Weight Annealing Heuristics for Solving Bin Packing Problems

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Outline of Presentation

- Introduction
- Concept of Weight Annealing
- One-Dimensional Bin Packing Problem
- Two-Dimensional Bin Packing Problem
- Conclusions

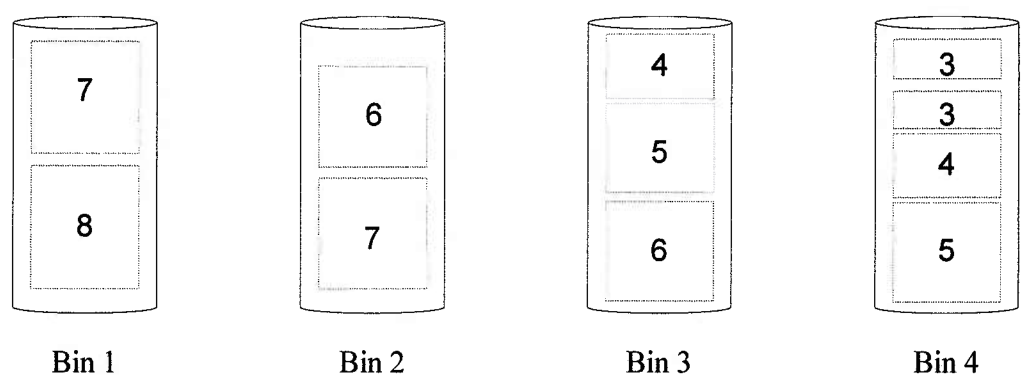
Weight Annealing Concept

- Assigning different weights to different parts of a combinatorial problem to guide computational effort to poorly solved regions.
  - Ninio and Schneider (2005)
  - Elidan et al. (2002)
- Allowing both uphill and downhill moves to escape from a poor local optimum.
- Tracking changes in the objective function value, as well as how well every region is being solved.
- Applied to the Traveling Salesman Problem. (Ninio and Schneider 2005)
  - Weight annealing led to mostly better results than simulated annealing.

One-Dimensional Bin Packing Problem

- Pack a set of $N = \{1, 2, \ldots, n\}$ items, each with size t_i, i=1, 2,..., n, into identical bins, each with capacity C.
- Minimize the number of bins without violating the capacity constraints.
- Large literature on solving this NP-hard problem.

Item List ={8,7,7,6,6,5,4,4,3,3}   Bin Capacity =15



Outline of Weight Annealing Algorithm

- Construct an initial solution using first-fit decreasing.
- Compute and assign weights to items to distort sizes according to the packing solutions of individual bins.
- Perform local search by swapping items between all pairs of bins.
- Carry out re-weighting based on the result of the previous optimization run.
- Reduce weight distortion according to a cooling schedule.

Neighborhood Search for Bin Packing Problem

- From a current solution, obtain the next solution by swapping items between bins with the following objective function (suggested by Fleszar and Hindi 2002)

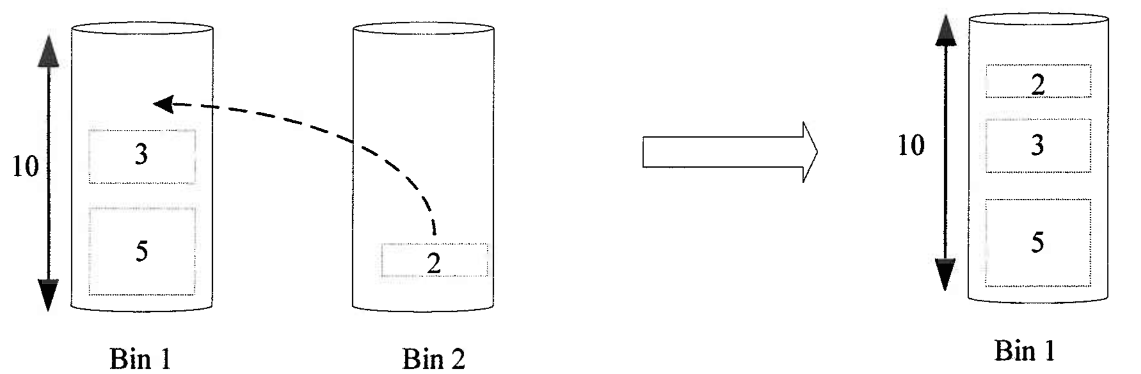
$$\text{Maximize} \quad f = \sum_{i=1}^{p} (l_i)^2$$

$l_i = \sum_{j=1}^{q_i} t_j$ bin load i

p = number of bins

q_i = number of items in bin i

t_j = size of item j

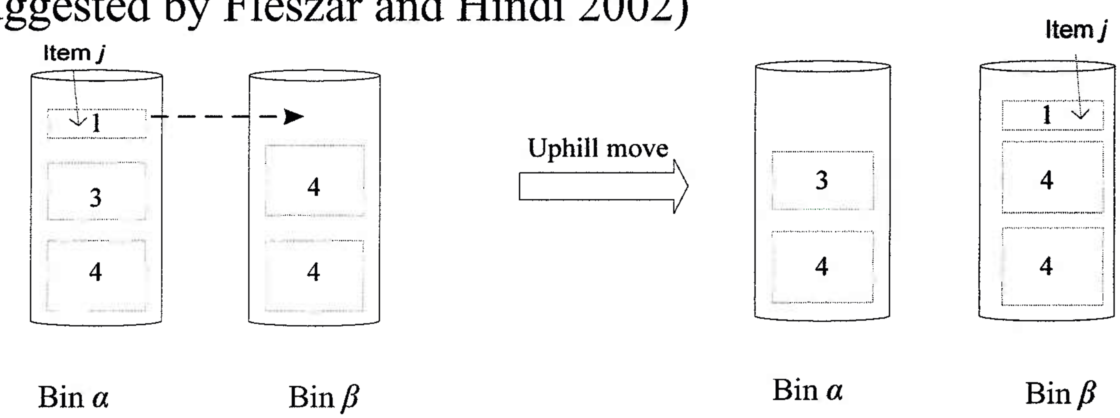


$$f = (5+3)^2 + 2^2 = 68 \qquad f_{new} = (5+3+2)^2 = 100$$

Neighborhood Search for Bin Packing Problem

- Swap schemes
  - Swap items between two bins.
  - Carry out Swap (1,0), Swap (1,1), Swap (1,2), Swap (2,2) for all pairs of bins.
  - Analogous to 2-Opt and 3-Opt.

- Swap (1,0) (suggested by Fleszar and Hindi 2002)



$$f = (1+3+4)^2 + (4+4)^2 = 128$$

$$f_{new} = (3+4)^2 + (1+4+4)^2 = 130$$

- Need to evaluate only the change in the objective function value.

