The Minimum Label Spanning Tree Problem: Illustrating the Power and Flexibility of Genetic Algorithms

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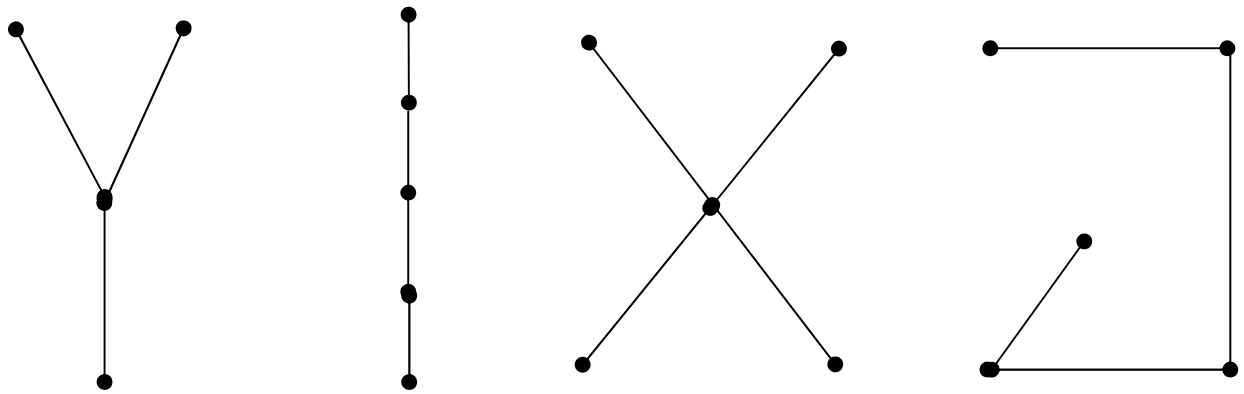
Outline of Lecture

- 10 - Minute Introduction to Graph Theory and Complexity
- Introduction to the MLST Problem
- A GA for the MLST Problem
- Four Modified Versions of the Benchmark Heuristic
- A Modified Genetic Algorithm
- Results and Conclusions

Defining Trees

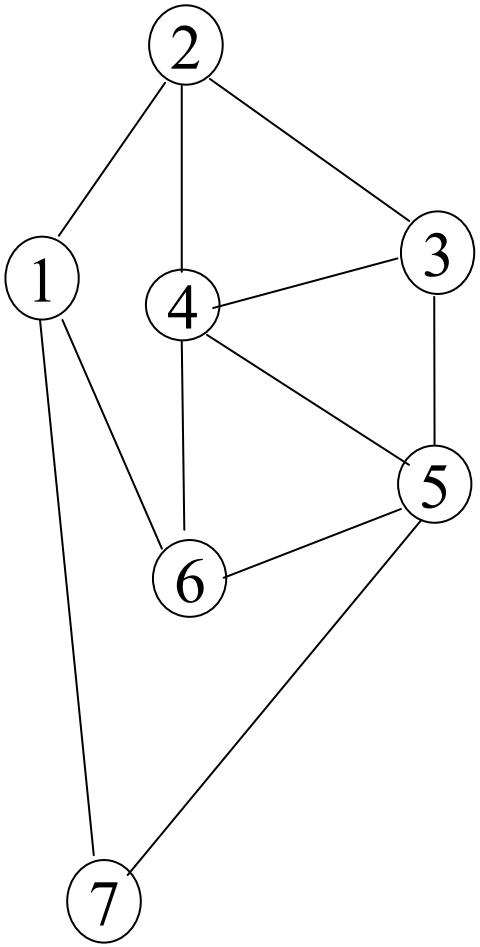
- A graph with no cycles is *acyclic*
- A *tree* is a connected acyclic graph



