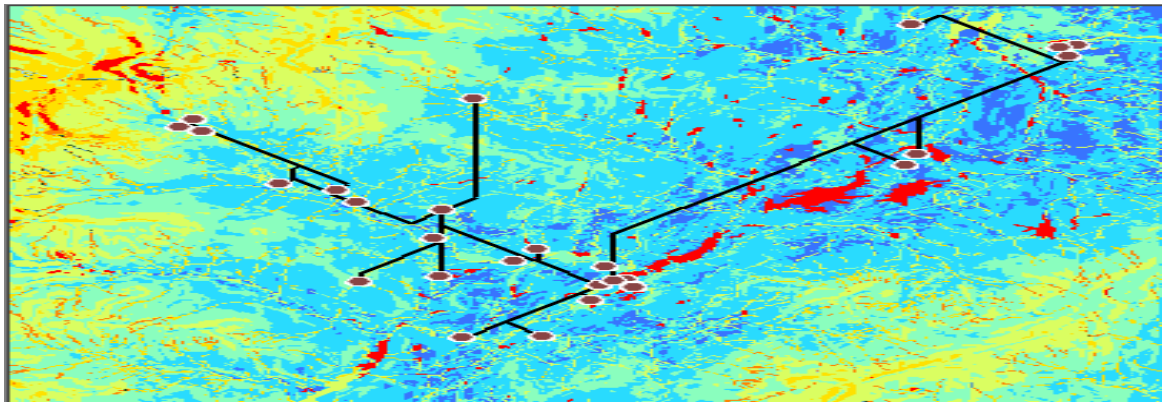
Solving the Euclidean Non-Uniform Steiner Tree Problem Using a Genetic Algorithm



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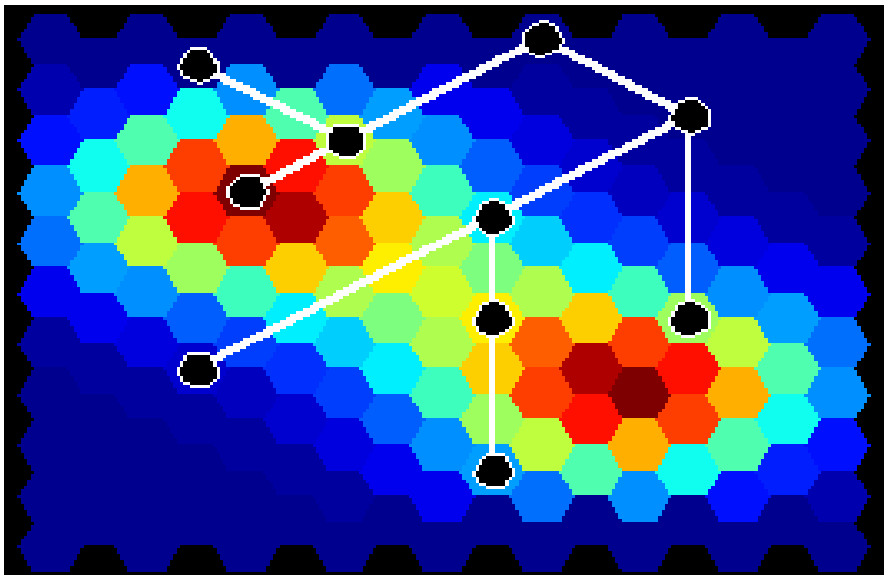
Background:
The Steiner Tree Problem (STP)

- **INPUT**: weighted graph, set of terminal nodes, additional nodes
- **OUTPUT**: minimal cost tree connecting all terminal nodes, where cost = sum of edge costs
- Additional nodes ("Steiner nodes") may be used to help minimize the tree cost

Euclidean Non-Uniform Problem

Edge cost depends on length and location

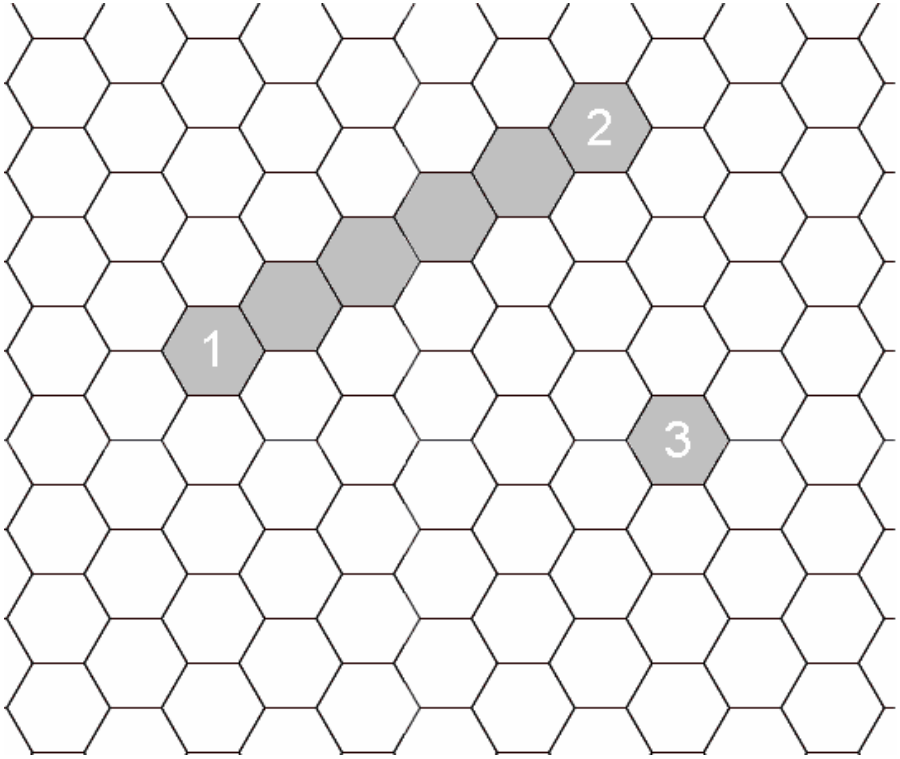


Applications: laying cable networks, routing transmission lines, design of communication networks, printed circuits

Problem Formulation

- Tile the plane with regular hexagons (helpful since distances between centers of adjacent cells are equal)
- Each hex cell is assigned a cost
- At most one node (terminal or Steiner) per cell
- Edge = straight line of cells

