The Split Delivery Vehicle Routing Problem: Applications, Algorithms, Test Problems, and Computational Results

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Introduction

- Split Delivery Vehicle Routing Problem (SDVRP)

  Variant of the standard, capacitated VRP

  Customer's demand can be split among several vehicles

  Potential to use fewer vehicles thereby reducing the total distance traveled by the fleet

  NP-hard problem

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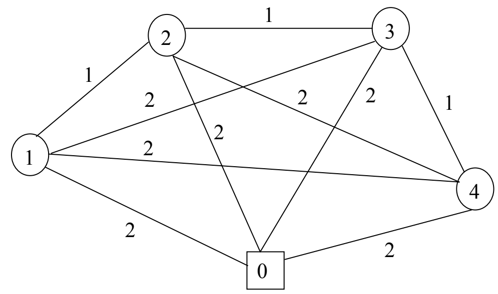
Example

▶ Archetti, Hertz, and Speranza (2006)



Node 0 is the depot

Each customer has a demand of 3 units

Vehicle capacity is 4 units

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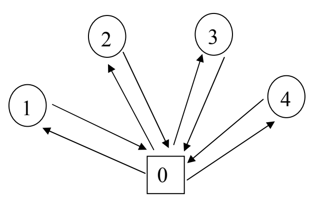
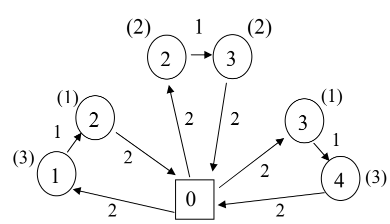
Example

▶ Archetti, Hertz, and Speranza (2006)





VRP optimal solution	SDVRP optimal solution
Four vehicles	Three vehicles
Total distance is 16	Total distance is 15

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Applications

- Mullaseril, Dror, and Leung (1997)

  Distribution of feed to cattle at a large livestock ranch in Arizona

  - 100,000 head of cattle, 5 types of feed
  - Six trucks deliver feed to pens
  - Last stop may not receive full load

- Sierksma and Tijssen (1998)

  Route helicopters for weekly crew exchanges at natural gas platforms in the North Sea

  - 51 platforms, crew of 20 to 60
  - Fuel capacity and seating capacity limits

Applications

- Song, Lee, and Kim (2002)

  Distributing bundles of newspapers in Seoul

  - 400 agents, 3 printing plants
  - Close-in agents receive split deliveries
  - Reduced delivery cost by 15% on average

- Levy (2006)

  Containerized sanitation pick up at commercial office buildings

  - Large bins may require several trucks to handle all of the trash

Solution Procedures

- Dror and Trudeau (1989, 1990)
  - Two-stage algorithm (DT)
    - Solve the VRP and improve the solution
    - Use *k*-split interchanges and route additions to solve the SDVRP
      - *k*-split interchange splits the demand of customer *i* among *k* routes
      - Add a route to eliminate a split delivery

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Solution Procedures

- Dror and Trudeau (1989, 1990)
  - Computational Results
    - Three problems with 75, 115, 150 nodes and a vehicle capacity of 160
    - Six demand scenarios
      [0.01 – 0.10], [0.01 – 0.30], [0.01 – 0.50]
      [0.10 – 0.90], [0.30 – 0.70], [0.70 – 0.90]
    - Demand for customer *i* randomly selected from a uniform distribution on [160α,160β]
    - 30 instances per scenario

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Solution Procedures

- Dror and Trudeau (1989, 1990)
  - Computational Results
    - Use DT to solve each problem twice – as an SDVRP and a VRP
    - When customer demand is low relative to vehicle capacity, there are almost no split deliveries
    - When customer demand is very large (e.g., [0.70 – 0.90]), split deliveries occur and produce a distance savings (average of 11.24% over the VRP solution for a 75-node problem)

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Solution Procedures

- Frizzell and Giffin (1992, 1995)
  - SDVRP on a grid network
    - Develop a construction heuristic
    - Solve problems with time windows
- Belenguer, Martinez, and Mota (2000)
  - Lower bound for the SDVRP
  - Develop a cutting-plane algorithm
    - Gap between upper bound and lower bound about 18% for random problems