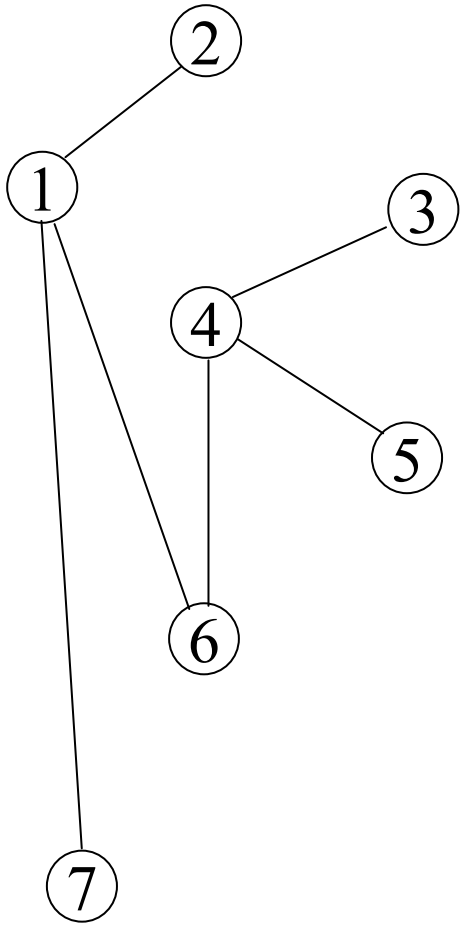
Some examples of trees

- A *spanning tree* of a graph G contains all the nodes of G

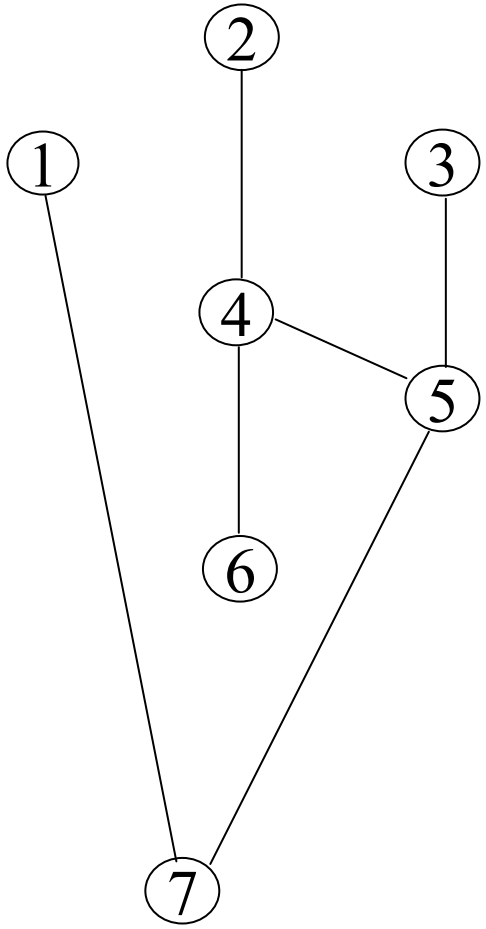
Spanning Trees



Graph G



A spanning tree of G



Another spanning tree of G

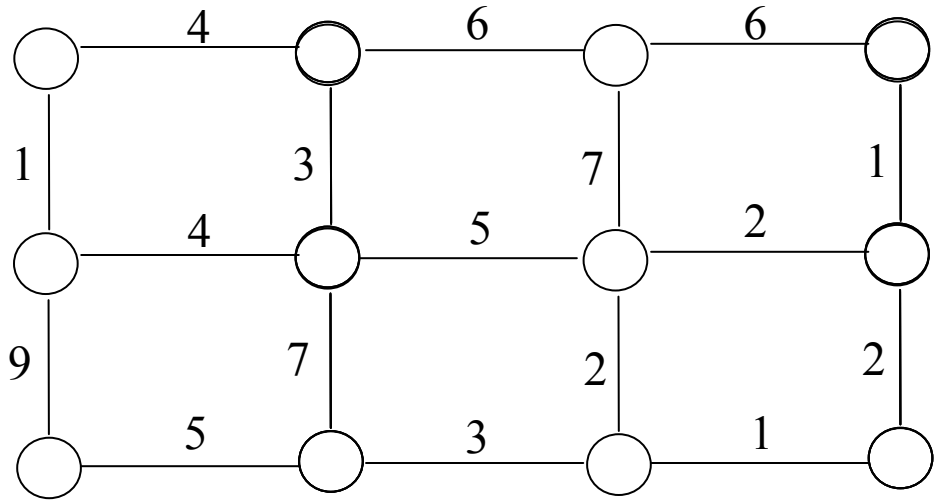
Minimal Spanning Trees

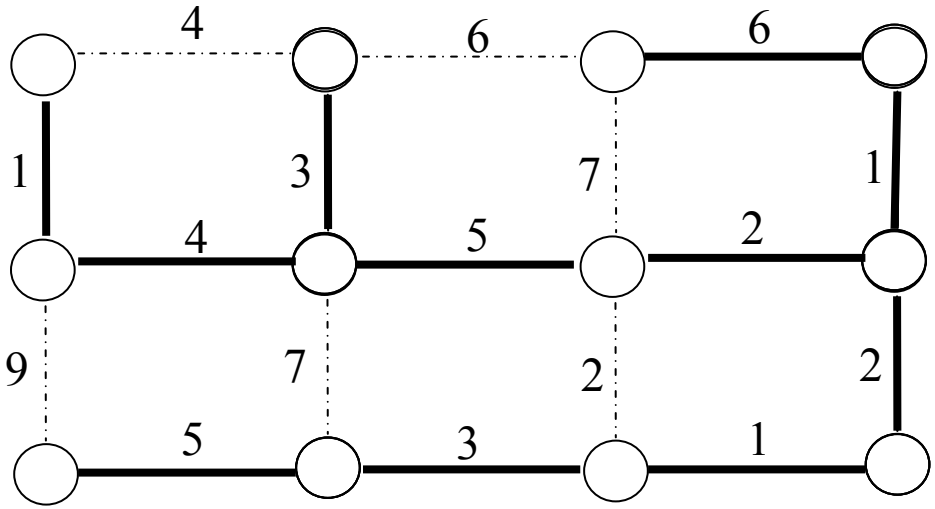
A network problem for which there is a simple solution method is the selection of a minimum spanning tree from an undirected network over *n* cities

- The cost of installing a communication link between cities i and j is $c_{ij} = c_{ji} \geq 0$
- Each city must be connected, directly or indirectly, to all others, and this is to be done at minimum total cost
- Attention can be confined to trees, because if the network contains a cycle, removing one link of the cycle leaves the network connected and reduces cost

A Minimal Spanning Tree

Original Network

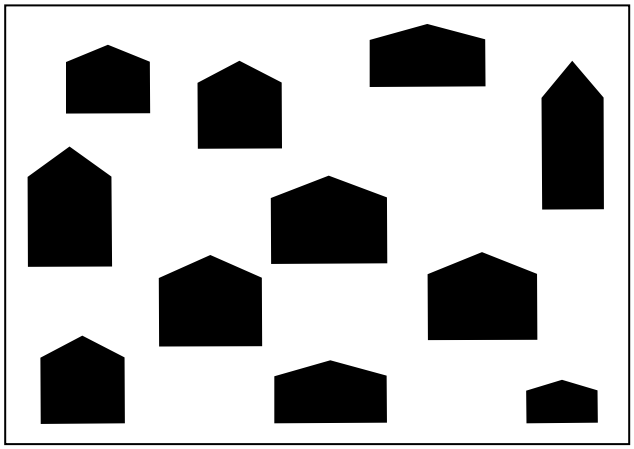




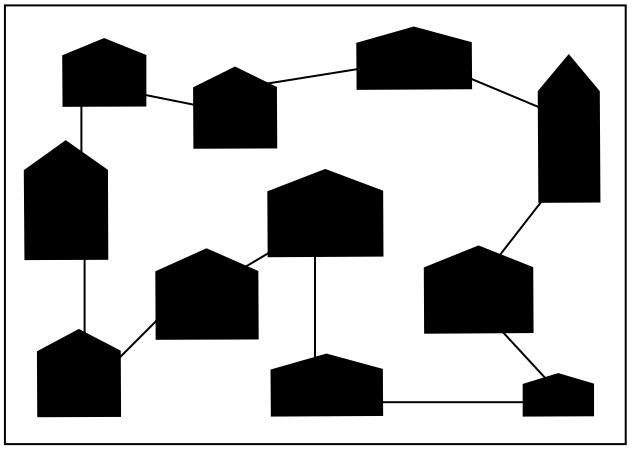
Minimum Spanning Tree

The Traveling Salesman Problem

- Imagine a suburban college campus with 140 separate buildings scattered over 800 acres of land
- To promote safety, a security guard must inspect each building every evening
- The goal is to sequence the 140 buildings so that the total time (travel time plus inspection time) is minimized
- This is an example of the well-known TSP