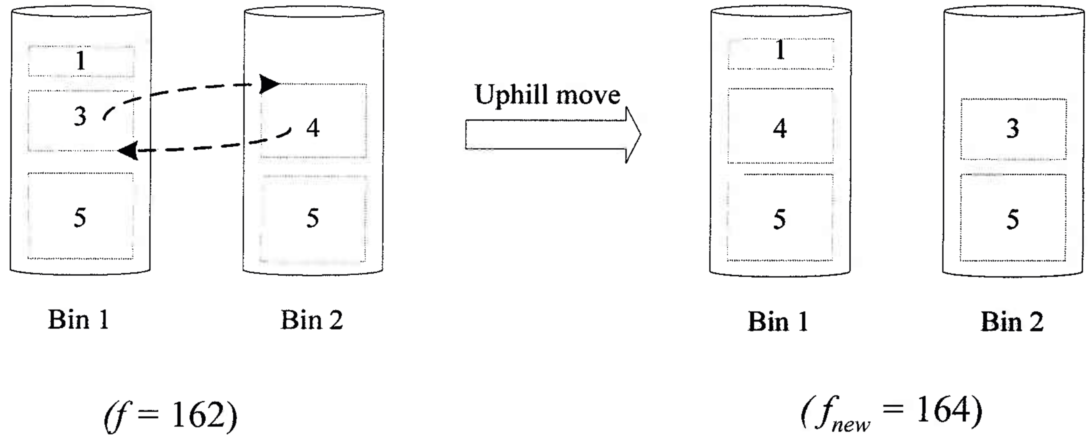
$$\Delta f = (l_\alpha - t_j)^2 + (l_\beta + t_j)^2 - l_\alpha^2 - l_\beta^2$$

l_α = total load of bin α

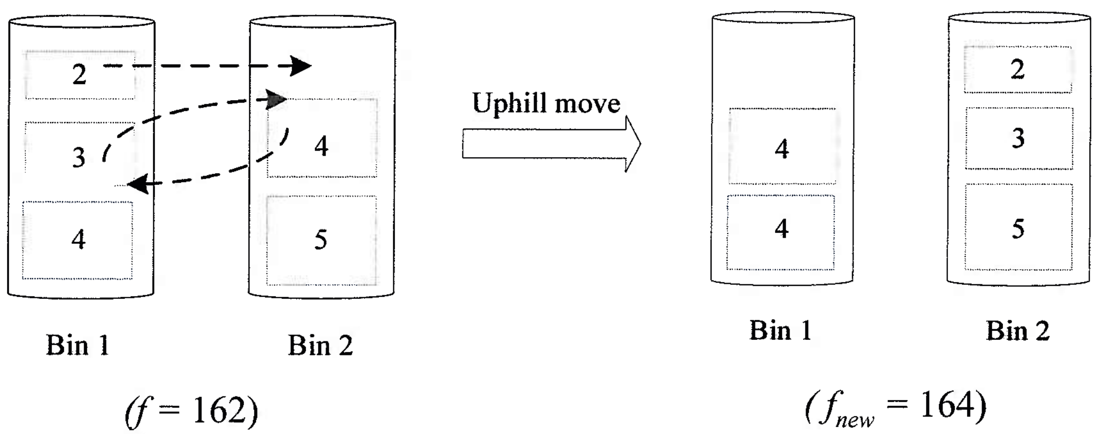
t_i = size of item i

Neighborhood Search for Bin Packing Problem

- Swap (1,1)



- Swap (1,2)



Weight Annealing for Bin Packing Problem

- Weight of item i

$$w_i = 1 + K r_i$$

residual capacity $r_i = \left(\frac{C - l_i}{C} \right)$

C = capacity

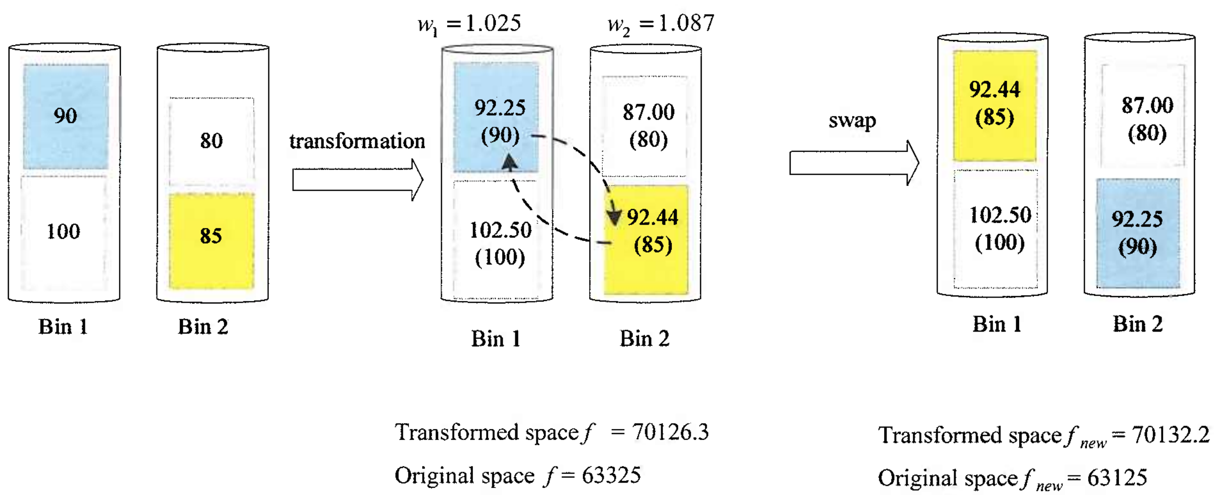
l_i = load of bin i

- An item in a not-so-well-packed bin, with large r_i, will have its size distorted by a large amount.
- No size distortions for items in fully packed bins.
- K controls the size distortion, given a fixed r_i .

Weight Annealing for Bin Packing Problem

- Weight annealing allows downhill moves in a maximization problem.

- Example $C = 200,\ K = 0.5,\ w_i = 1 + 0.5\left(\frac{200 - l_i}{200}\right)$



  - Transformed space - uphill move
  - Original space - downhill move

Solution Procedures (1BP)

- BISON (Scholl, Klein, and Jürgens 1997)
  - Hybrid method combining tabu search and branch-and-bound
  - New branch schemes
- MTPCS (Schwerin and Wäscher 1999)
  - Bounding procedures based on a cutting stock problem (CS)
  - Integrating the lower bound into Martello and Toth procedure (MTP)
- PMBS' +VNS (Fleszar and Hindi 2002)
  - Minimum bin slack heuristic
  - Variable neighborhood search
- HI_BP (Alvim, Ribeiro, Glover, and Aloise 2004)
  - Sophisticated hybrid improvement heuristic
  - Tabu search to move items between bins
- WA1BP
  - Weight annealing heuristic that creates dimension distortions to different parts of the search space during the local search.

