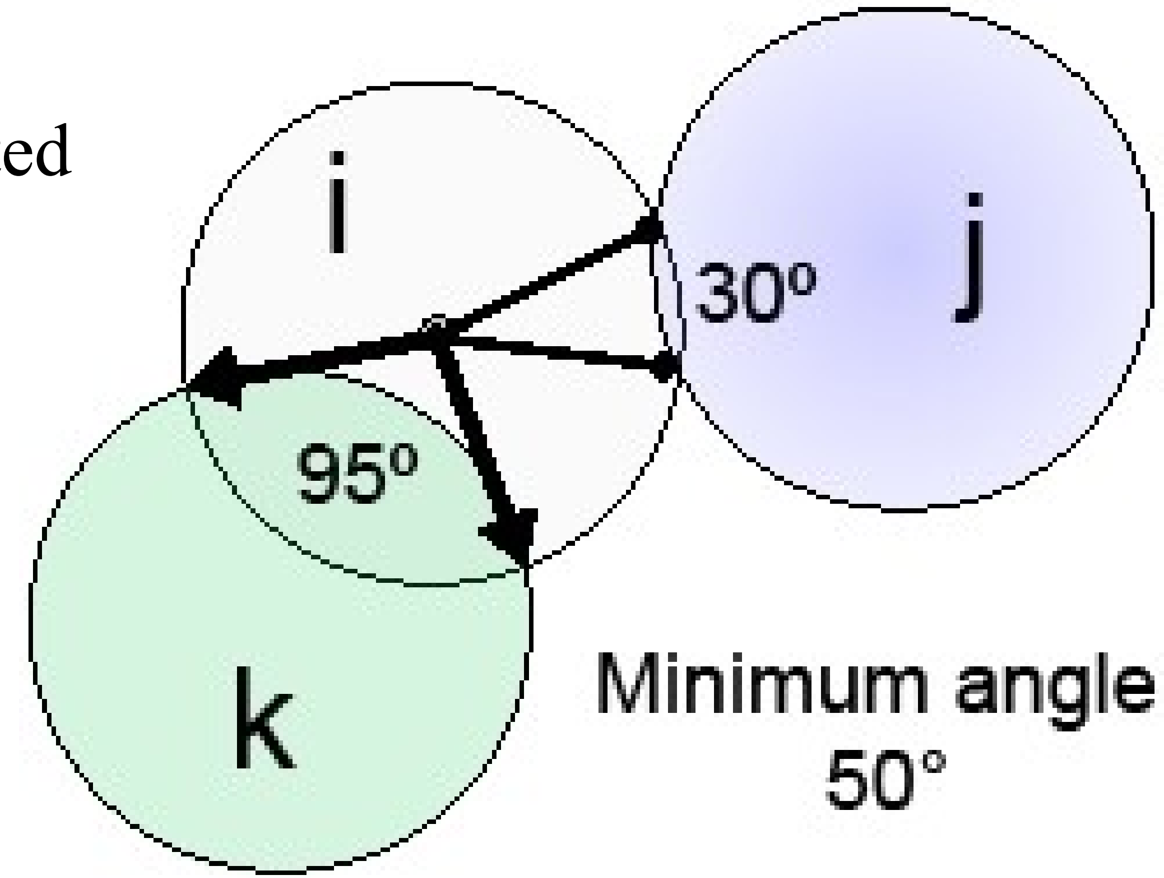
1. Restrict SZ Degree

- Experiments showed that placing an upper bound on the degree of Steiner zones did not work well

- We placed a lower bound on the total angle of intersection for a Steiner zone
  - A larger lower bound results in SZs that have a larger area
  - More flexibility on where to locate the point in each SZ
  - We call this the minimum angle of intersection heuristic (MAH)

Minimum Angle of Intersection

- SZij is not created
  - Total angle of intersection is too small
  - Much less flexibility for point location
- SZik is created



