The Euclidean Non-uniform Steiner Tree Problem

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Introduction

- The Steiner Tree Problem (STP)

  - We are given a set of terminal nodes
  - We want to find edges to connect these nodes at minimum cost
  - Additional nodes (Steiner nodes) may be added to the terminal nodes in order to reduce overall cost
  - Applications: laying cable networks, printed circuits, routing of transmission lines, design of communication networks

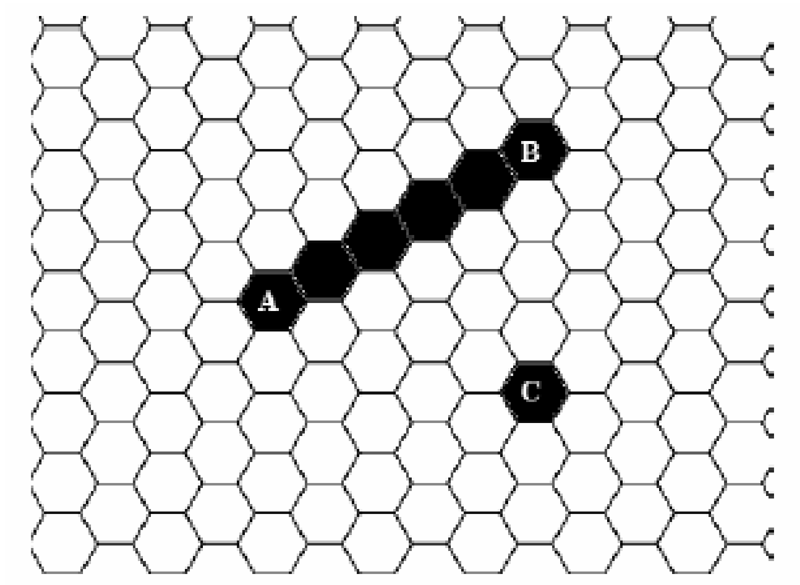
Introduction

- The Euclidean Non-uniform Steiner Tree Problem
  - Many STP variants have been studied
  - They have been shown to be NP-hard
  - In the Euclidean Non-uniform STP (ENSTP), the cost of an edge depends on its location as well as its distance
  - Certain streets are more expensive to rip apart and re-build than others
  - The ENSTP was introduced by Coulston (2003)

Description of New Variant

- Grid Structure
  - Use a hexagonal tiling of the plane
  - Each tile or cell has six adjacent neighbors
  - The distances between centers of adjacent cells are equal
  - Each cell has a cost and it may contain at most one of the nodes in the graph
  - Two nodes can be connected directly only if a straight line of cells can be drawn between the cells containing the two nodes

Hexagonal Grid



- A and B are directly connected, A and C are not

Determining Cost

- When an edge connects cells A and B, the cost of the edge is the sum of the costs associated with all the intermediate cells
- The cost of the tree includes the edge costs plus the costs corresponding to each node (terminal and Steiner) in the tree
- We may charge an additional fee for each Steiner node
- In some applications, Steiner nodes may require the installation of additional hardware

Genetic Algorithm for the ENSTP

1. Input: terminal node set, grid cost structure

2. Generate Initial Population randomly

   Repeat Steps 3 to 7 for TMAX iterations

   3. Find Fitness (= cost) of each individual

   4. Select parents via Queen Bee Selection

   5. Apply Spatial-Horizontal Crossover to parents to produce offspring

   6. Mutation 1 – add Steiner nodes at edge crossings

   7. Mutation 2 – randomly move Steiner nodes

Initial Population

- We use a population size of 40
- Each individual is a fixed-length chromosome of Steiner node locations (coordinates)
- Checks are performed to ensure that Steiner node locations and terminal node locations do not coincide
- Otherwise, locations are randomly selected