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Solution Procedures

- Archetti, Hertz, and Speranza (2006)
  - Three-phase algorithm (SPLITABU)
    - Use GENIUS algorithm, tabu search, followed by final improvement
    - Variant (SPLITABU-DT) generates high-quality solutions to seven classical problems with 50 to 199 customers
- Recent dissertations
  - Liu (2005)
  - Nowak (2005)

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New IP Approach

- Endpoint Mixed Integer Program (EMIP)
  - Start with an initial solution (Clarke-Wright)
  - For each route in the solution, consider its one or two *endpoints* and the c closest neighbors to each endpoint
  - Each endpoint is allowed to allocate its demand among its neighbors
  - After reallocation, there are three possibilities for each endpoint (symmetric distances)
    - No change is made

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New IP Approach

▶ Endpoint Mixed Integer Program

Endpoint *i* is removed from its current route(s) and all of its demand is moved to another route or routes

savings = $(l_{i\text{-}1,i} + l_{i,0} - l_{i\text{-}1,0}) - (l_{0,i} + l_{i,j} - l_{0,j})$

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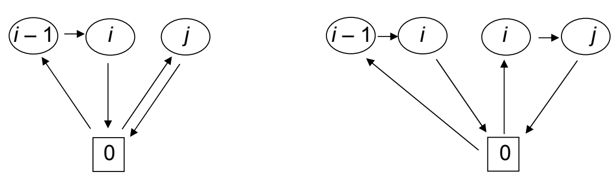
New IP Approach

▶ Endpoint Mixed Integer Program

Endpoint *i* is partially removed from its current route(s) and part of its demand is moved to another route or routes



savings = $-(l_{0,i} + l_{i,j} - l_{0,j})$

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New IP Approach

▶ EMIP Formulation

Definitions

i, j endpoints of current routes

l_{ij} distance between i and j

R_i residual capacity on the route with i as an endpoint

D_i demand of endpoint i carried on its route

R set of routes

N set of endpoints

$NC(i)$ set of c closest neighbors of endpoint i

$p(i)$ predecessor of endpoint i

$s(i)$ successor of endpoint i

New IP Approach

▶ EMIP Formulation

Decision variables

d_{ij} amount of endpoint i's demand moved before endpoint j

m_{ij} = 1 if endpoint i is inserted before endpoint j; 0 otherwise

b_i = 1 if endpoint i's entire demand is removed from the route on which it was an endpoint; 0 otherwise

Objective function

maximize the total savings from the reallocation process

New IP Approach

- EMIP Formulation

  Constraints

  Amount added to a route minus the amount taken away from a route ≤ residual capacity

  Amount diverted from an endpoint on a route ≤ demand of that endpoint on the route

  If an endpoint is removed from a route, then all of its demand must be diverted to other routes

  If we move some of *i*'s demand before *j* ($d_{ij} > 0$) then *i* is inserted before *j* ($m_{ij} = 1$)

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New IP Approach

- EMIP Formulation

  Constraints

  If node *i* is removed from a route, then no node can be inserted before node *i*

  If the predecessor of node *i* is removed from a route, then no node can be inserted before node *i*

  If a route has only two endpoints, then both endpoints cannot be removed at the same time

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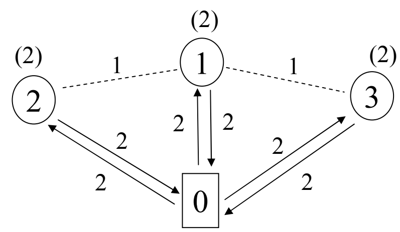
New IP Approach

▶ EMIP Formulation Example



Node 0 is the depot
Customers 1, 2, and 3 have a demand of two units each
Vehicle capacity is three units
Clarke-Wright solution (solid edges) has a total distance of 12