Original problem



Possible solution

Analysis of Algorithms

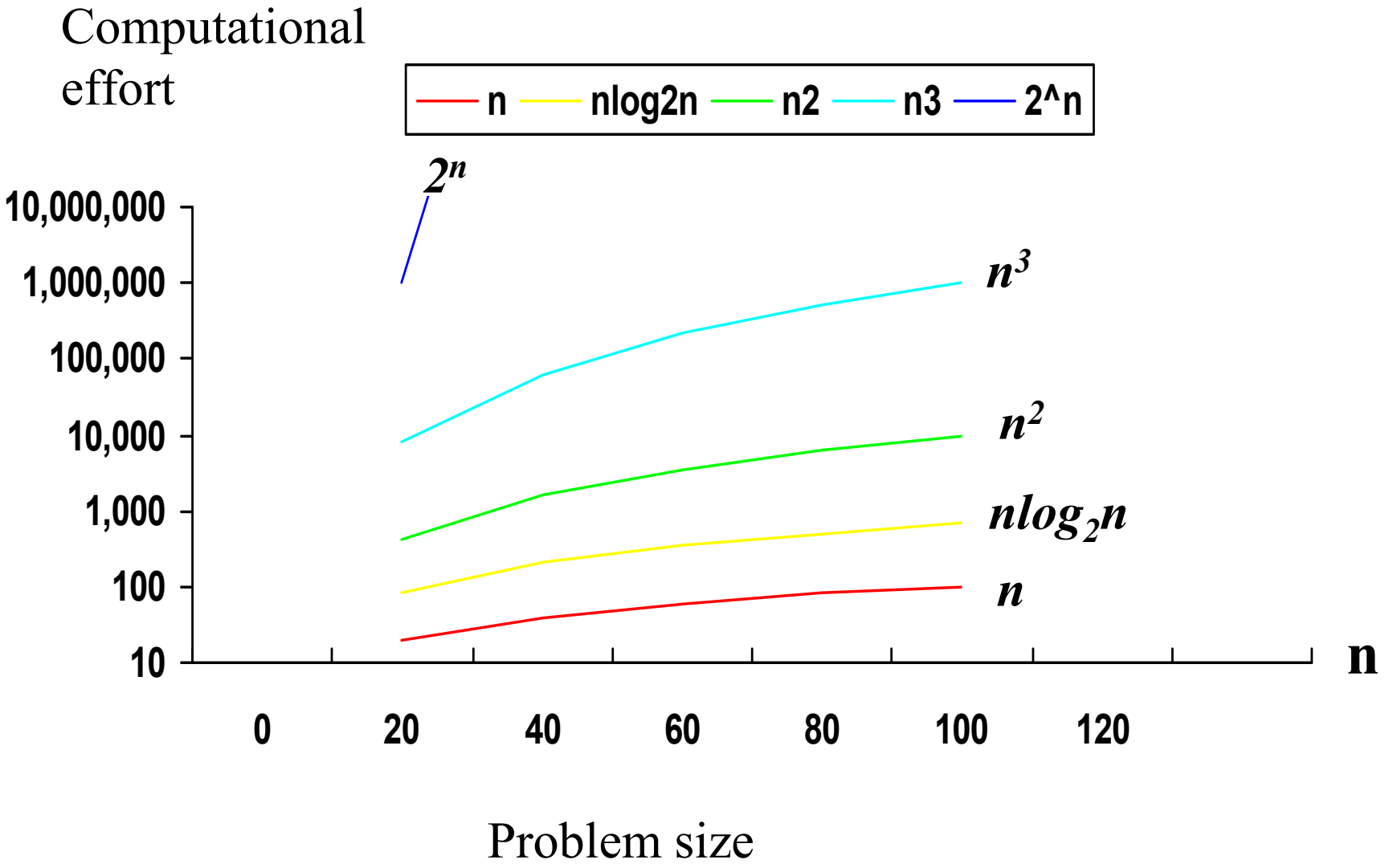
- Definitions
  - Algorithm- method for solving a class of problems on a computer
  - Optimal algorithm –verifiable optimal solution
  - Heuristic algorithm –feasible solution
- Performance Measures
  - Number of basic computations / Running time
  - Computational effort

    --- Problem size

    --- Player one

    --- Player two

Computational Effort as a Function of Problem Size



Good vs. Bad Algorithms

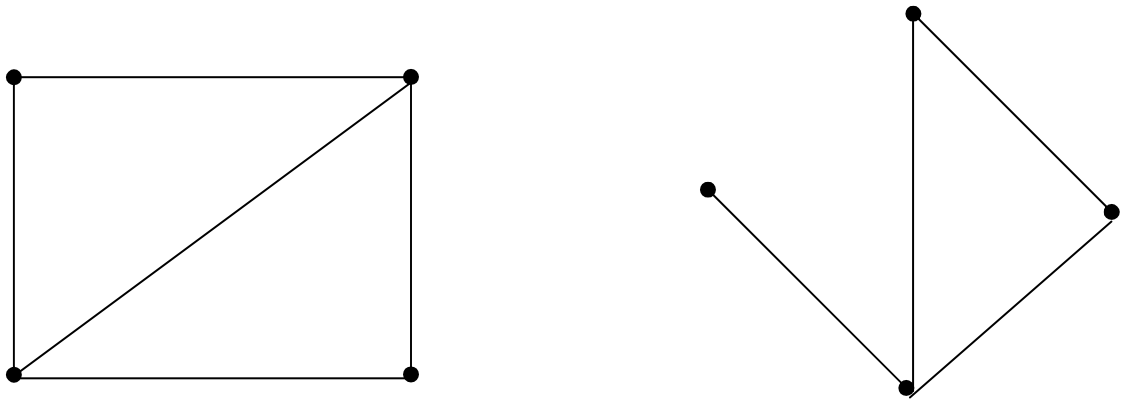
- Terminology
  - Researchers have emphasized the importance of finding polynomial time algorithms, by referring to all such polynomial algorithms as inherently good
  - Algorithms that are *not* polynomially bounded, are labeled inherently bad
- Good Optimal Algorithms Exist for these Problems
  - Transportation problem
  - Minimal spanning tree problem
  - Shortest path problem
  - Linear programming