Hexagonal Grid



1 & 2 can be connected directly
1 & 3 and 2 & 3 can not

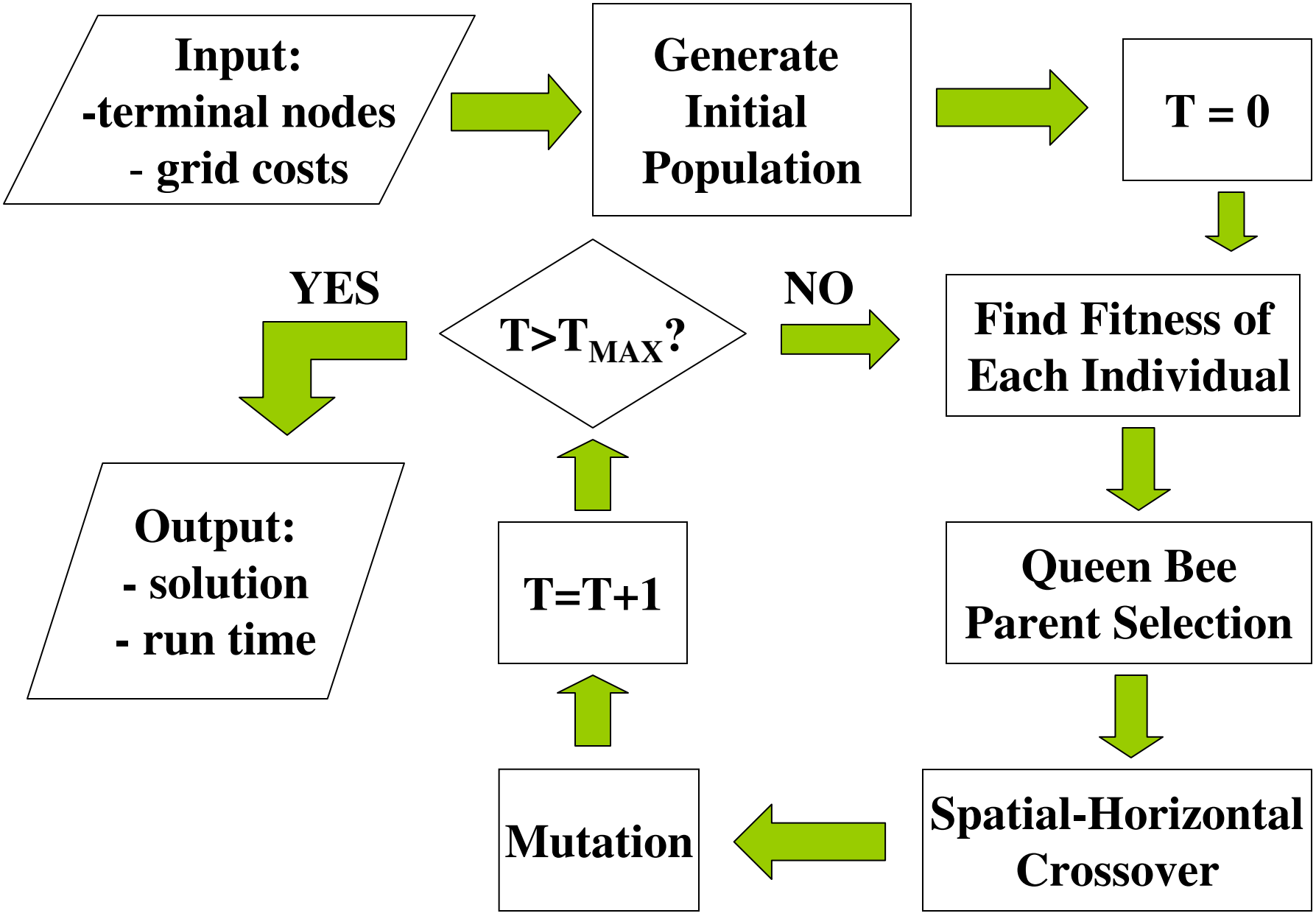
Determining Cost

- Tree cost = sum of edge costs of tree edges
- Edge cost = sum of costs of cells connecting the 2 nodes + .5 * (each node's cell cost)
- Could also charge extra for each Steiner node (the node-weighted Steiner Tree Problem)

Genetic Algorithms (GAs)

- Based on biological evolution and natural selection
- Use a **population** of solutions (list of Steiner node locations in our case)
- Generate new solutions from existing ones using a **crossover mating** procedure
- Pass the **fittest** solutions (least cost in our case) on to the next generation and repeat

Flow of Our GA



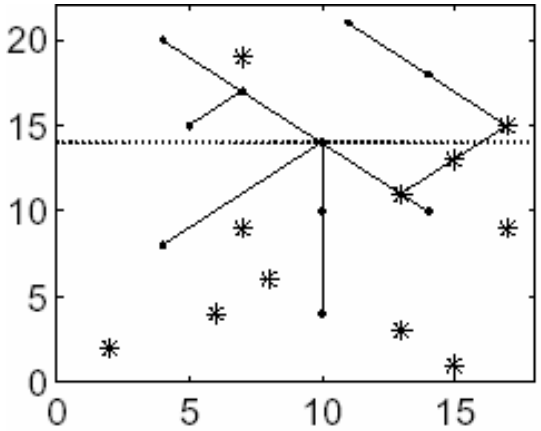
Source: Wikipedia

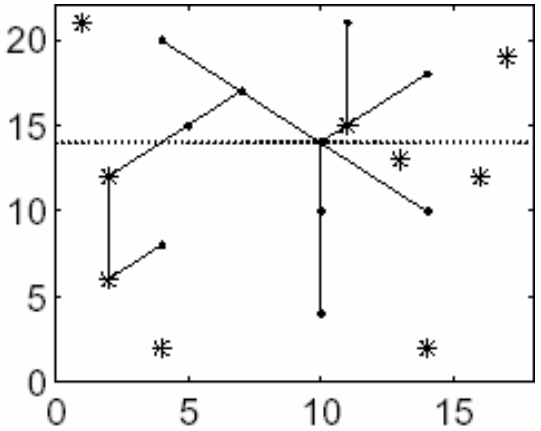
Queen Bee Selection

- Mate the fittest individual (**the queen bee**) with each other individual to produce two offspring per mating
- For population size of 40, this creates 78 offspring
- Choose the 40 fittest out of the offspring and parents to survive to the next generation

Spatial-Horizontal Crossover

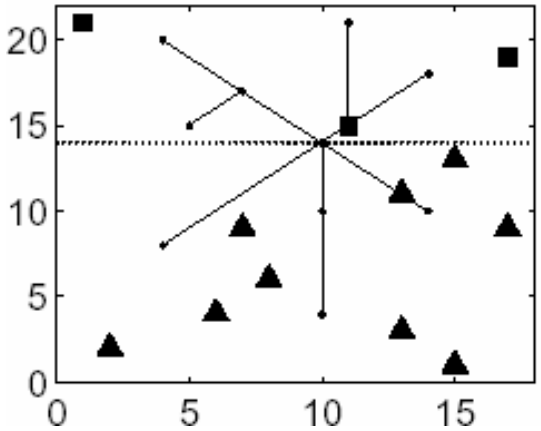
Parent 1:
Fitness = 24

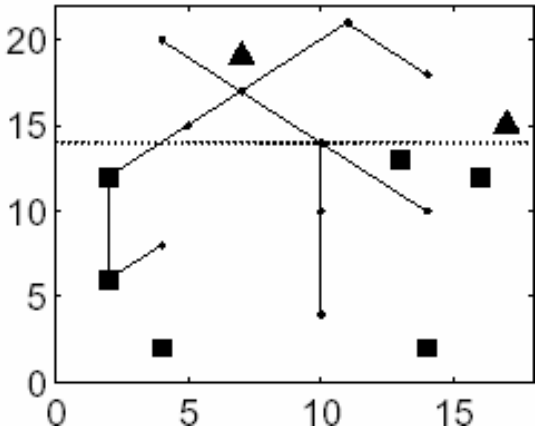




Parent 2:
Fitness = 23

Offspring 1:
Fitness = 21





Offspring 2:
Fitness = 23

Small Problem Results

Problem	Optimal	GA best	% Gap	GA mean	% Gap	GA stdev
1	11.138	11.138	0.0	11.155	0.15	0.01
2	10.001	10.001	0.0	10.012	0.12	0.03
3	4.806	4.806	0.0	4.806	0.0	0.0
4	4.158	4.158	0.0	4.158	0.0	0.0
5	5.605	5.605	0.0	5.605	0.0	0.0