New IP Approach

▶ EMIP Formulation Example

$$\begin{aligned} \text{maximize } & 2b_1 l_{01} + 2b_2 l_{02} + 2b_3 l_{03} - m_{12}(l_{01} + l_{12} - l_{02}) \\ & - m_{13}(l_{01} + l_{13} - l_{03}) - m_{21}(l_{02} + l_{21} - l_{01}) \\ & - m_{23}(l_{02} + l_{23} - l_{03}) - m_{31}(l_{03} + l_{31} - l_{01}) \\ & - m_{32}(l_{03} + l_{32} - l_{02}) \end{aligned}$$

subject to

$$d_{21} + d_{31} - d_{12} - d_{13} \le R_1$$
$$d_{12} + d_{32} - d_{21} - d_{23} \le R_2$$
$$d_{13} + d_{23} - d_{31} - d_{32} \le R_3$$

$$d_{12} + d_{13} \le D_1$$
$$d_{21} + d_{23} \le D_2$$
$$d_{31} + d_{32} \le D_3$$

New IP Approach

- EMIP Formulation Example

$d_{12} + d_{13} \geq D_1 b_1$
$d_{21} + d_{23} \geq D_2 b_2$
$d_{31} + d_{32} \geq D_3 b_3$

$D_1 m_{12} \geq d_{12}$
$D_1 m_{13} \geq d_{13}$
$D_2 m_{21} \geq d_{21}$
$D_2 m_{23} \geq d_{23}$
$D_3 m_{31} \geq d_{31}$
$D_3 m_{32} \geq d_{32}$

$1 - b_1 \geq m_{21} + m_{31}$
$1 - b_2 \geq m_{32} + m_{12}$
$1 - b_3 \geq m_{23} + m_{13}$

$d_{ij} \geq 0$ for $i, j = 1, 2, 3$
$b_i = 0, 1$ for $i = 1, 2, 3$
$m_{ij} = 0, 1$ for $i, j = 1, 2, 3$

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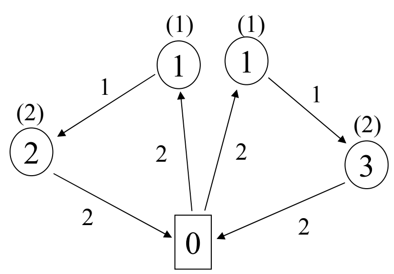
New IP Approach

- EMIP Formulation Example



Optimal solution
Demand for customer 1 is split between two routes
Total distance is 10

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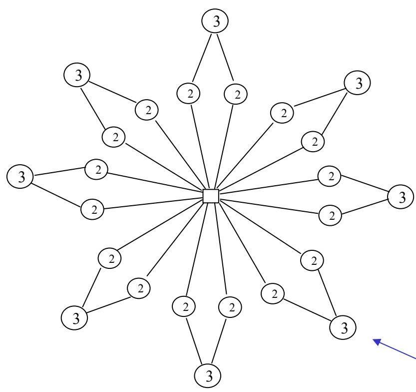
New IP Approach