High Quality Heuristic Algorithms

- Good Optimal Algorithms Don't Exist for these Problems
  - Traveling salesman problem (TSP)
  - Minimum label spanning tree problem (MLST)
- Why Focus on Heuristic Algorithms?
  - For the above problems, optimal algorithms are not practical
  - Efficient, near optimal heuristics are needed to solve real-world problems
  - The key is to find fast, high-quality heuristic algorithms

One More Concept from Graph Theory

- A disconnected graph consists of two or more connected graphs
- Each of these connected subgraphs is called a component



A disconnected graph with two components

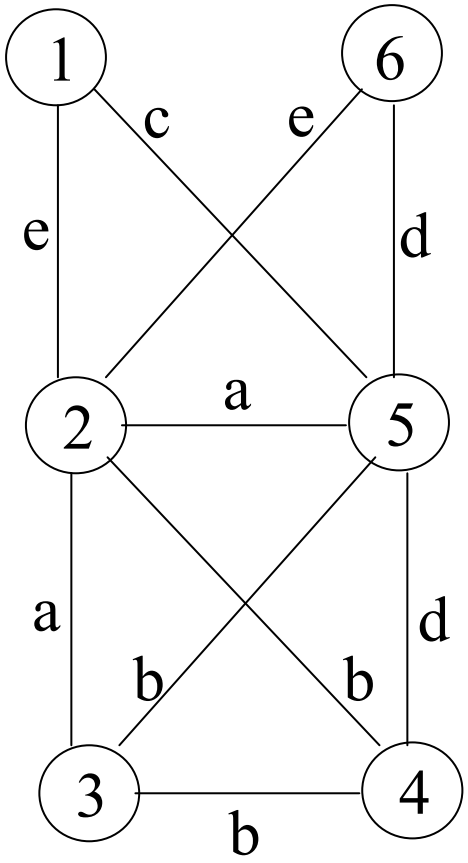
Introduction

- The Minimum Label Spanning Tree (MLST) Problem
  - Communications network design
  - Edges may be of different types or media (e.g., fiber optics, cable, microwave, telephone lines, etc.)
  - Each edge type is denoted by a unique letter or color
  - Construct a spanning tree that minimizes the number of colors

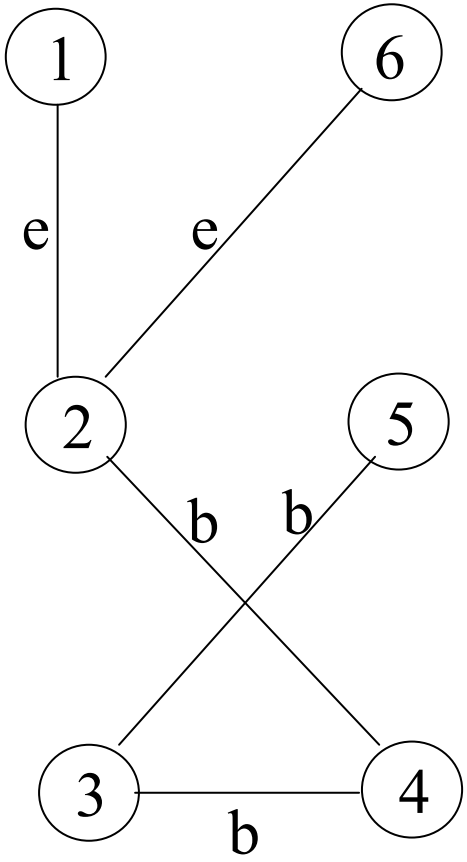
Introduction

- A Small Example

Input



Solution



Literature Review

- Where did we start?
  - Proposed by Chang & Leu (1997)
  - The MLST Problem is NP-hard
  - Several heuristics had been proposed
  - One of these, MVCA (maximum vertex covering algorithm), was very fast and effective
  - Worst-case bounds for MVCA had been obtained

Literature Review

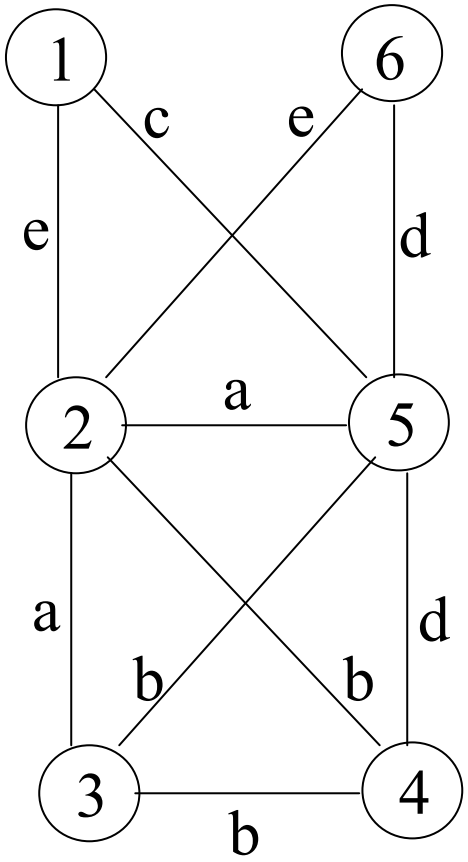
- An optimal algorithm (using backtrack search) had been proposed
- On small problems, MVCA consistently obtained nearly optimal solutions
- A description of MVCA follows