2. Phase IV

- Post-process a tour generated by SZH

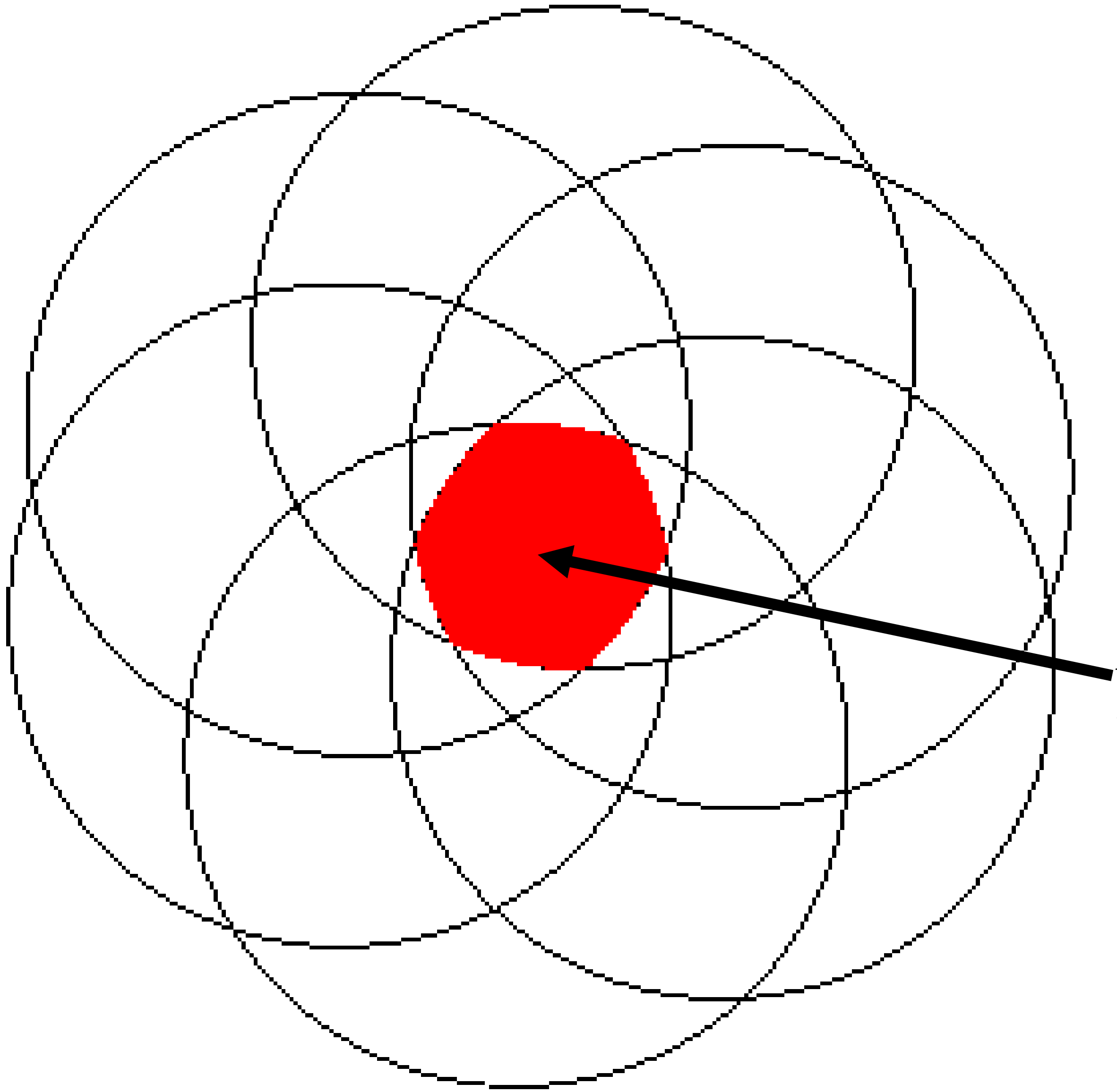
1. Sum the turn angles on the Steiner zones and remove the largest 10%

1. Systematically break each of the SZs in this 10% group into SZs with degree one or two
    - Give each SZ a unique new representative point