- Limitation of EMIP

  Not all feasible solutions can be reached



Initial solution uses 8 vehicles

Vehicle capacity is 8

Distance of 1 for each edge

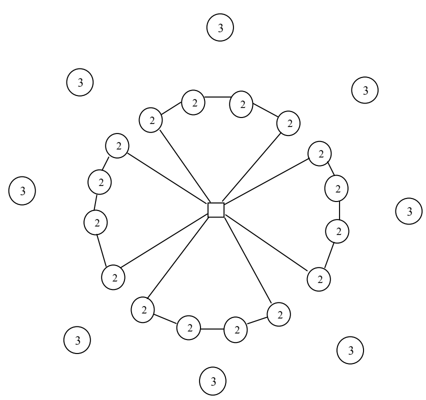
Total distance is 32

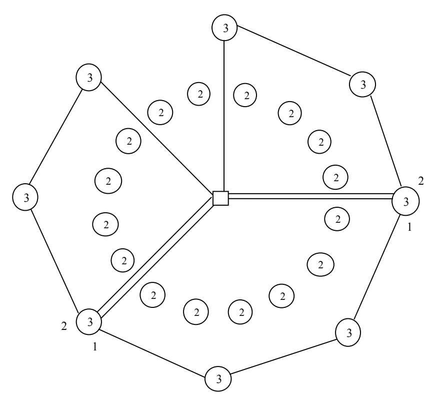
Not an endpoint
Cannot be split

New IP Approach

- Limitation of EMIP

  Improved solution: 7 vehicles, splits two demands





New SDVRP Heuristic

- Combine EMIP and Record-to-record Travel Algorithm
  - Use Clarke-and-Wright to generate a starting solution
  - Using the starting solution, formulate an EMIP
    - 200-node problem with vehicle capacity of 200 and demands between 140 and 180 has about 200 endpoints, 2,200 integer variables, 2,000 continuous variables, and 2,800 constraints
  - Set a time limit T and solve EMIP
    - Save the best feasible solution (E1)
  - Use E1 to formulate and solve a second EMIP (E2)
    - Larger neighbor list
    - Smaller running time limit

New SDVRP Heuristic

- Combine EMIP and Record-to-record Travel Algorithm
  - Improve the E2 solution
    - Post process with the variable length record-to-record travel algorithm (VRTR) of Li, Golden, and Wasil (2005)
    - Consider one-point, two-point, and two-opt moves within and between routes
  - Heuristic denoted by EMIP+VRTR

Computational Results

- Six Benchmark Problems

  Taken from Christofides and Eilon (1969) and Christofides, Mingozzi, and Toth (1979)

  50 to 199 customers

  Customer demands selected from six scenarios

  [0.01*k*, 0.10*k*], …, [0.70*k*, 0.90*k*]

  Vehicle capacity (*k*) varies

  160 for 50 customers
  200 for 199 customers

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Computational Results

- Computational Comparison

  Compare the results of EMIP+VRTR to the results of SPLITABU-DT

  EMIP+VRTR run on 30 instances of each scenario

  Use CPLEX 9.0 with Visual C++ (v6.0)
  1.7 GHz Pentium 4 with 512 MB of RAM

  SPLITABU-DT run five times on one instance of each scenario

  2.4 GHz Pentium 4 with 256 MB of RAM

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Computational Results

▶ 50 customers with vehicle capacity 160

		SPLITABU-DT				
Scenario	EMIP+VRTR	1	2	3	4	5
[0.01 – 0.1]	457.21	464.64	464.64	466.19	460.79	462.54
[0.1 – 0.3]	723.57	751.60	767.46	752.84	760.57	774.56
[0.1 – 0.5]	943.86	1013.00	1015.15	997.22	1007.13	1010.86
[0.1 – 0.9]	1408.34	1461.01	1473.29	1470.11	1443.84	1501.39
[0.3 – 0.7]	1408.68	1507.60	1491.92	1490.73	1487.02	1507.25
[0.7 – 0.9]	2056.01	2166.34	2174.81	2166.11	2170.43	2148.38

Median value from 30 instances

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Computational Results

▶ 199 customers with vehicle capacity 200

		SPLITABU-DT				
Scenario	EMIP+VRTR	1	2	3	4	5
[0.01 – 0.1]	1040.20	1051.61	1058.60	1060.41	1047.88	1062.87
[0.1 – 0.3]	2258.66	2383.90	2378.06	2386.29	2389.44	2383.11
[0.1 – 0.5]	3202.57	3298.49	3265.60	3247.32	3333.66	3277.32
[0.1 – 0.9]	5094.61	4737.47	4902.00	4893.66	4835.13	4900.89
[0.3 – 0.7]	5088.08	5184.25	5103.60	5001.46	5066.96	5157.95
[0.7 – 0.9]	7207.04	8065.69	7676.12	8007.30	8022.49	7951.60

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Computational Results

▶ Observations for EMIP+VRTR

Scenarios with small customer demands

Most of the savings attributable to VRTR

Greater emphasis is on *routing* the vehicles

50-node problem, scenario 1

E2 averages 0.61% savings over CW

After VRTR, 8.11% savings over E2

Scenarios with large customer demands

Most of the savings attributable to EMIP

Greater emphasis is on *packing* the vehicles

50-node problem, scenario 6

E2 averages 13.89% savings over CW

After VRTR, 0.60% savings over E2

Computational Results

▶ Statistical Test

Observation

If EMIP+VRTR and SPLITABU-DT are equally good with respect to solution quality, then SPLITABU-DT would beat the median EMIP+VRTR result about half the time