Description of MVCA

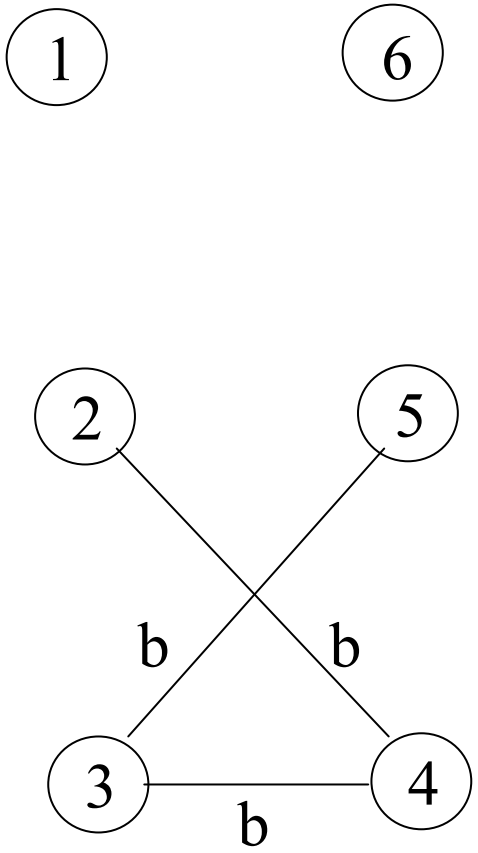
0. Input: $G\,(V, E, L)$.
1. Let $C \leftarrow \{\,\}$ be the set of used labels.
2. repeat
3. Let H be the subgraph of G restricted to V and edges with labels from C.
4. for all $i \in L - C$ do
5. Determine the number of connected components when inserting all edges with label i in H.
6. end for
7. Choose label i with the smallest resulting number of components and do: $C \leftarrow C \cup \{i\}$.
8. Until H is connected.

How MVCA Works

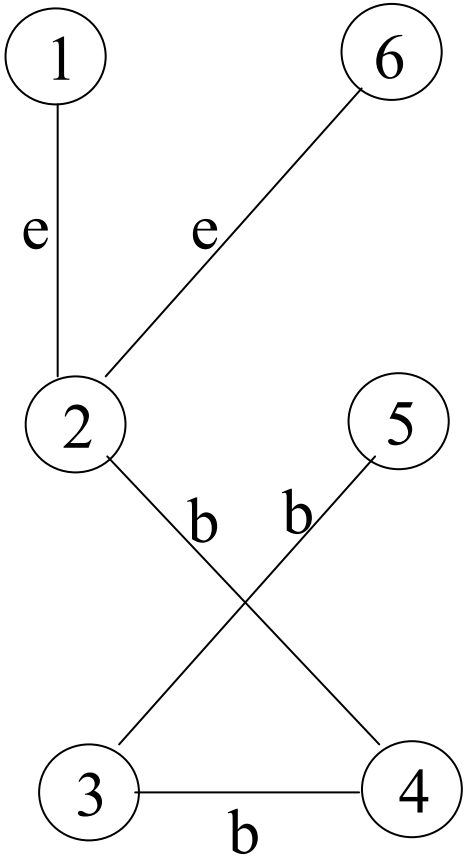
Input



Intermediate Solution



Solution



Worst-Case Results

1. Krumke, Wirth (1998): $\frac{\text{MVCA}}{\text{OPT}} \le 1 + 2\ln n$

2. Wan, Chen, Xu (2002): $\frac{\text{MVCA}}{\text{OPT}} \le 1 + \ln(n-1)$

3. Xiong, Golden, Wasil (2005): $\frac{\text{MVCA}}{\text{OPT}} \le H_b = \sum_{i=1}^{b} \frac{1}{i} < 1 + \ln b$

where b = max label frequency, and

H_b= bth harmonic number

Some Observations

- The Xiong, Golden, Wasil worst-case bound is tight
- Unlike the MST, where we focus on the edges, here it makes sense to focus on the labels or colors
- Next, we present a genetic algorithm (GA) for the MLST problem

Genetic Algorithm: Overview

- Randomly choose p solutions to serve as the initial population
- Suppose s [0], s [1], … , s [p – 1] are the individuals (solutions) in generation 0
- Build generation k from generation k – 1 as below

  For each j between 0 and p – 1, do:

  t [j] = crossover { s [j], s [(j + k) mod p] }

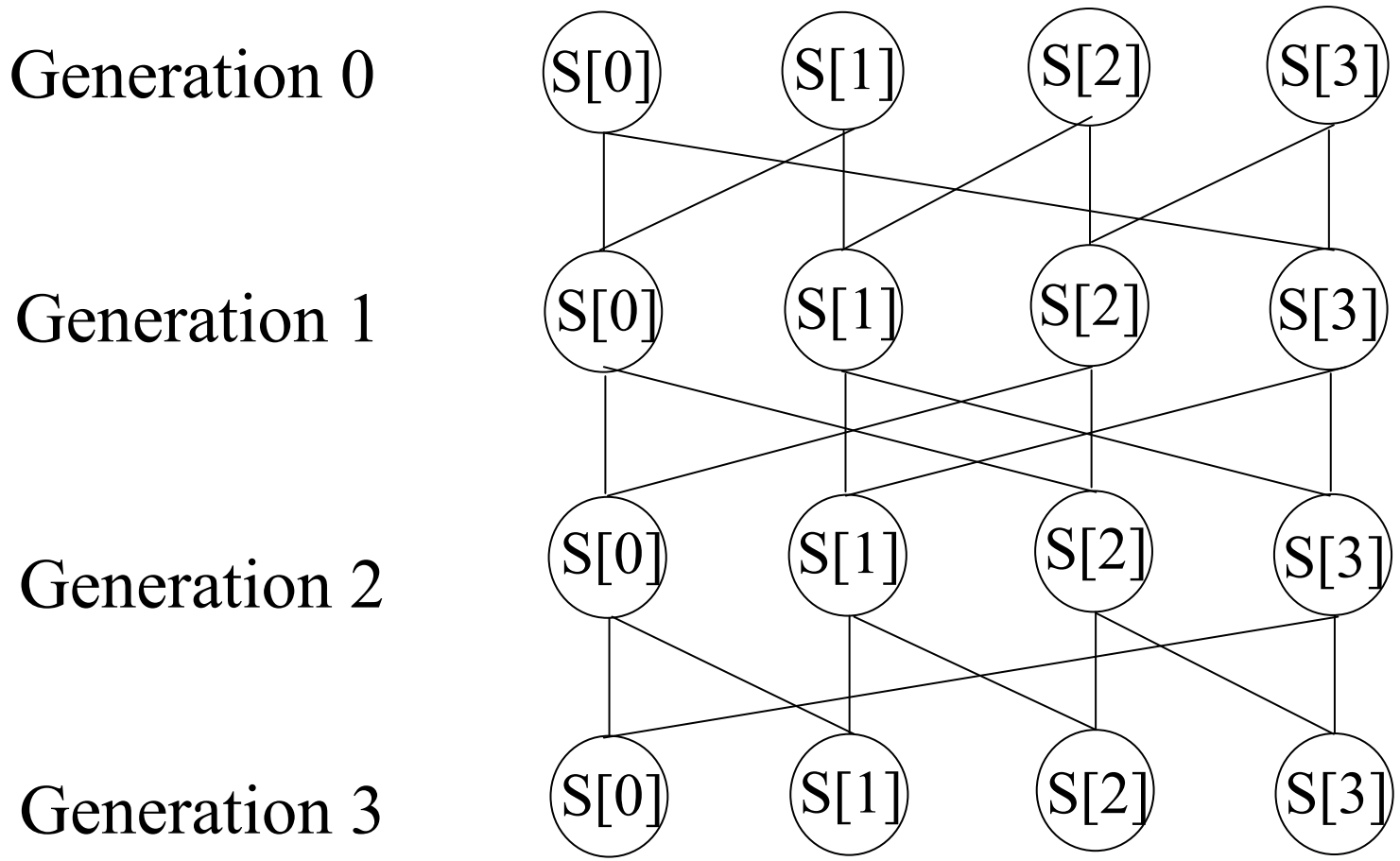
  t [j] = mutation { t [j] }

  s [j] = the better solution of s [j] and t [j]

  End For

- Run until generation p – 1 and output the best solution from the final generation

Crossover Schematic (p = 4)



Crossover

- Given two solutions s [1] and s [2], find the child T = crossover { s [1], s [2] }
- Define each solution by its labels or colors
- Description of Crossover

    a. Let S = s [1] $\cup$ s [2] and T be the empty set

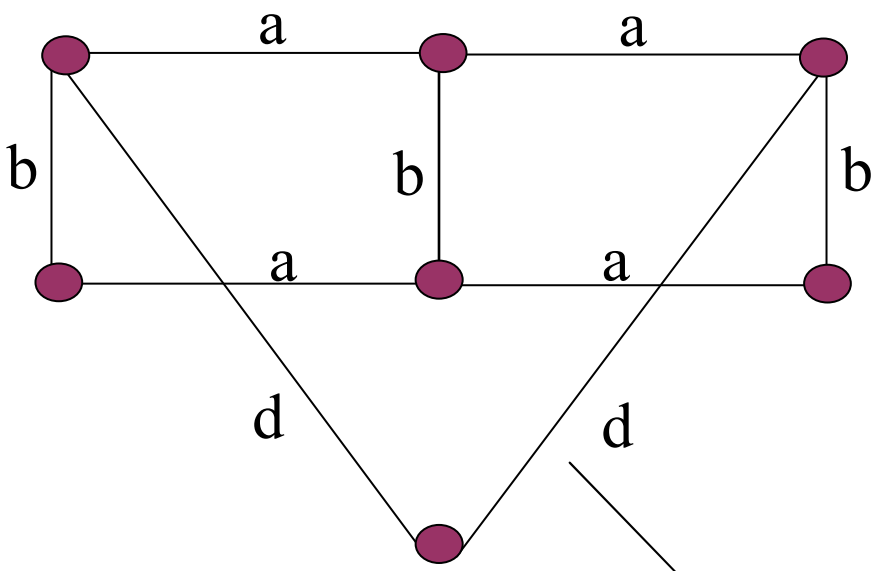
    b. Sort S in decreasing order of the frequency of labels in G

    c. Add labels of S, from the first to the last, to T until T represents a feasible solution