1. Calculate the new TSP tour and reoptimize with respect to the Steiner zones (Phases II and III)

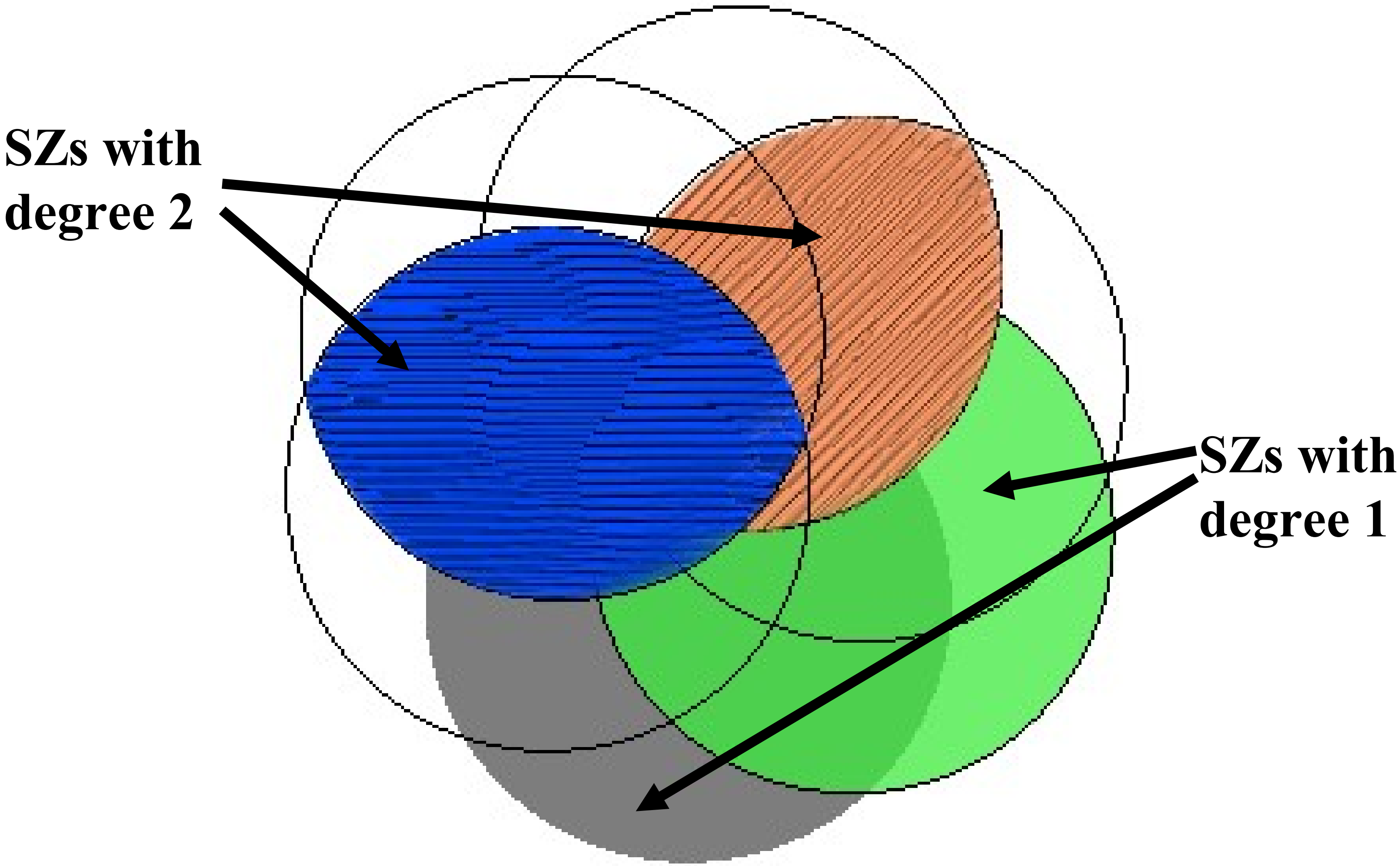
1. Repeat until the solution has only Steiner zones with degree one or two