Test of Hypotheses

H_0: $p = 0.50$ (two methods equally good)

H_a: $p < 0.50$ (SPLITABU-DT performs worse than EMIP+VRTR)

Decision Rule

Reject H_0 when $(\hat{p}-0.5)/\sqrt{(0.5)(0.5)/36} \leq -2.33$ ($\hat{p} \leq 0.3058$)

If SPLITABU-DT performs better than the median value of EMIP+VRTR in fewer than (0.3058)(36) = 11 cases, then we reject H_0

Computational Results

▶ Statistical Test

Conclusion

Over 36 cases, number of times SPLITABU-DT solution is better than the median solution of EMIP+VRTR

Runs of SPLITABU-DT

1	2	3	4	5
5	4	5	6	4

Each count < 11, so we reject H_0 and conclude that SPLITABU-DT performs worse than EMIP+VRTR

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Computational Results

▶ Average Running Times

Set values of *NL* and *T* in EMIP+VRTR to equalize running times with SPLITABU-DT

50 customers with vehicle capacity 160

Scenario	EMIP+VRTR	SPLITABU-DT
[0.01 – 0.1]	1.9	4.8
[0.1 – 0.3]	3.4	21.8
[0.1 – 0.5]	14.7	28.2
[0.1 – 0.9]	55.4	60.8
[0.3 – 0.7]	47.9	48.6
[0.7 – 0.9]	135.4	106.0

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Computational Results

- Average Running Times

199 customers with vehicle capacity 200

Scenario	EMIP+VRTR	SPLITABU-DT
[0.01 – 0.1]	413.4	525.8
[0.1 – 0.3]	618.5	754.8
[0.1 – 0.5]	1,775.7	2,668.0
[0.1 – 0.9]	3,038.1	3,297.2
[0.3 – 0.7]	3,035.7	3,565.6
[0.7 – 0.9]	12,542.3	21,849.0

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Computational Results

- Five Benchmark Problems with Lower Bounds

Taken from Belenguer, Martinez, and Mota (2000)

50 customers

Customer demands selected from [0.10*k*, 0.90*k*]

Problem	Belenguer et al. Lower Bound	EMIP+VRTR	Time (s)	% Above Lower Bound
S51D4	1520.67	1586.5	201.74	4.33
S51D5	1272.86	1355.5	201.62	6.49
S51D6	2113.03	2197.8	301.90	4.01
S76D4	2011.64	2136.4	601.92	6.20
S101D5	2630.43	2846.2	645.99	8.20

Average 5.85%

May not be a tight bound

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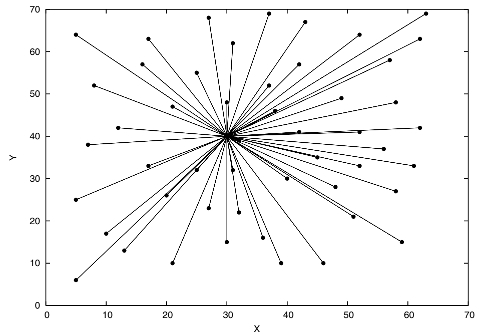
Computational Results

- Solutions to Problem S51D6

Clarke-and-Wright solution



Total distance is 2402.34 with 50 vehicles

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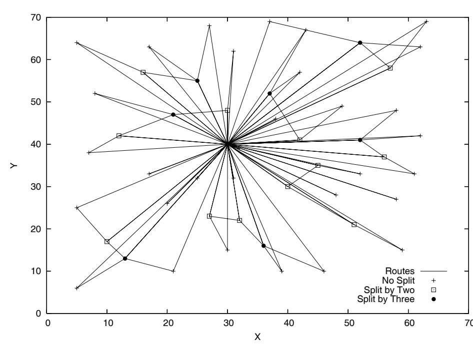
Computational Results