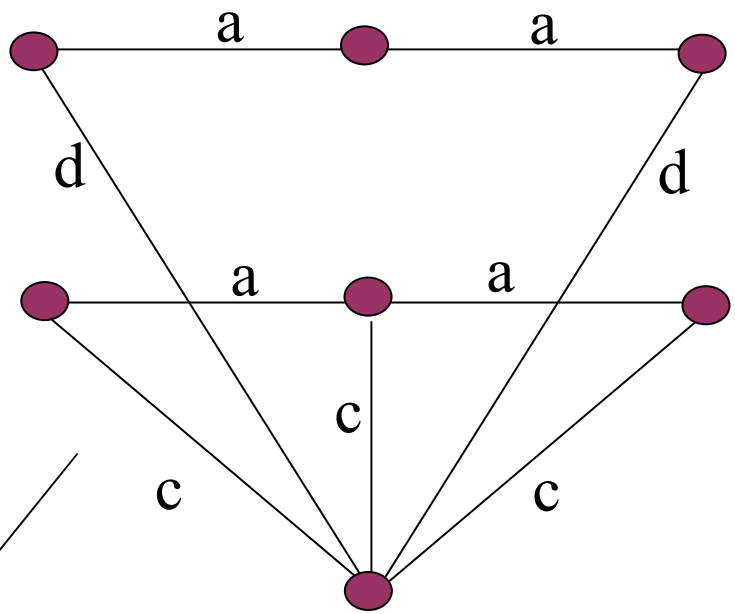
    d. Output T

An Example of Crossover

s [1] = { a, b, d }



s [2] = { a, c, d }



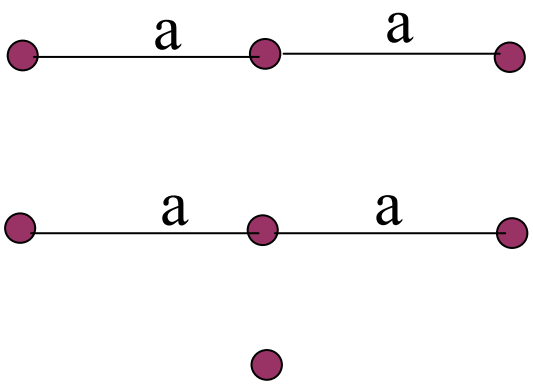
T = { }

S = { a, b, c, d }

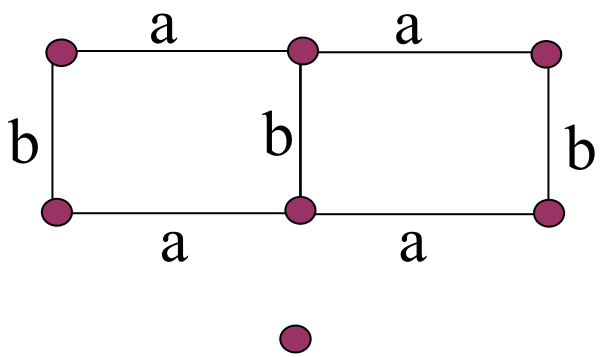
Ordering: a, b, c, d

An Example of Crossover

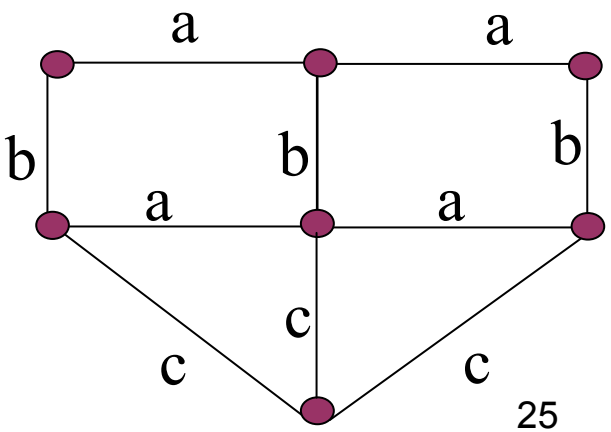
T = { a }



T = { a, b }



T = { a, b, c }



Mutation

- Given a solution S, find a mutation T
- Description of Mutation

  a. Randomly select c not in S and let T = S $\cup$ c

  b. Sort T in decreasing order of the frequency of the labels in G

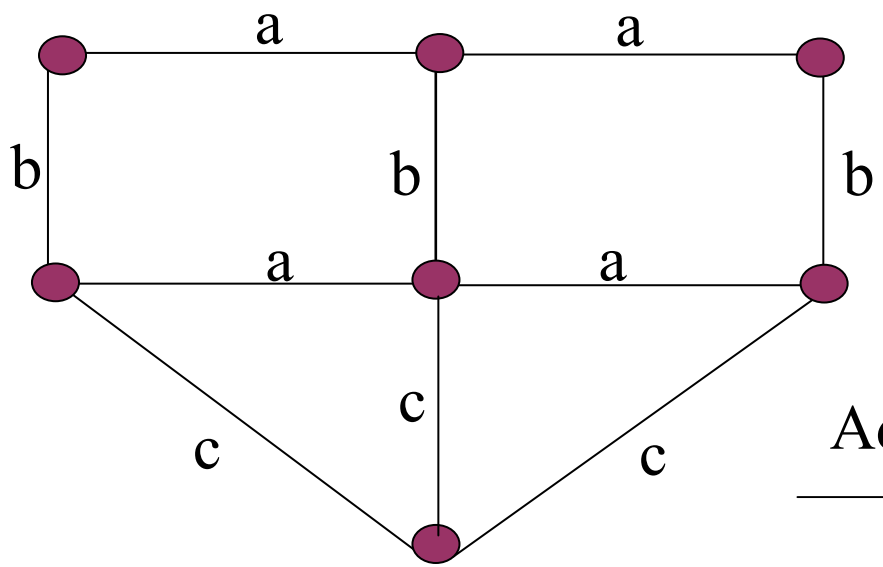
  c. From the last label on the above list to the first, try to remove one label from T and keep T as a feasible solution