The GA was run 10 times on each problem

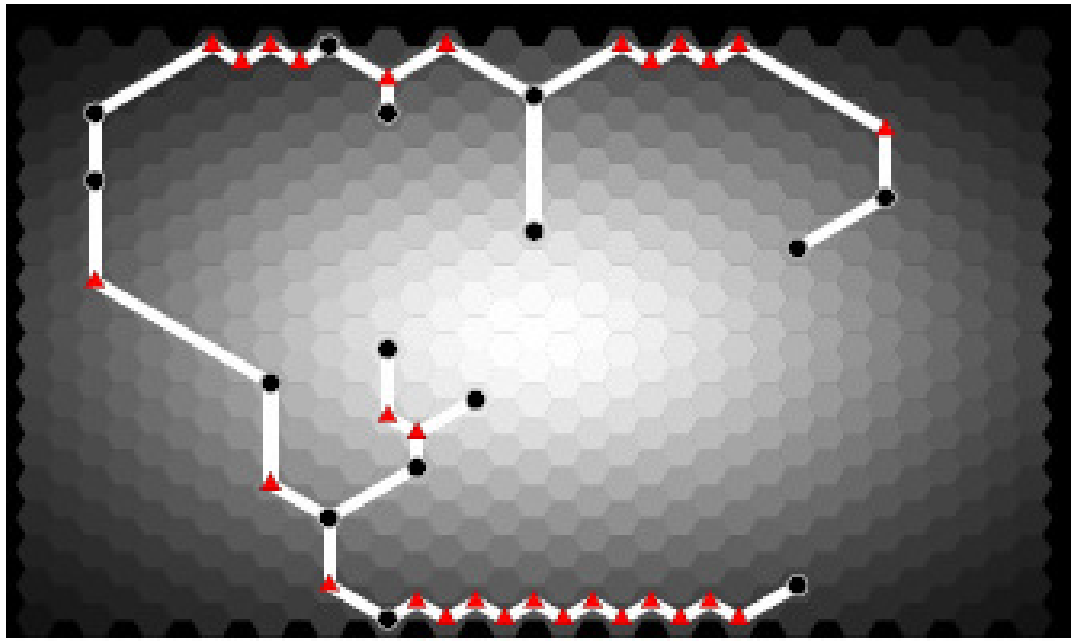
Medium Problem Results

Problem	Optimal	GA best	% Gap	GA mean	% Gap	GA stdev
1	26.606	26.653	0.18	27.104	1.87	0.21
2	31.310	31.405	0.30	31.513	0.65	0.07
3	31.965	32.324	1.12	32.574	1.91	0.19
4	32.744	32.744	0.00	32.752	0.02	0.02
5	19.435	19.494	0.31	19.730	1.52	0.26

The GA was run 10 times on each problem

Computational Notes

- **Grid size**: 21 by 17 (small) or 35 by 35 (medium)
- **Terminal nodes**: 7 or 10 (small), 15 (medium)
- Coded in MATLAB, run on a 3 GHz machine with 3 GB of RAM
- **GA run times**: 1 minute (small), 15 minutes (medium)
- **Optimal algorithm run times:** 1.5 min (7- node small), 20 min. (10- node small); 12 days (medium)