Computational Results (1BP)

- Benchmark problems
  - Five sets of test problems
    - Uniform U120, U205, U500, U1000
    - Triplet T60, T120, T249, T501
    - Set Set1, Set2, Set3
    - Was Was1, Was2
    - Gau Gau1
  - A total of 1587 problem instances

Computational Results (1BP)

- Weight annealing performed slightly better than HI_BP.
  - Generated more optimal solutions to the Gau set (17 versus 14).

- Weight annealing performed much better than BISON, PMBS' +VNS, and MTPCS.
  - Generated more optimal solutions to Set benchmark problems.
    - Weigh annealing found optimal solutions to all 1210 instances.
    - BISON, PMBS' +VNS and MTPCS fell short (by 37, 40, and 94 instances).
  - Was faster than BISON and MTPCS (0.18s versus 31.5s - 118.2s).

- Overall Performance of the weight annealing algorithm
  - Found 1582 optimal solutions to 1587 problem instances.
  - Found three new optimal solutions to the Gau set.
  - Took 0.16s on average to solve an instance.

Two-Dimensional Bin Packing Problems

Problem statement

- Allocate, without overlapping, n rectangular items to identical rectangular bins.
- Pack items such that the edges of bins and items are parallel to each other.
- Minimize the total number of rectangular bins (NP-hard).

Classifications

- Guillotine Cutting
  - 2BP|O|G — Fixed Orientation (O), Guillotine Cutting (G)
  - 2BP|R|G — Allowable 90° Rotation (R), Guillotine Cutting (G)
- Free Cutting
  - 2BP|O|F — Fixed Orientation (O), Free Cutting (F)
  - 2BP|R|F — Allowable 90° Rotation (R), Free Cutting (F)

Two-Dimensional Bin Packing Problems (2BP|O|G)

Hybrid first-fit algorithm

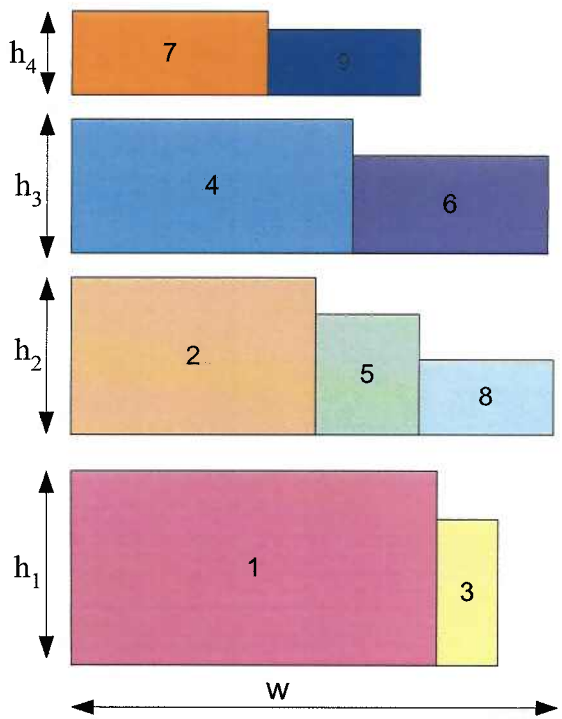
- Phase One (one-dimensional horizontal level packing)
  - Arrange the items in the order of non-increasing height.
  - Pack the items from left to right into levels, each level i with the same width W.
  - Pack an item (left justified) on the first level that can accommodate it; start a new level if no level can accommodate it.

- Phase Two (one-dimensional vertical bin packing)
  - Arrange the levels in the order of non-increasing height h_i; this is the height of the first item on the left.
  - Solve one-dimensional bin packing problems, each item i with size h_i, and bin size H.

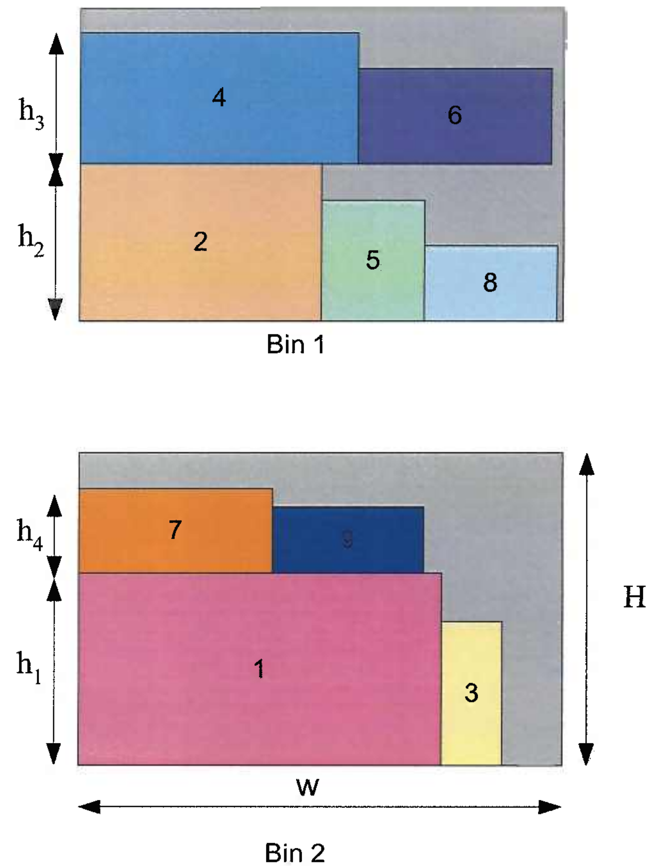
Two-Dimensional Bin Packing Problems (2BP|O|G)