SZ(6) before Phase IV



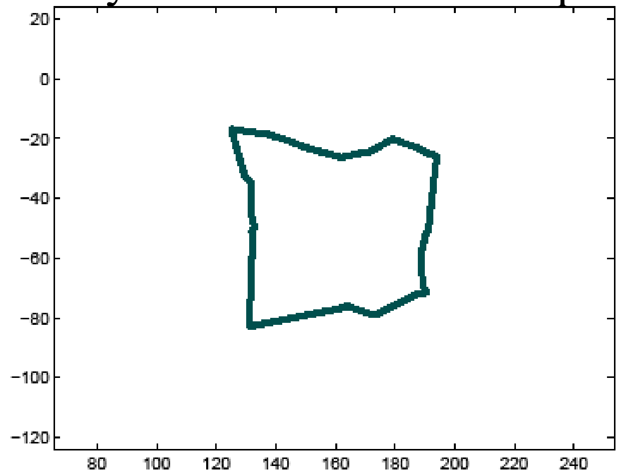
All 6 circles
intersect here

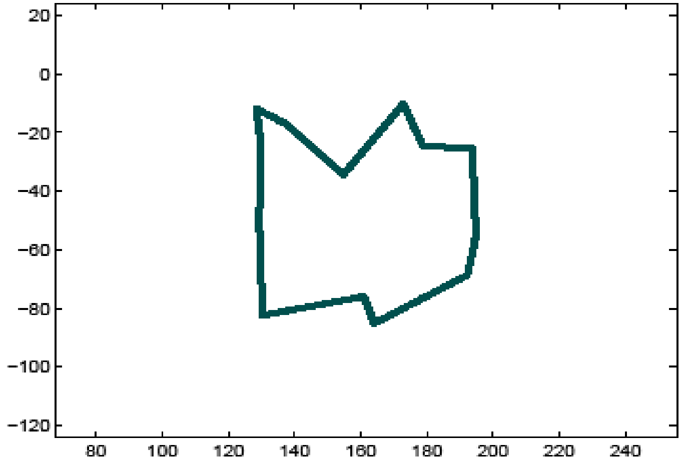
After Phase IV, 4 SZs



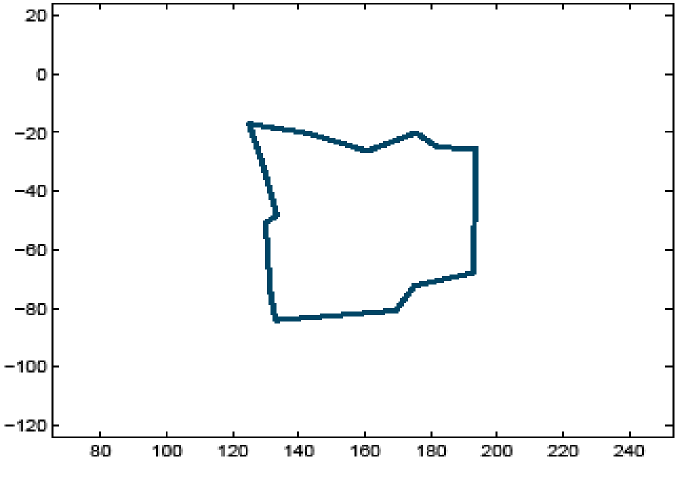
Three Solutions

Best solution found by GA for the 200-node problem; distance is 249





An SZH solution; distance is about 284



After applying Phase IV, the SZH distance is now 250

Results

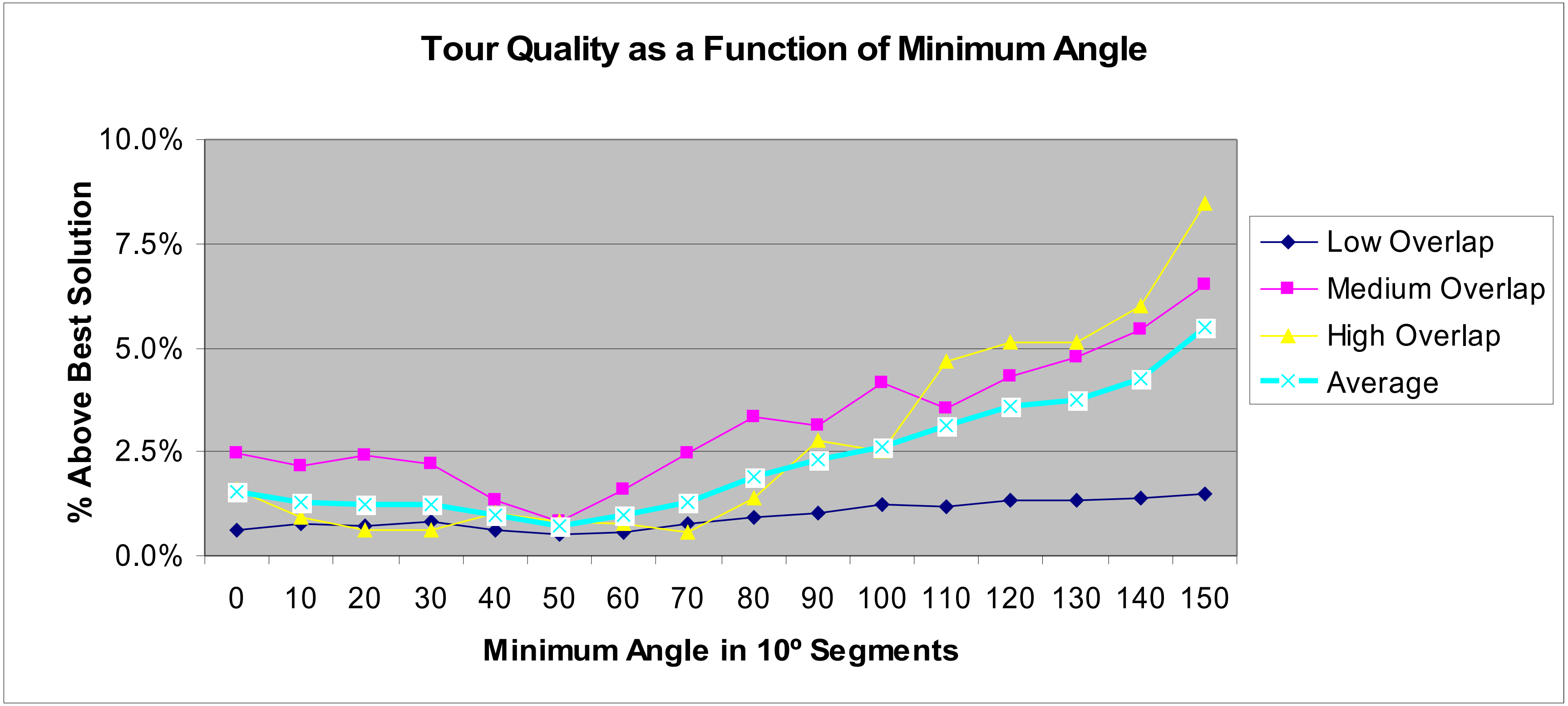
- Machine and test problems
  - 3.2 GHz, 3 GB RAM
  - Seven test problems from TSPLIB
  - Each problem was run in three different scenarios – low, medium, and high overlap
  - Each problem was solved with both Euclidean and Manhattan norms
- In previous work, almost all best-known solutions for the low and medium overlap problems were found by GA
- All best-known solutions for high overlap problems were found by SZH

MAH Results

- We solved each problem for minimum angle, k, where $k = 1,\ldots,360$
  - For the low and medium overlap problems, solutions averaged 3% above the best-known solutions
    - Previously, we were 4% above for low overlap and 7% above for medium overlap
  - Average solution time is now 650 CPU seconds
    - Previously, it was 0.75 seconds
- Could easily use k processors in parallel to test k different minimum angles
  - We divided 360º into groups of 10º ($k = 10$)
  - Tried to determine which 10º group produced the shortest tours on average over all problems

MAH Results

- Range of 50-59º produced the best solutions on average



Phase IV Results

- For the two difficult classes of problems, we now average 2% above the best-known solutions