- Solutions to Problem S51D6

EMIP+VRTR solution



Total distance is 2197.8 with 42 vehicles

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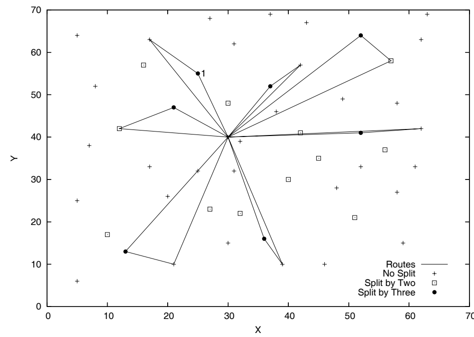
Computational Results

▶ Solutions to Problem S51D6

EMIP+VRTR solution



Seven customers (●) are split among three routes

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Computational Results

▶ New Test Problems

Generate 21 new test problems

8 to 288 customers

Vehicle capacity is 100

Customer demands selected from [0.7*k*, 0.9*k*]

Customers located in concentric circles around the depot

Visually estimate a near-optimal solution

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Computational Results

- New Test Problems

Apply EMIP+VRTR with no fine tuning

Problem	*n*	ES	EMIP+VRTR	Time (s)	% Above ES
SD1	8	228.28	228.28	0.7	0.00
SD2	16	708.28	714.40	54.4	0.86
SD3	16	430.61	430.61	67.3	0.00
SD4	24	631.06	631.06	400.0	0.00
SD5	32	1390.61	1408.12	402.7	1.26
SD6	32	831.21	831.21	408.3	0.00
SD7	40	3640.00	3714.40	403.2	2.04
SD8	48	5068.28	5200.00	404.1	2.60
SD9	48	2044.23	2059.84	404.3	0.76
SD10	64	2684.85	2749.11	400.0	2.39

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Computational Results

- New Test Problems

Problem	*n*	ES	EMIP+VRTR	Time (s)	% Above ES
SD11	80	13280.00	13612.12	400.1	2.50
SD12	80	7280.00	7399.06	408.3	1.64
SD13	96	10110.60	10367.06	404.5	2.54
SD14	120	10920.00	11023.00	5021.7	0.94
SD15	144	15151.10	15271.77	5042.3	0.80
SD16	144	3381.32	3449.05	5014.7	2.00
SD17	160	26560.00	26665.76	5023.6	0.40
SD18	160	14380.30	14546.58	5028.6	1.16
SD19	192	20191.20	20559.21	5034.2	1.82
SD20	240	39840.00	40408.22	5053.0	1.43
SD21	288	11271.10	11491.67	5051.0	1.96
Average					1.29

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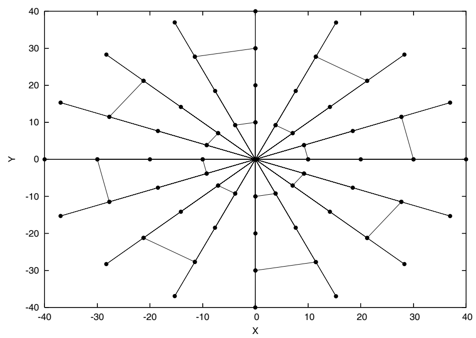
Computational Results

▶ Solutions to Problem SD10

Visually estimated solution

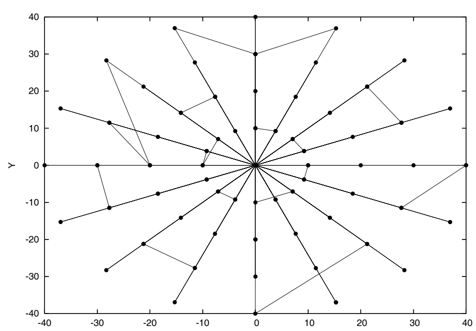


Total distance is 2684.85 with 48 vehicles and 64 customers

Computational Results

▶ Solutions to Problem SD10

EMIP+VRTR solution



Total distance is 2749.11 with 49 vehicles

Conclusions

- New Effective Heuristic
  - Combined a mixed integer program with a record-to-record travel algorithm
    - EMIP+VRTR has only two parameters
    - Applied to six benchmark problems
      - Outperformed tabu search
    - Performed well on five other problems
- New Problem Set
  - Developed 21 problems with 8 to 288 customers
    - Visually estimate near-optimal solutions
    - EMIP+VRTR produced high-quality results

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