  d. Repeat the above step until no labels can be removed

  e. Output T

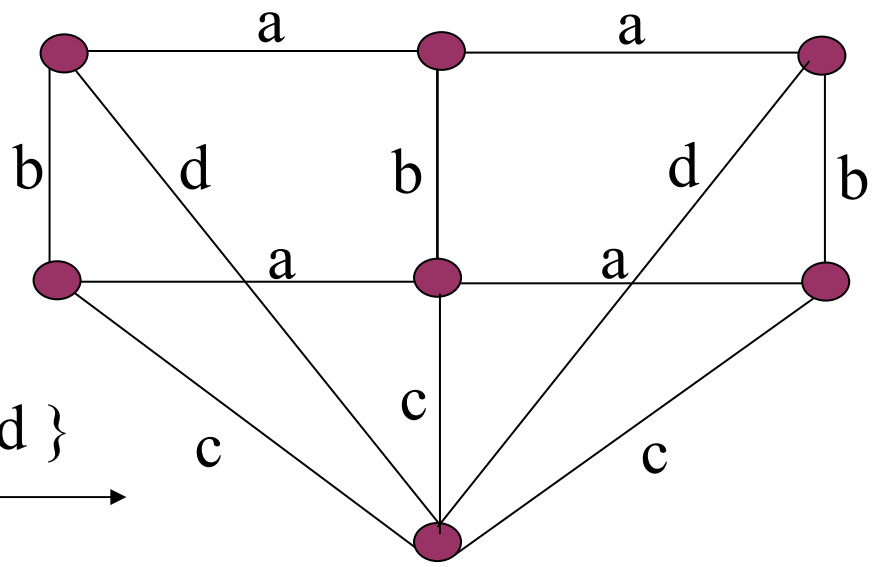
An Example of Mutation

S = { a, b, c }



Add { d }

S = { a, b, c, d }



Ordering: a, b, c, d

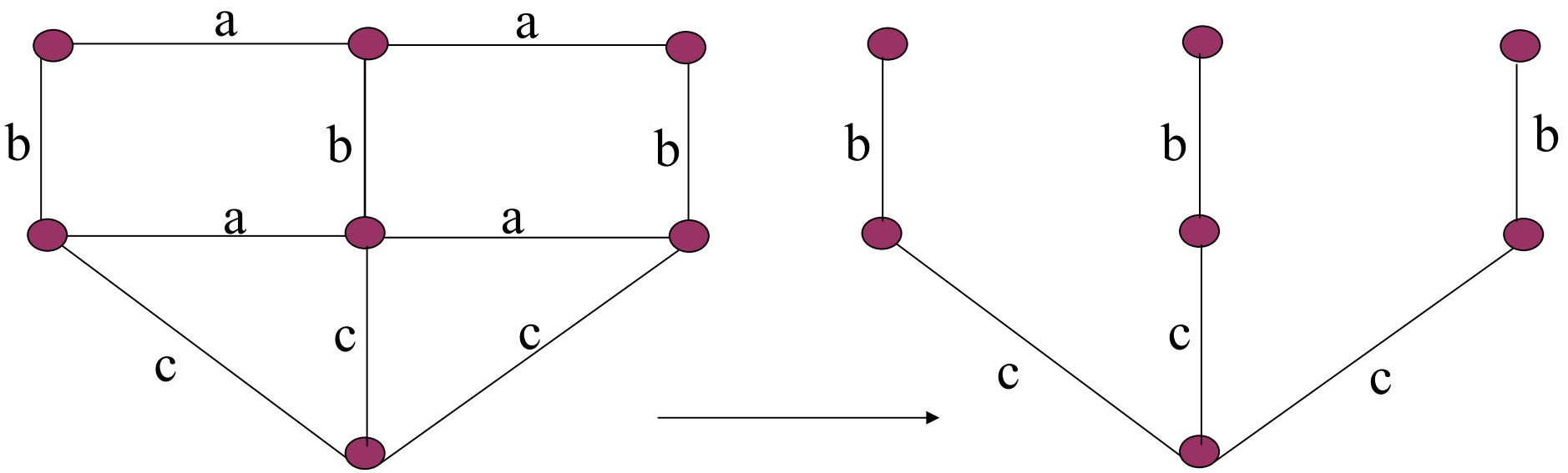
An Example of Mutation

Remove { d }

S = { a, b, c }

Remove { a }

S = { b, c }



T = { b, c }

Three Modified Versions of MVCA

- Voss et al. (2005) implement MVCA using their pilot method
- The results were quite time-consuming
- We added a parameter (%) to improve the results
- Three modified versions of MVCA
  - MVCA1 uses % = 100
  - MVCA2 uses % = 10
  - MVCA3 uses % = 30

MVCA1

- We try each label in L (% = 100) as the first or pilot label
- Run MVCA to determine the remaining labels
- We output the best solution of the l solutions obtained
- For large l, we expect MVCA1 to be very slow

MVCA2 (and MVCA3)

- We sort all labels by their frequencies in G, from highest to lowest
- We select each of the top 10% (% = 10) of the labels to serve as the pilot label
- Run MVCA to determine the remaining labels
- We output the best solution of the $l/10$ solutions obtained
- MVCA2 will be faster than MVCA1, but not as effective
- MVCA3 selects the top 30% (% = 30) and examines $3l/10$ solutions
- MVCA3 is a compromise approach

A Randomized Version of MVCA (RMVCA)

- We follow MVCA in spirit
- At each step, we consider the three most promising labels as candidates
- We select one of the three labels
  - The best label is selected with prob. = 0.4
  - The second best label is selected with prob. = 0.3
  - The third best label is selected with prob. = 0.3
- We run RMVCA 50 times for each instance and output the best solution

A Modified Genetic Algorithm (MGA)

- We modify the crossover operation described earlier
- We take the union of the parents (i.e., $S = S_1 \cup S_2$) as before
- Next, apply MVCA to the subgraph of G with label set S ($S \subseteq L$), node set V, and the edge set E' ($E' \subseteq E$) associated with S
- The new crossover operation is more time-consuming than the old one
- The mutation operation remains as before

Computational Results

- 48 combinations: n = 50 to 200 / l = 12 to 250 / density = 0.2, 0.5, 0.8
- 20 sample graphs for each combination
- The average number of labels is compared

Performance Comparison

	MVCA	GA	MGA	MVCA1	MVCA2	MVCA3	RMVCA	Row Total
MVCA	-	3	0	0	0	0	0	3
GA	30	-	0	1	9	4	2	46
MGA	33	30	-	10	20	16	16	125
MVCA1	35	30	10	-	24	20	18	137
MVCA2	31	20	5	0	-	0	6	62
MVCA3	34	27	8	0	23	-	11	103
RMVCA	35	30	7	3	20	10	-	105

Summary of computational results with respect to accuracy for seven heuristics on 48 cases. The entry (i, j) represents the number of cases heuristic i generates a solution that is better than the solution generated by heuristic j.

Running Times

	MVCA	GA	MGA	MVCA1	MVCA2	MVCA3	RMVCA
n = 100, l = 125, d = 0.2	0.05	1.80	7.50	8.25	0.80	2.30	3.85
n = 150, l = 150, d = 0.5	0.10	1.85	4.90	11.85	1.15	3.45	4.75
n = 150, l = 150, d = 0.2	0.15	3.45	13.55	21.95	2.15	6.35	8.45
n = 150, l = 187, d = 0.5	0.15	2.20	6.70	21.70	2.00	6.15	7.50
n = 150, l = 187, d = 0.2	0.20	3.95	17.55	39.35	3.60	11.20	11.90
n = 200, l = 100, d = 0.2	0.15	3.75	11.40	11.25	1.15	3.35	6.75
n = 200, l = 200, d = 0.8	0.25	2.45	5.80	26.70	2.70	8.00	8.65
n = 200, l = 200, d = 0.5	0.25	3.45	10.15	38.65	3.90	10.15	12.00
n = 200, l = 200, d = 0.2	0.35	6.20	26.65	68.25	6.85	20.35	20.55
n = 200, l = 250, d = 0.8	0.30	3.05	7.55	52.25	5.25	15.35	12.95
n = 200, l = 250, d = 0.5	0.30	3.95	12.60	69.90	6.80	20.35	16.70
n = 200, l = 250, d = 0.2	0.50	6.90	33.15	124.35	12.10	35.80	28.80
Average running time	0.23	3.58	13.13	41.20	4.04	11.90	11.90

Running times for 12 demanding cases (in seconds).

One Final Experiment for Small Graphs

- 240 instances for n = 20 to 50 are solved by the seven heuristics
- Backtrack search solves each instance to optimality
- The seven heuristics are compared based on how often each obtains an optimal solution

Procedure	OPT	MVCA	GA	MGA	MVCA1	MVCA2	MVCA3	RMVCA
% optimal	100.00	75.42	96.67	99.58	95.42	87.08	93.75	97.50

Conclusions

- We presented three modified (deterministic) versions of MVCA, a randomized version of MVCA, and a modified GA
- All five of the modified procedures generated better results than MVCA and GA, but were more time-consuming
- With respect to running time and performance, MGA seems to be the best

Related Work

- The Label-Constrained Minimum Spanning Tree (LCMST) Problem
  - We show the LCMST problem is NP-hard
  - We introduce two local search methods
  - We present an effective genetic algorithm
  - We formulate the LCMST as a MIP and solve for small cases
  - We introduce a dual problem