- Over all problems, average 1% above the best-known solutions
- 15 - 25 times slower than standard Steiner zone heuristic (Phases 1 through 3 only)
  - Average solution time is now 21 CPU seconds
  - Much faster than the GA
- Try combining MAH and Phase IV to produce better solutions

MAH + Phase IV Results

- For the most difficult class of problems, we now average 1% above the best-known solutions

- Overall, we average 0.44% above the best-known solutions

- Average of 11,000 CPU seconds per problem
  - Much faster than GA (1.1 million CPU second average)
  - Impractical using serial computing

MAH + Phase IV Results

- Naïve parallel implementation makes this method fast and accurate
  - Several processors could run SZH + Phase IV for many different minimum angles of intersection
  - Running time would be same as running SZH + Phase IV serially for only one minimum angle (average of 21 CPU seconds)

- We divided 360º into groups of 10º ($k = 10$)
  - Each class of problems was found to have a different minimum angle set that performed best on average.

Low overlap:	0º - 9º
Medium overlap:	20º - 29º
High overlap:	50º - 59º

Conclusions

- When computation speed is paramount, MAH reduces the performance (accuracy) gap with respect to GA with only a small increase in time
- Phase IV is still very fast (relative to GA) and almost completely closes the performance gap
- MAH + Phase IV is an accurate but slow heuristic
- MAH and MAH + Phase IV would achieve linear speedup in a parallel environment
  - *k* processors for *k* different minimum angles of intersection