<u>An example of hybrid first-fit</u>

Phase 1 - One Dimensional
Horizontal Level Packing

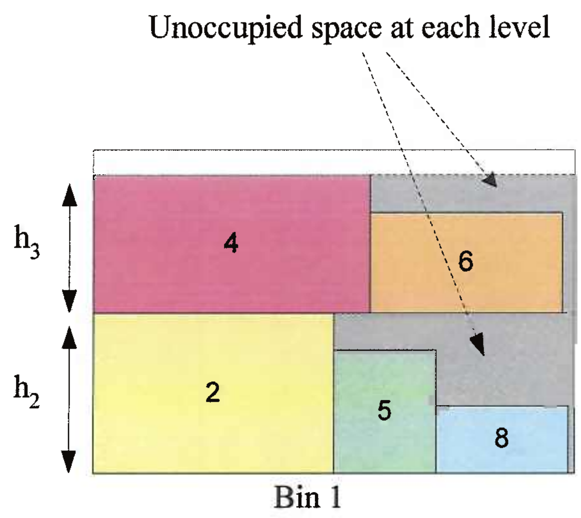


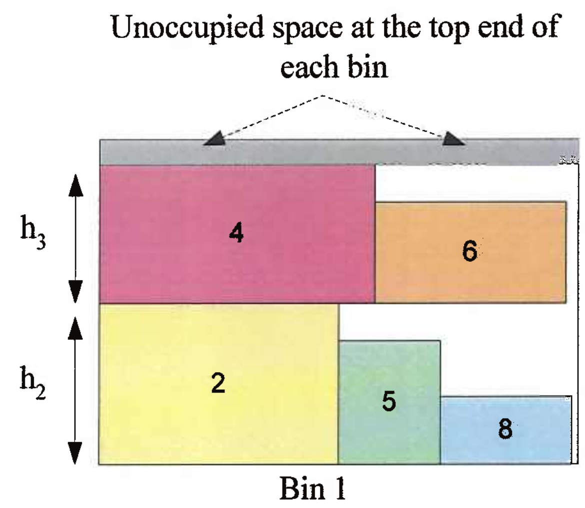
Phase 2 - One Dimensional
Vertical Bin Packing



Two-Dimensional Bin Packing Problems (2BP|O|G)

- Weakness of hybrid first-fit





Weight Annealing Algorithm (2BP|O|G)

- Phase One (one-dimensional horizontal level packing)
  - Construct an initial solution.
    - Arrange the items in the order of non-increasing height.
    - Introduce randomness in the insertion order to generate different starting solutions, if necessary.
  - Swap items between levels to minimize the number of levels.
    - Objective function

$$\text{Maximize} \quad f = \sum_{i=1}^{p} (b_i)^2 \; - \; \sum_{i=1}^{p} (Wh_i - A_i)$$

$$b_i = \sum_{j=1}^{m_i} t_{ij}$$

$t_{ij} =$ width of item j in level i

$m_i =$ number of items in level i

$p =$ number of levels

$A_i =$ sum of item areas in level i

$h_i =$ height of level i

$W =$ bin width

Weight Annealing Algorithm (2BP|O|G)

- Phase Two (one-dimensional vertical bin packing)
  - Construct initial solution with first-fit decreasing using level height h_i as item sizes and bin height H.
  - Swap levels between bins to minimize the number of bins.
    - Objective function

$$\text{Maximize} \quad f = \sum_{i=1}^{q} (d_i)^2$$

$$d_i = \sum_{j=1}^{m_i} h_{ij}$$

h_{ij} = height of level j in bin i

m_i = number of levels in bin i

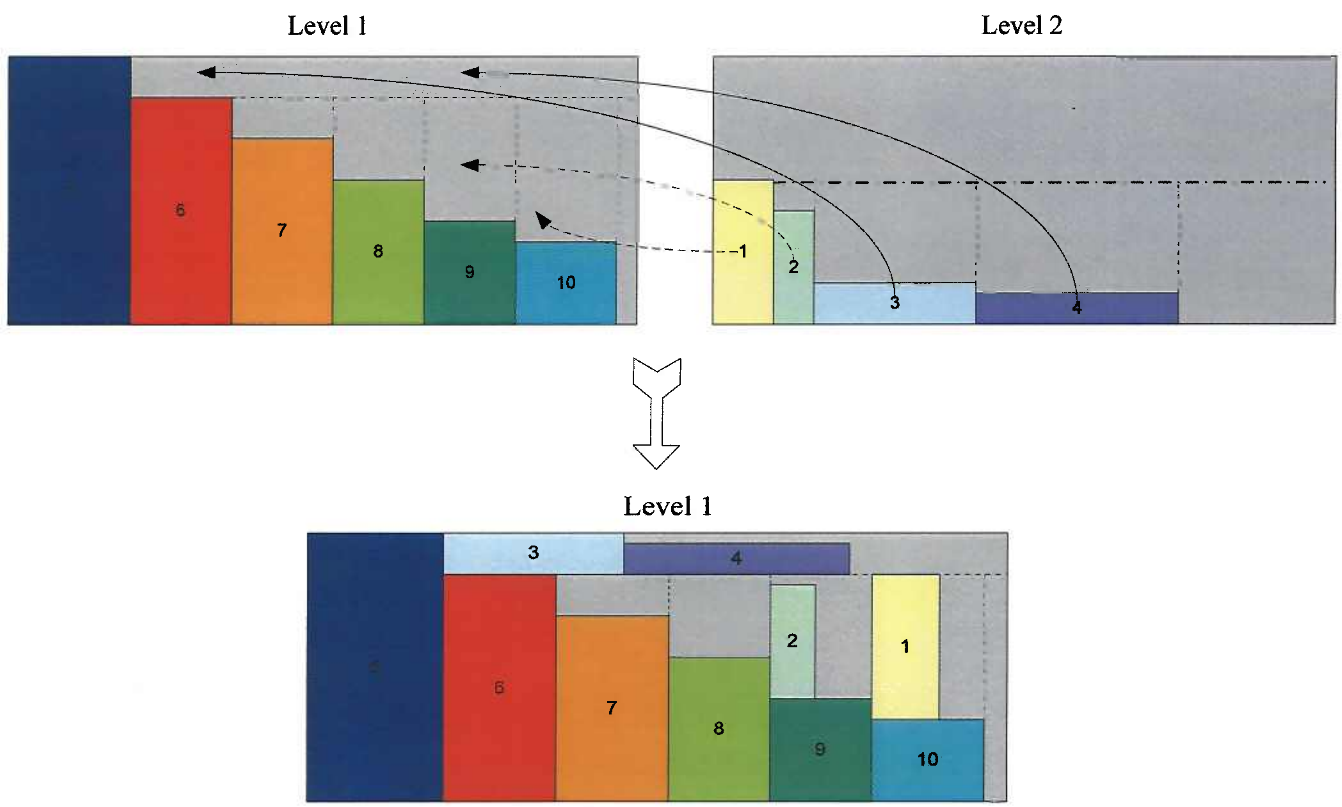
q = number of bins

Weight Annealing Algorithm (2BP|O|G)

- Phase Three
  - Filling unused space in each level.