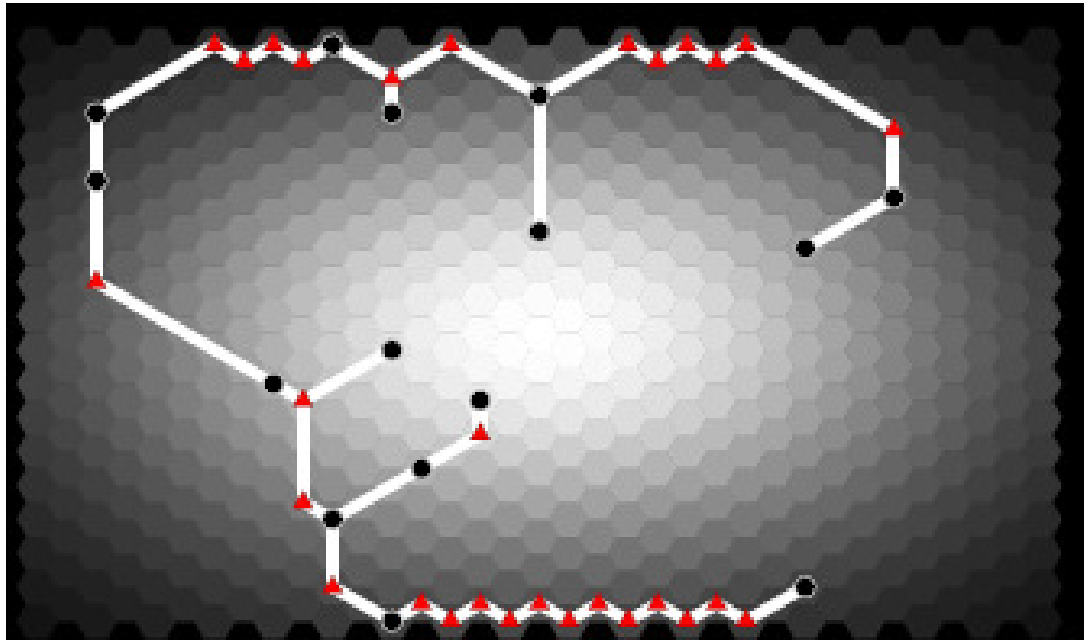


Optimal Solution

Cost = 31.310

● Terminal nodes

▲ Steiner nodes



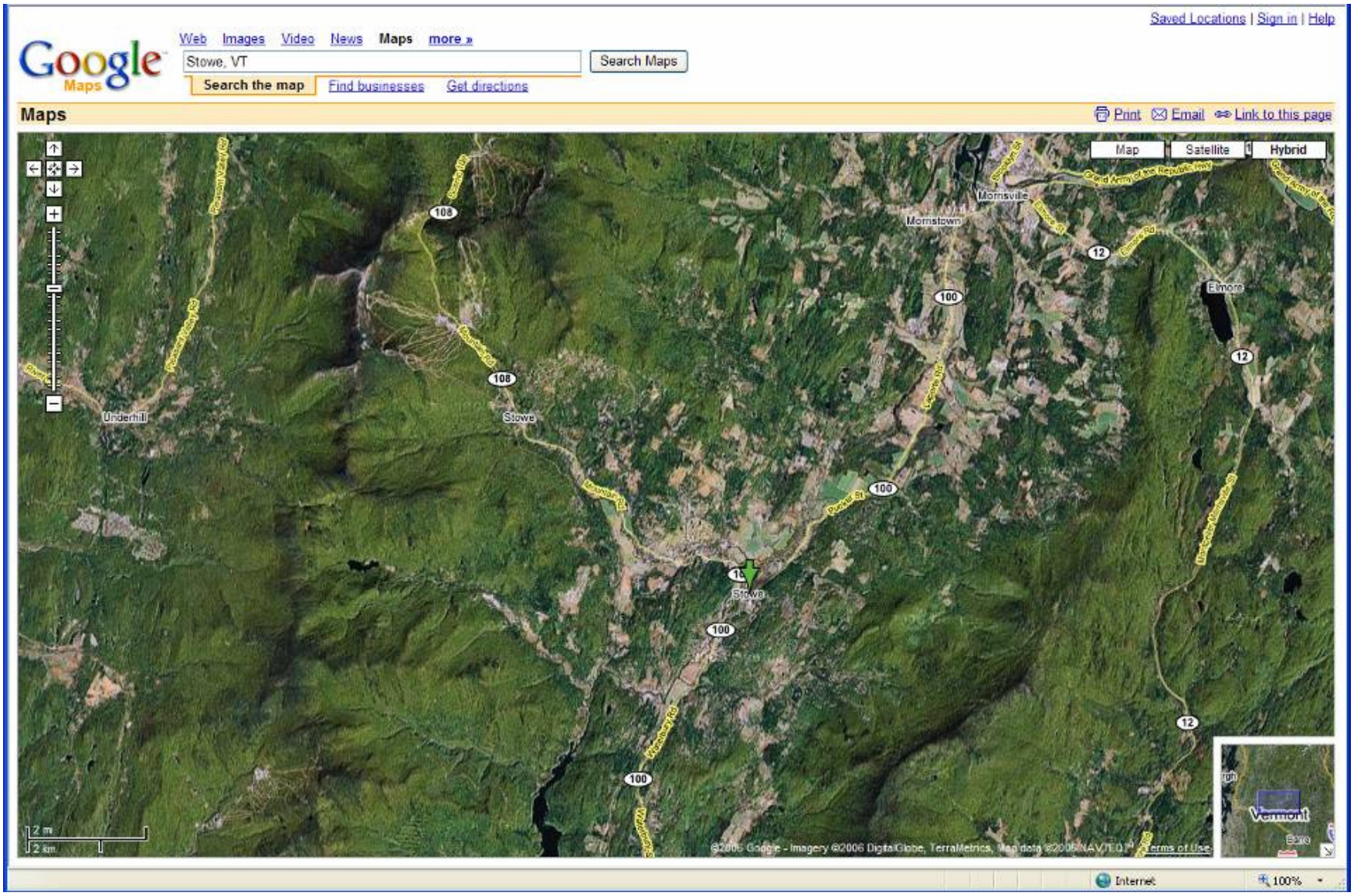
Genetic Algorithm Solution

Cost = 31.405

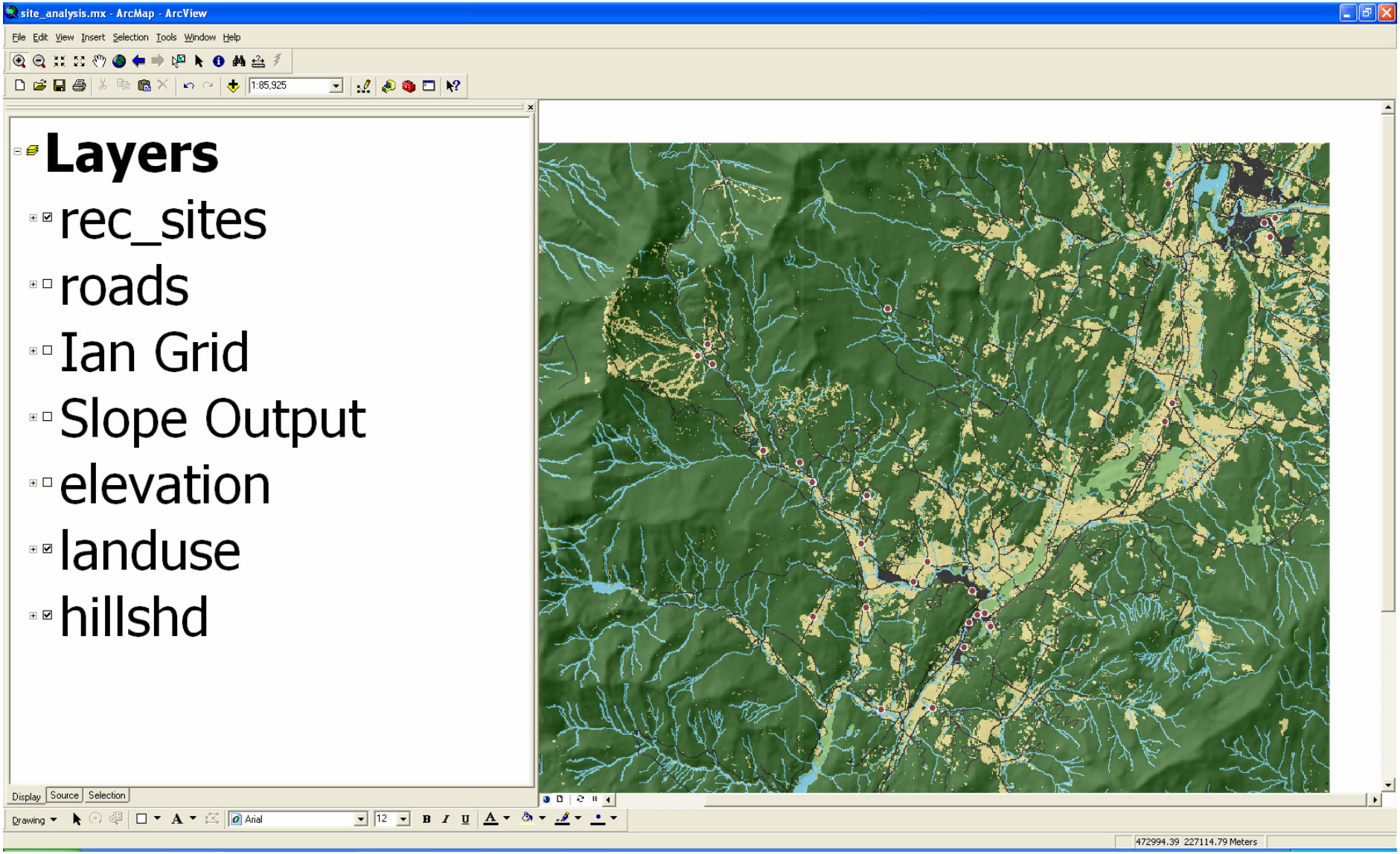
GIS

- **GIS** = Geographic Information Systems
- GIS is a kind of computerized geography
- Combines geographic data, hardware and software (we'll use **ArcMap** by ESRI)
- Capture, manage, map, analyze, display layers of geographic information
- See http://www.gis.com

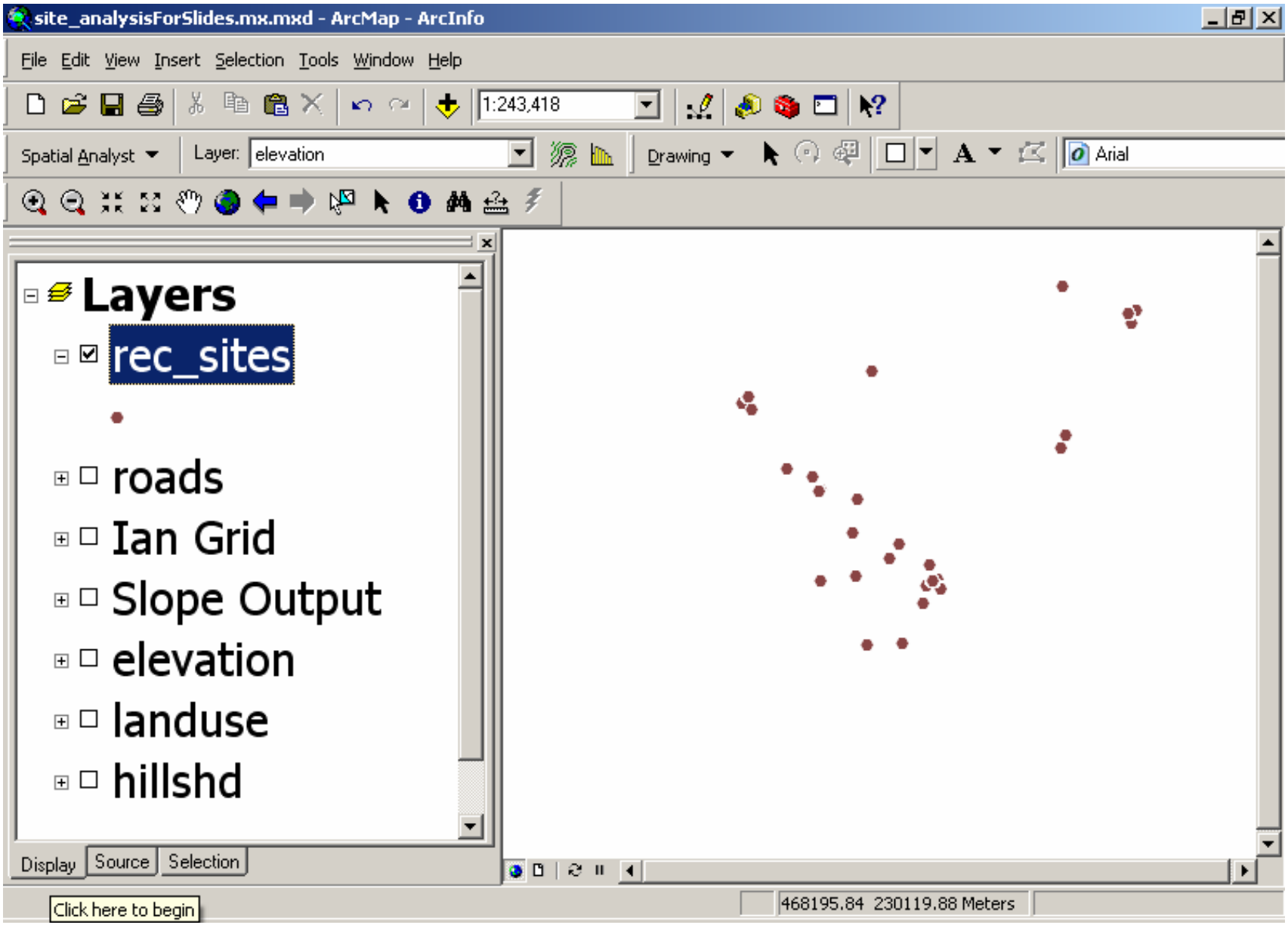
Application: Link up Recreational Sites in Stowe, Vermont



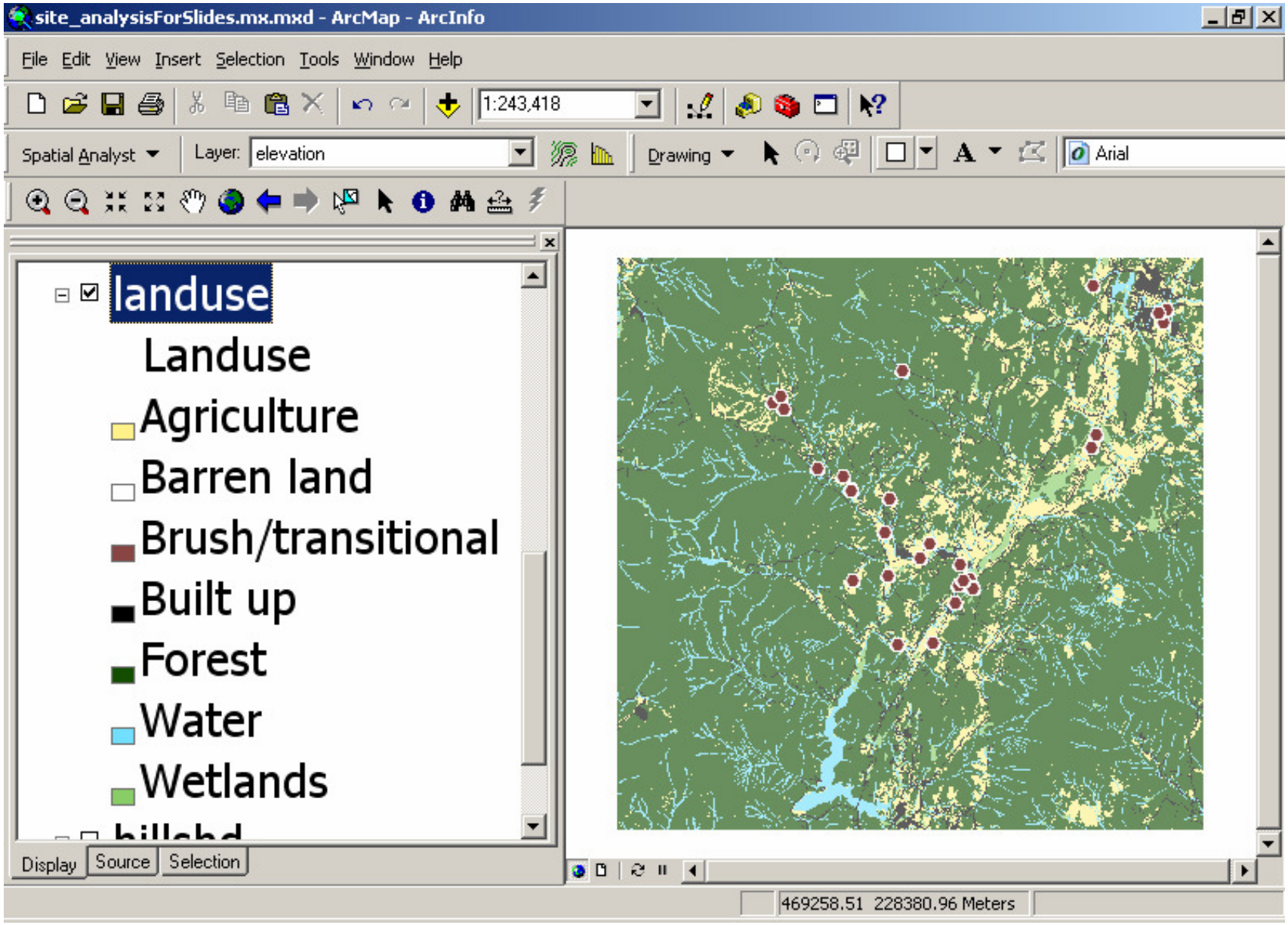
View Layers of GIS Data for Stowe, VT in ArcMap