Maximize $f = \sum_{i=1}^{p} (A_i)^2$

A_i = sum of item areas in level i

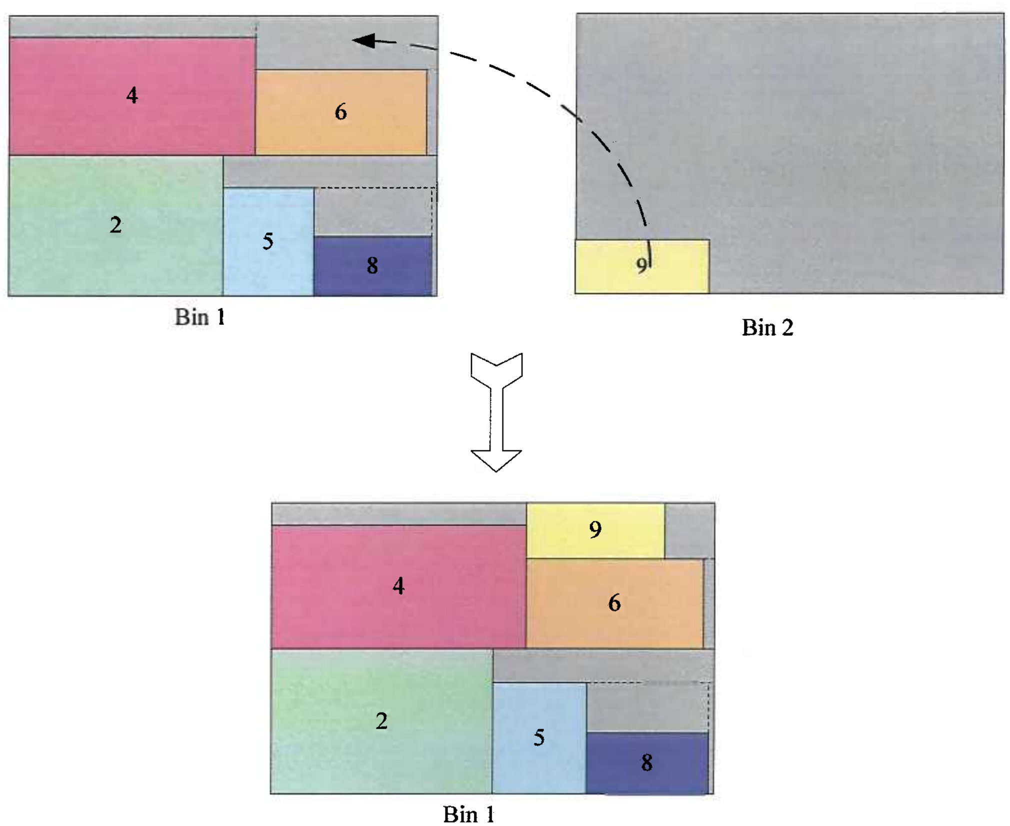


Weight Annealing Algorithm (2BP|O|G)

- Phase Three
  - Filling unused space at the top of each bin.

Maximize $f = \sum_{i=1}^{q} (A_i)^2$

$A_i =$ sum of item areas in bin i

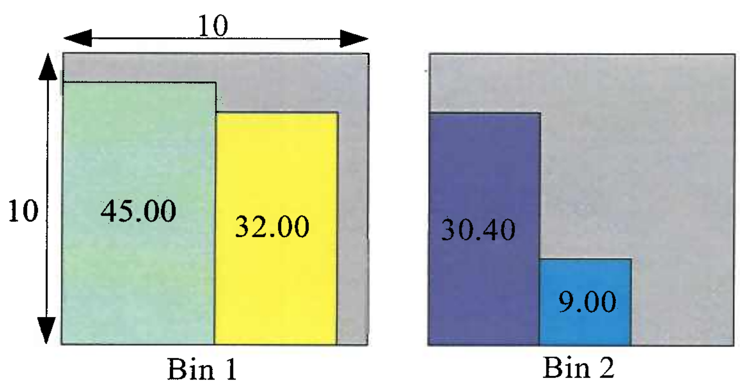


Weight Annealing Algorithm (2BP|O|G)

- Weight assignments
  - Phase One $w_i = 1 + Kr_i$ $r_i = \left(\frac{W - b_i}{W}\right)$
  - Phase Two $w_i = 1 + Kr_i$ $r_i = \left(\frac{H - d_i}{H}\right)$
  - Phase Three $w_i = 1 + Kr_i$ $r_i = \left(\frac{HW - A_i}{HW}\right)$

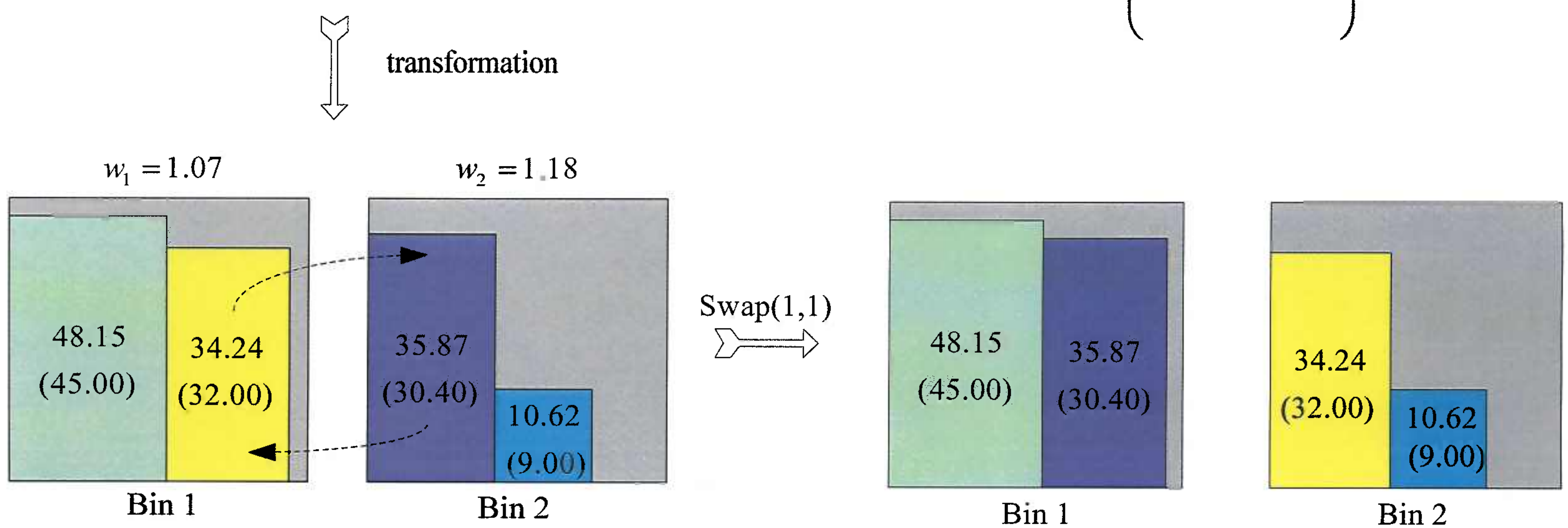
Weight Annealing Algorithm (2BP|R|G)

- Example: Weight Annealing allows downhill move in the maximization problem.