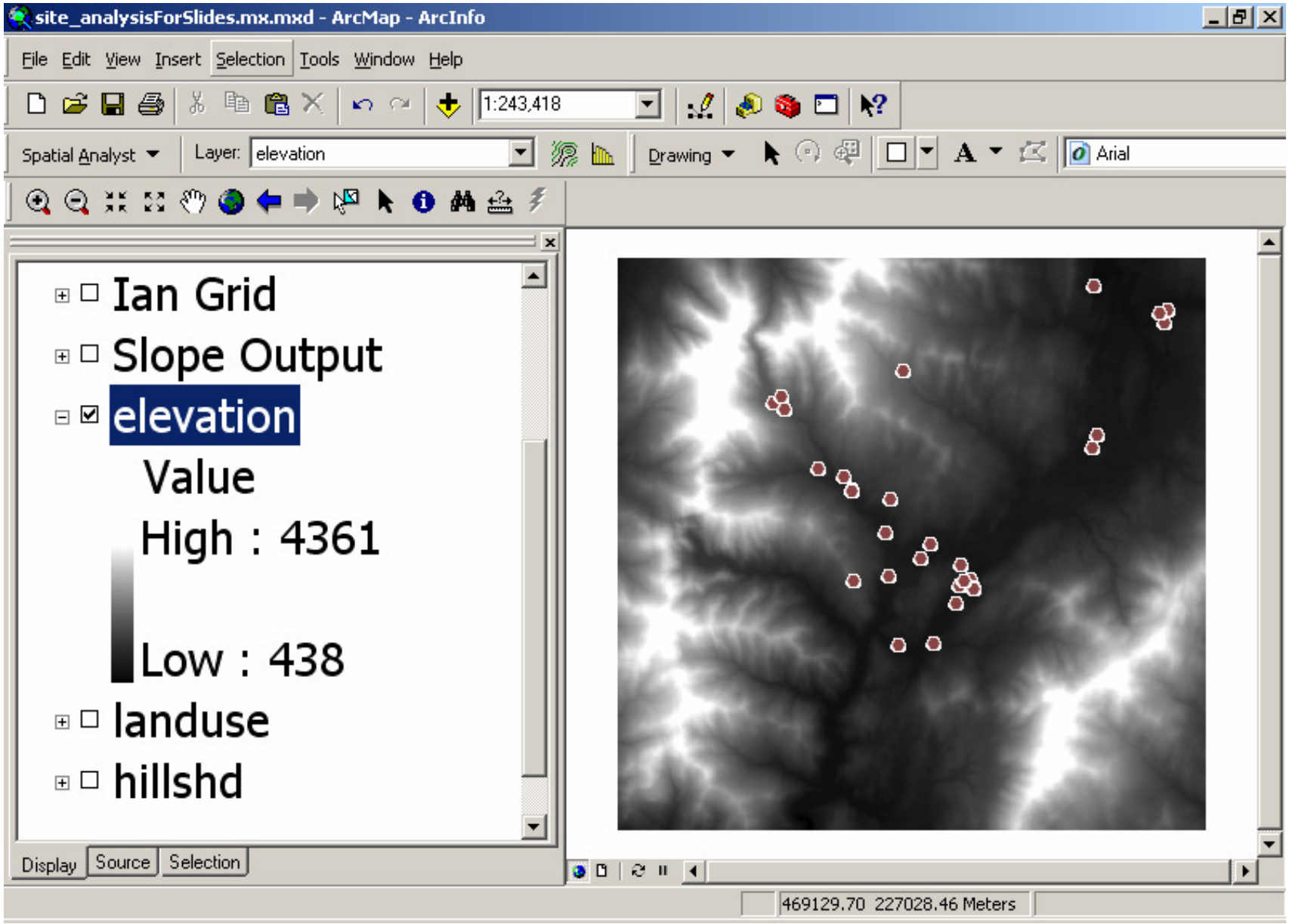
Recreational Sites Layer



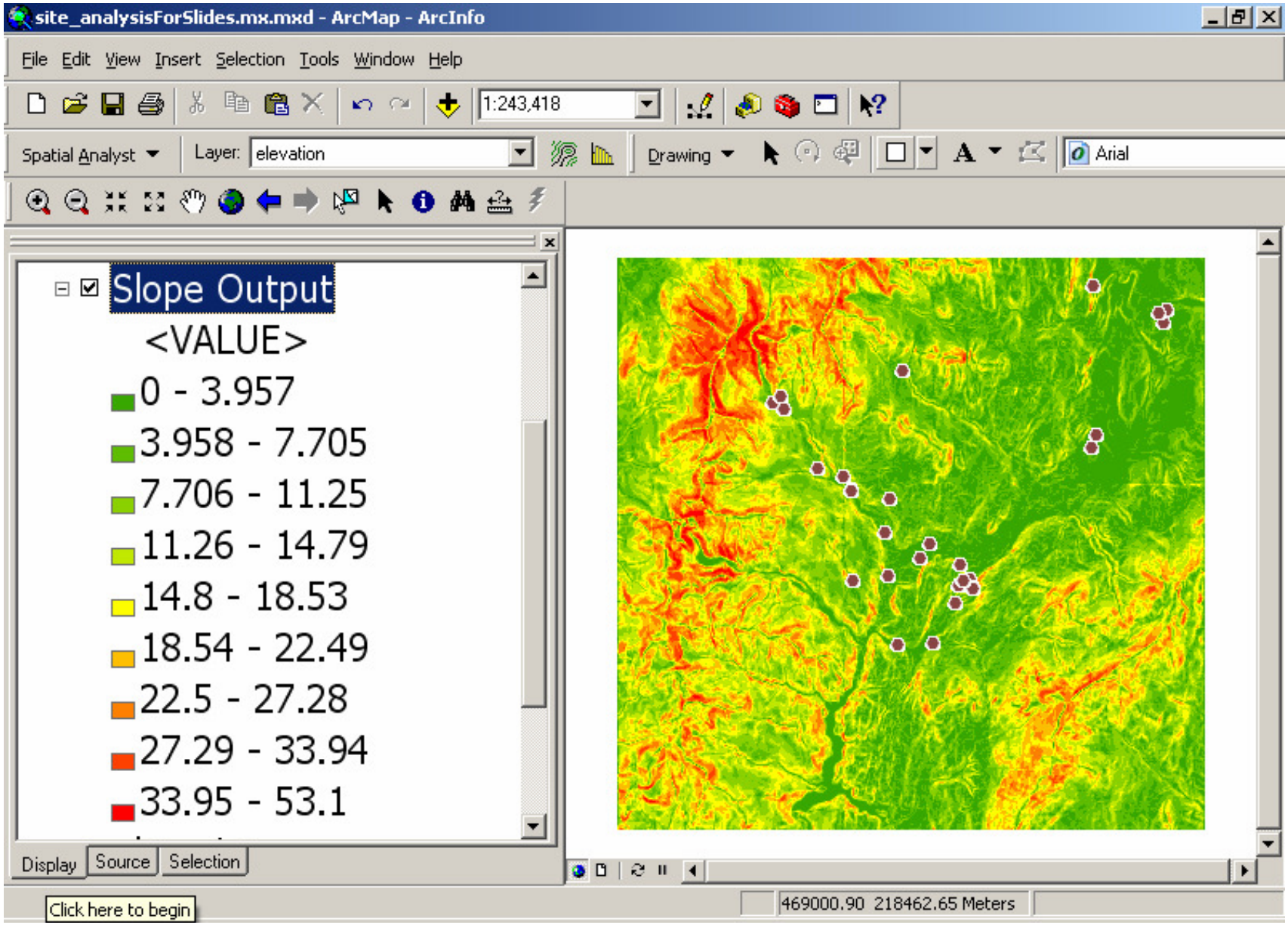
Land Use Layer



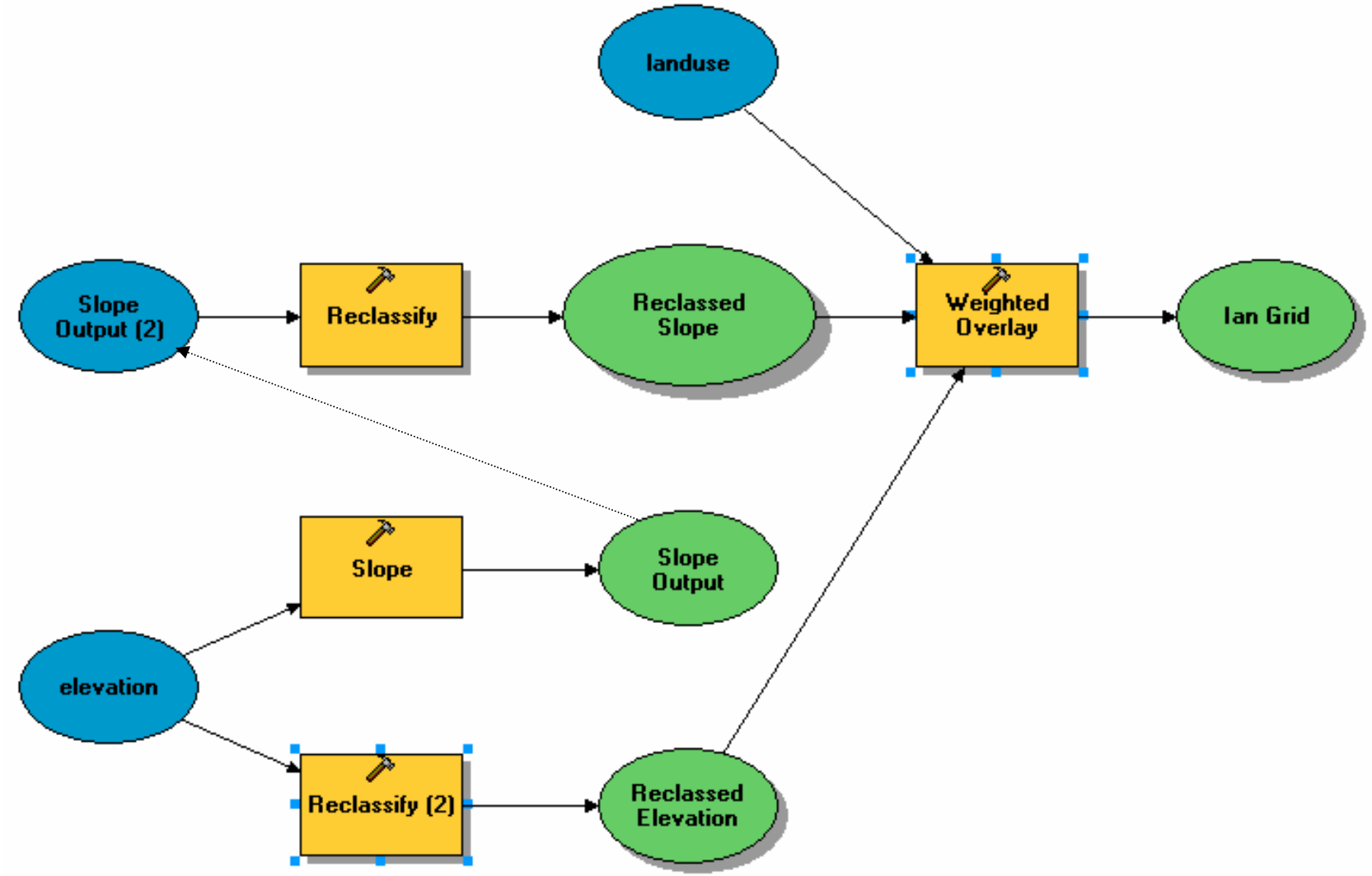
Elevation Layer



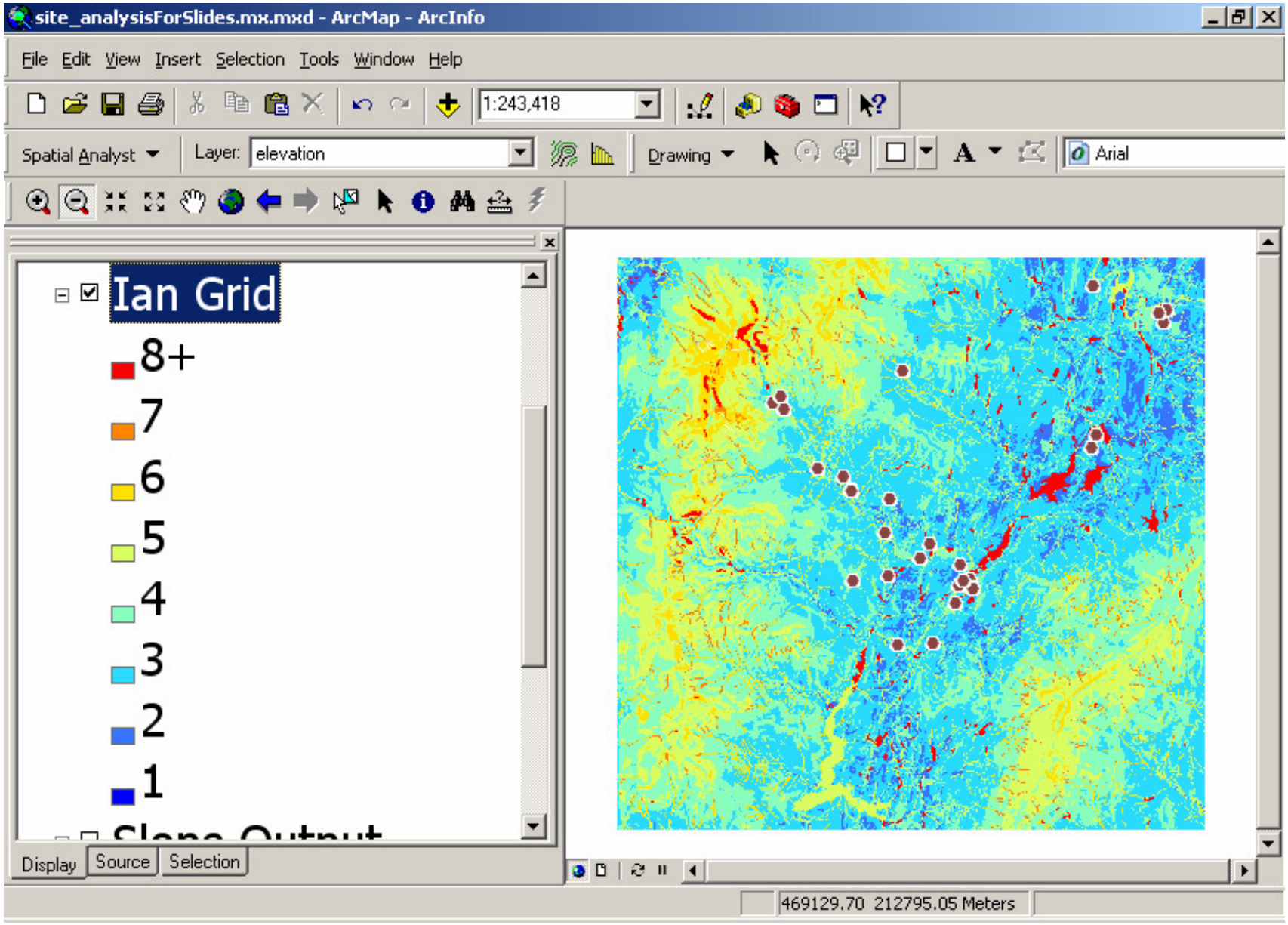
Slope Layer (derived from elevation layer)



Model in ArcMap Used to Generate Grid Cell Costs