bin area = 100 $\quad K = 0.3$

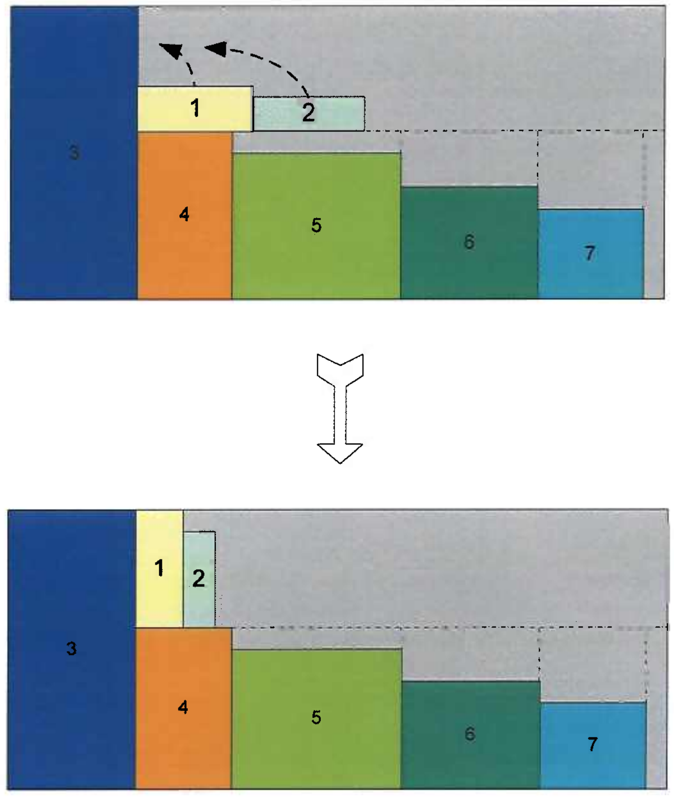
$$w_i = 1 + 0.3\left(\frac{100 - \sum_{j=1}^{m_i} a_{ij}}{100}\right)$$



- Transformed space - uphill move
- Original space - downhill move

Weight Annealing Algorithm (2BP|R|G)

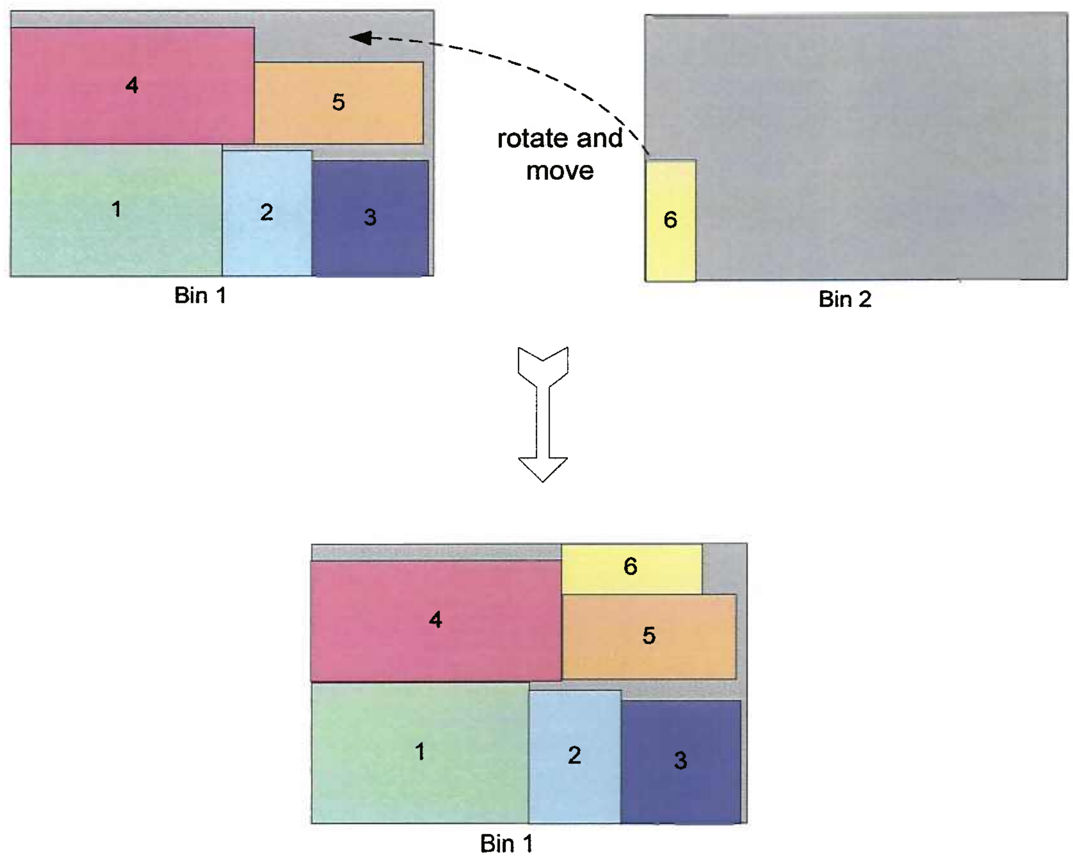
- Rotating an item through 90° to achieve a better packing solution.



Weight Annealing Algorithm (2BP|R|G)

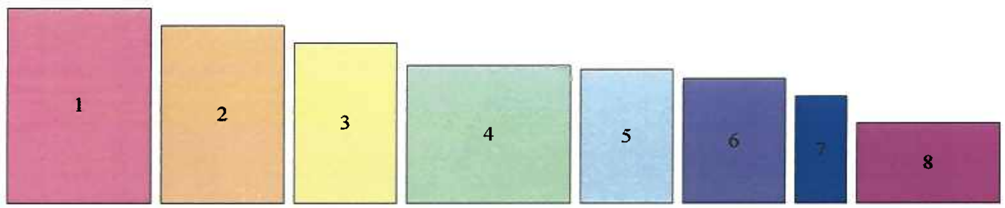
- Rotate an item through 90° and move it to another bin.

Maximize $f = \sum_{i=1}^{q} (A_i)^2$

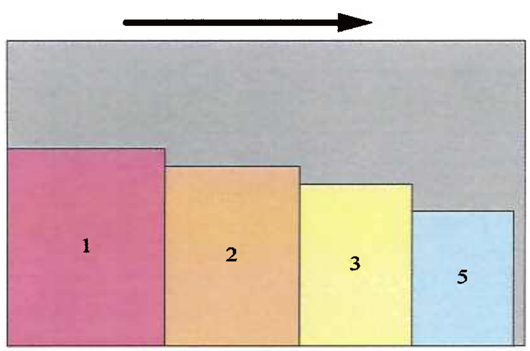


Weight Annealing Algorithm (2BP|O|F)

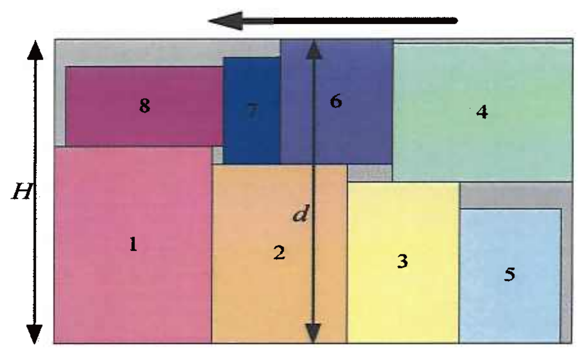
- Alternate direction algorithm
  - Arrange items in the order of non-increasing height.