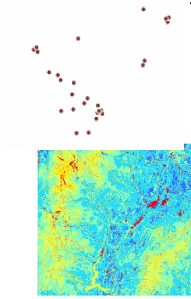


Weighted Combination Layer



ArcMap ↔ Matlab

I. ArcMap → Matlab

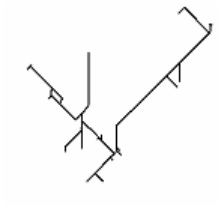


Terminal nodes (rec. sites)

Grid cell costs (weighted map)

II. Run the GA in Matlab

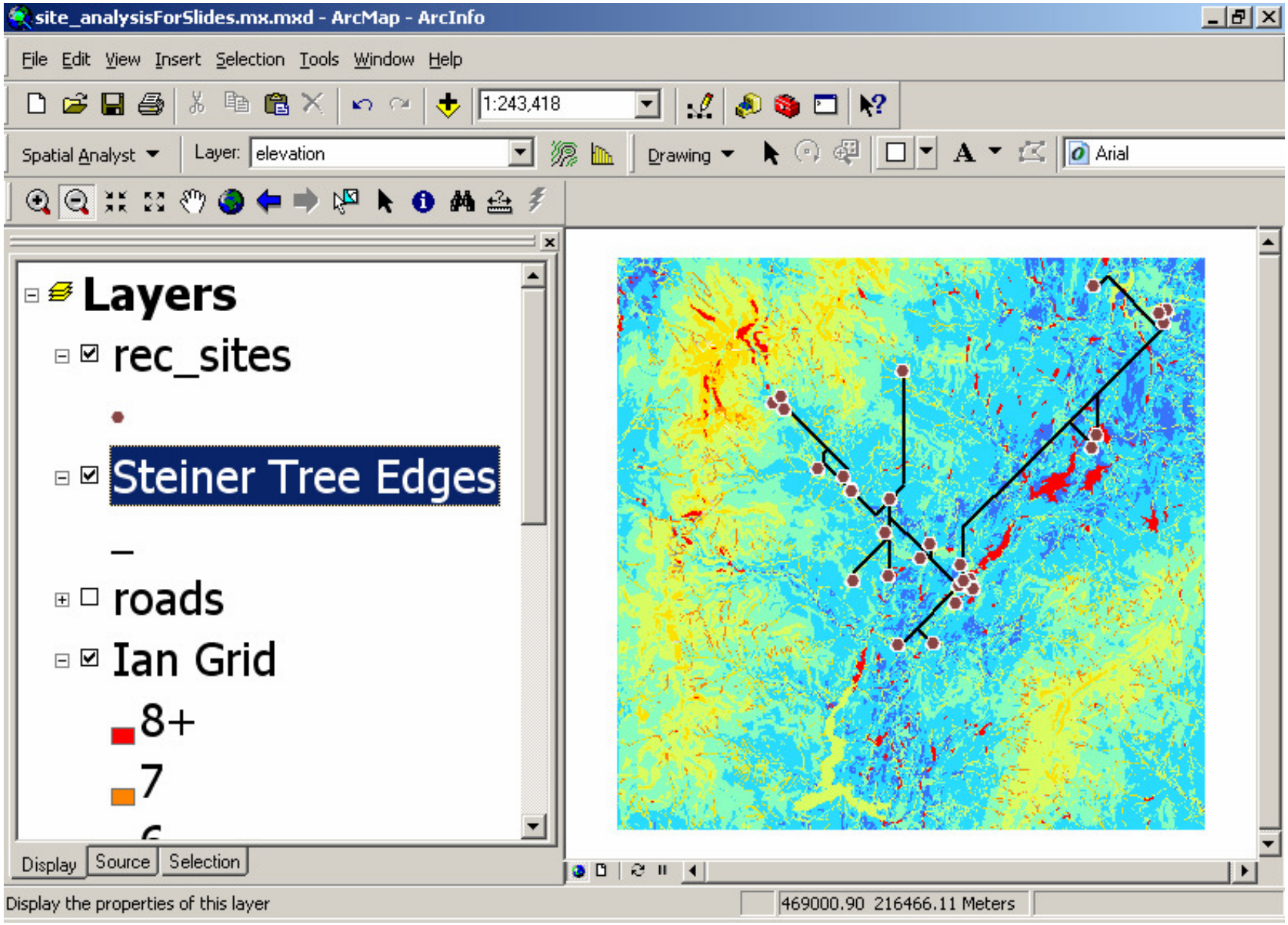
III. Matlab → ArcMap



Steiner tree edges

(Steiner nodes, tree cost)

Steiner Tree Edges Layer



Conclusion

- Applied a Genetic Algorithm to the Non-Uniform Steiner Tree Problem
- GA finds optimal or near optimal solutions much more quickly than an exact algorithm
- GA can also solve the node-weighted variant
- Illustrated linkage between algorithm and GIS