  - Packing items left to right.



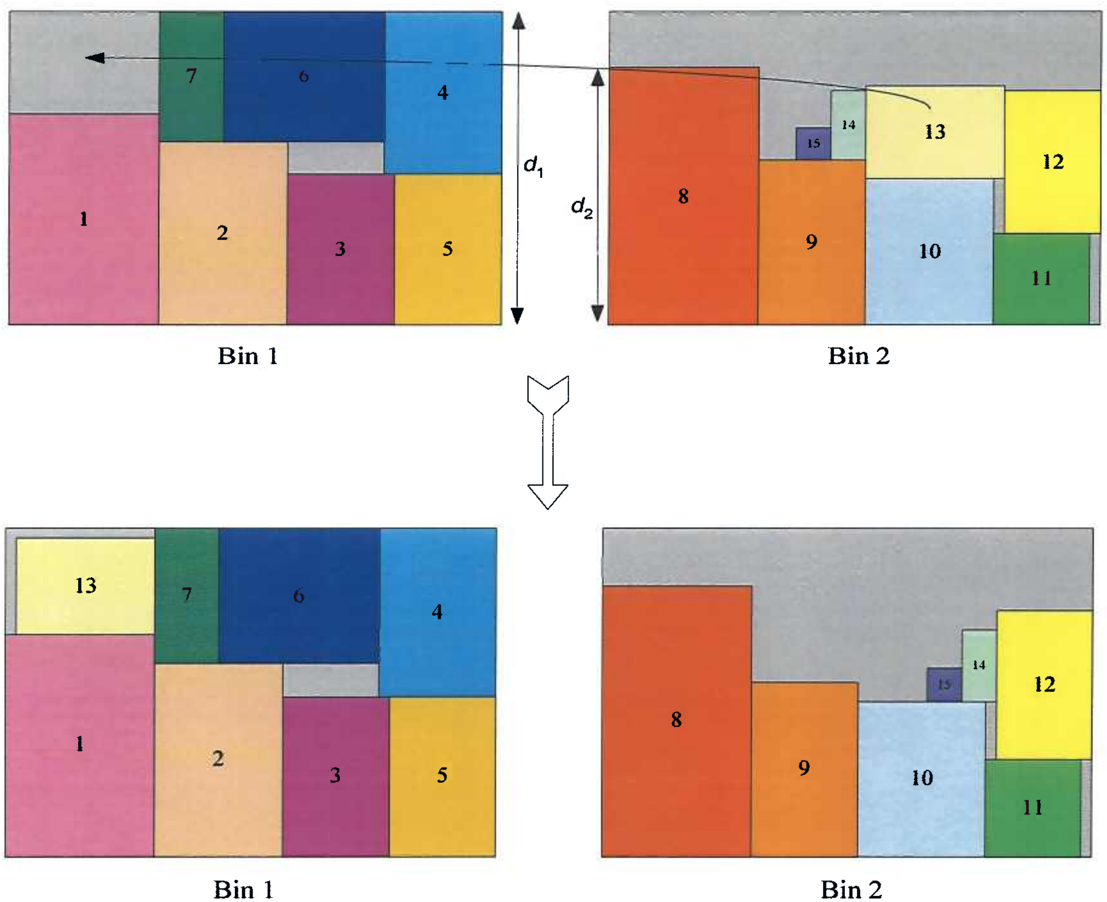
  - Packing items right to left.



Weight Annealing Algorithm (2BP|O|F)

- Moving an item from one bin to another and repacking.

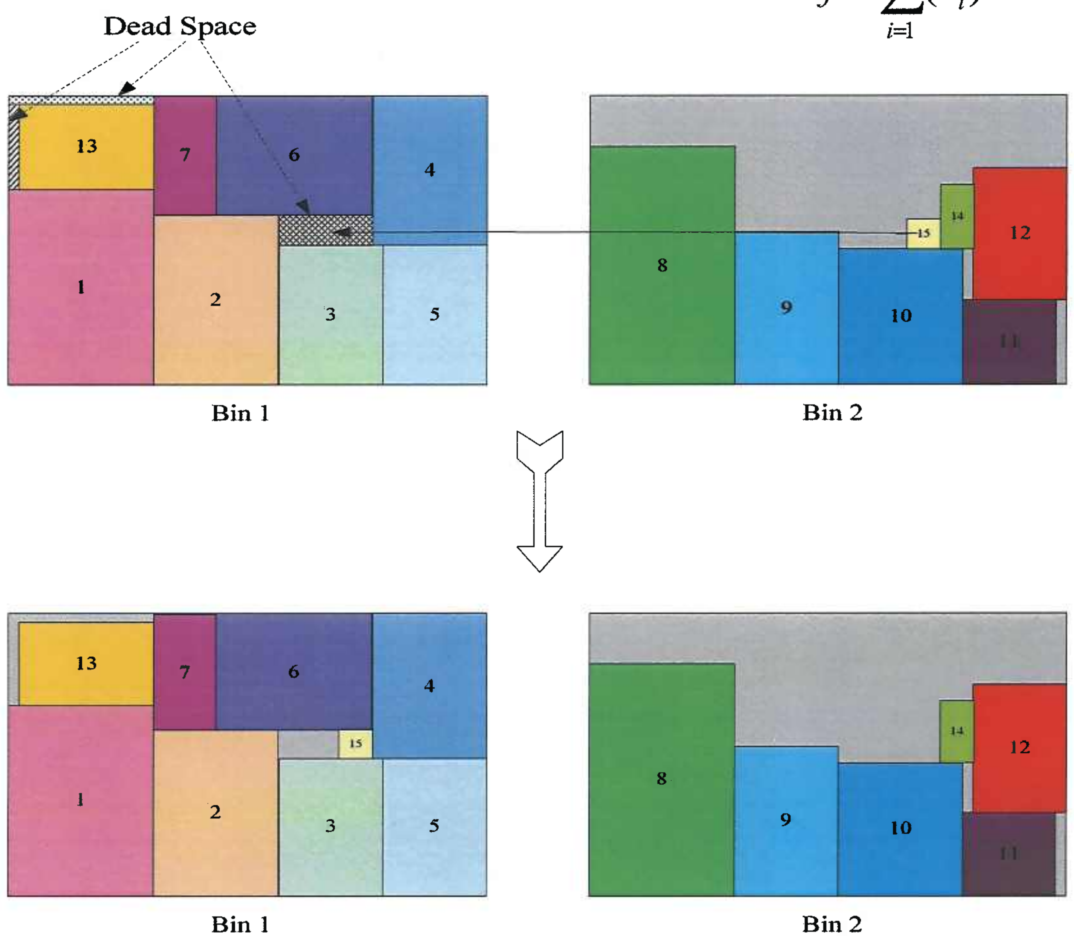
$$\text{Maximize} \quad f = \sum_{i=1}^{q} (A_i)^2$$



Weight Annealing Algorithm (2BP|O|F)

- Post-optimization processing

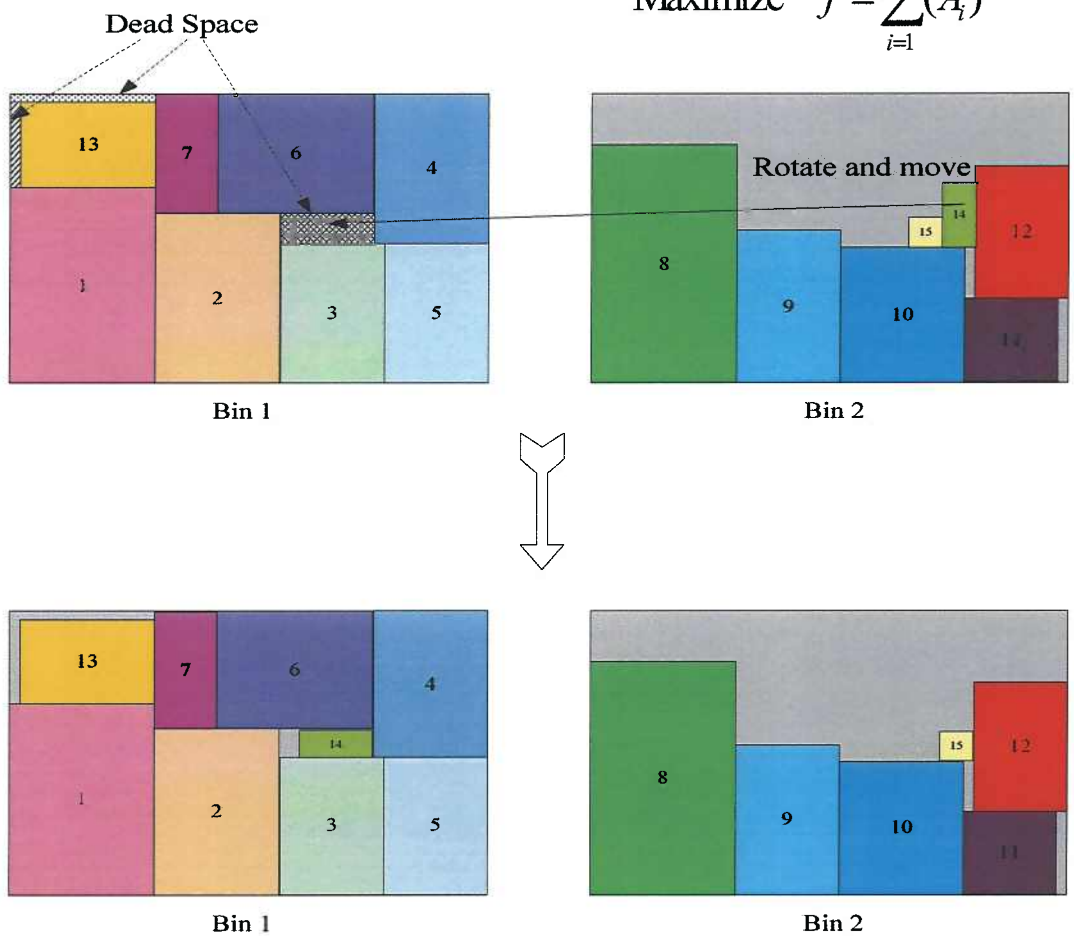
$$\text{Maximize} \quad f = \sum_{i=1}^{q} (A_i)^2$$



Weight Annealing Algorithm (2BP|R|F)

- Rotate an item through 90° to occupy dead space in another bin.

$$\text{Maximize} \quad f = \sum_{i=1}^{q} (A_i)^2$$



Solution Procedures (2BP)

- Exact algorithm by Martello and Vigo (1998) for 2BP|O|F
- Tabu search by Lodi, Martello and Toth (1999) for 2BP|O|G, 2BP|R|G, 2BP|O|F, 2BP|R|F
- Guided local search by Faroe, Pisinger, and Zachariasen (2003) for 2BP|O|F
- Constructive algorithm (HBP) by Boschetti and Mingozzi (2003) for 2BP|O|F
- Set covering heuristic by Monaci and Toth (2006) for 2BP|O|F

Computational Results of Weight Annealing (2BP)

- Benchmark problems
  - 300 problem instances of Berkey and Wang (1987)
  - 200 problem instances of Martello and Toth (1998)
- Comparing computational results (2BP|O|F) is not a straight-forward task.
  - Tabu search results
    - Average ratios (TS solution value/ lower bound) over 10 instances are reported.
    - Lower bounds not given in the papers.
  - Computational results and lower bounds quoted in journals were inconsistent.
  - Guided local search results did not include the running times.

Computational Results for 2BP|O|F

- Results for the 500 problem instances (summary measures).

Procedures	Total Number of Bins	Total Running Time(s)
Tabu Search	7364	1436.1
Guided Local Search	7302	-
Exact Algorithm	7313	524.7
Constructive Algorithm (HBP)	7265	345.9
Set Covering Heuristic	7248	148.5
Weight Annealing	7253	119.3

- The results of weight annealing and set covering heuristic are comparable.
  - The total number of bins are about 1.1 % above the best lower bound (7173 bins).
  - Both use fewer number of bins, and are faster than the other procedures.

Computational Results of Weight Annealing (2BP)

- Results for the 500 problem instances (summary measures).

2BP Variants	Total Number of Bins	Total Running Time (sec)
2BP|O|F	7253	119.3
2BP|R|F	7222	66.7
2BP|O|G	7373	24.8
2BP|R|G	7279	44.0

Conclusions

- The application of weight annealing to bin packing problems is new.
  - One-dimensional bin packing problem
  - Two-dimensional bin packing problem (four versions)
- Weight annealing algorithms produce high-quality solutions.
- Weight annealing algorithms are fast and competitive.
  - Easy to understand
  - Simple to code
  - Small number of parameters