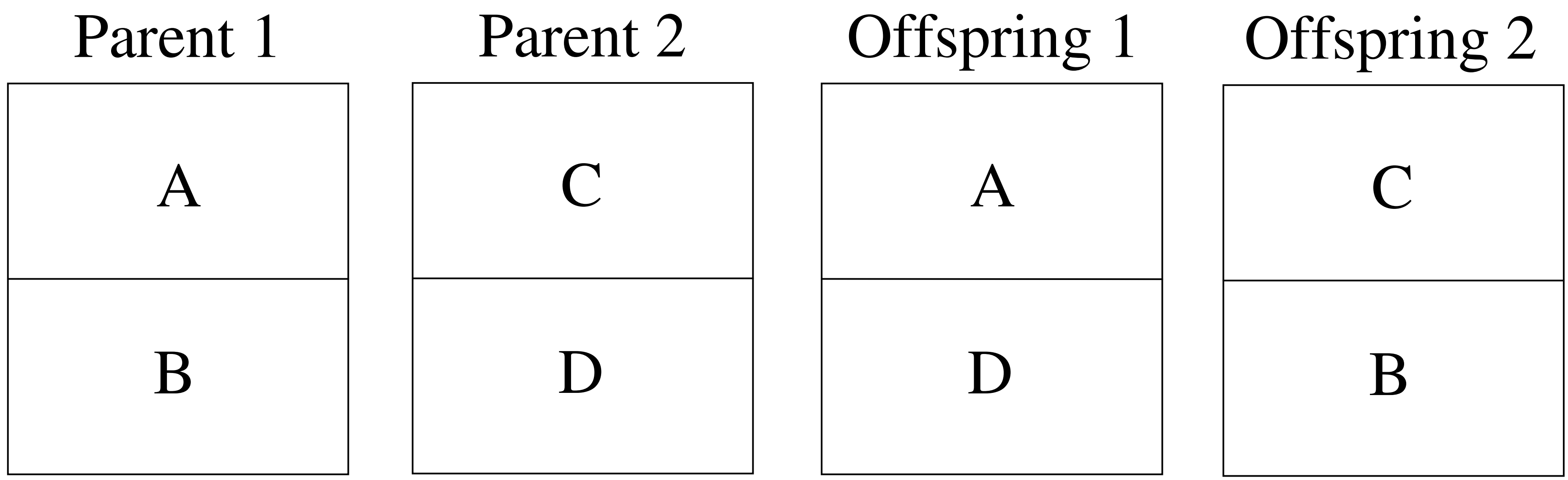
Fitness

- For each individual, form the complete graph over the terminal nodes and Steiner nodes
- Find a MST solution
- Remove degree-1 Steiner nodes and their incident edges
- Fitness is the cost of the resulting Steiner tree

Queen Bee Selection

- The fittest individual (the Queen Bee) mates with each other member of the population to produce two offspring
- This adds 78 offspring
- The 40 fittest of the 40 parents plus 78 offspring are chosen to survive to the next generation

Spatial-Horizontal Crossover



- A, B, C, and D are sets of Steiner nodes
- A and C can have some common nodes, as can B and D
- Terminal nodes are not shown above

More About the GA

- Mutation operations add and move Steiner nodes
- The ENSTP can be converted to a Steiner problem in graphs
- We have implemented the Dreyfus & Wagner algorithm to solve the Steiner problem in graphs optimally
- In preliminary computational experiments, we compare the GA solution to the optimal solution in small and medium-size problems

Preliminary Computational Results

Problem	Optimal	GA (best)	% Gap	GA (average)
1	11.138	11.138	0.0	11.230
2	10.001	10.102	1.0	10.284
3	4.806	4.811	0.1	4.819
4	4.158	4.206	1.2	4.236
5	5.605	5.605	0.0	5.605

- The GA was run 10 times on each problem

Computational Notes

- Grid size: 21 by 17
- 7 or 10 terminal nodes
- Coded in MATLAB, run on a 3.0 GHz machine with 1.5 GB of RAM
- Each GA run required about a minute
- The optimal algorithm also required a minute per problem

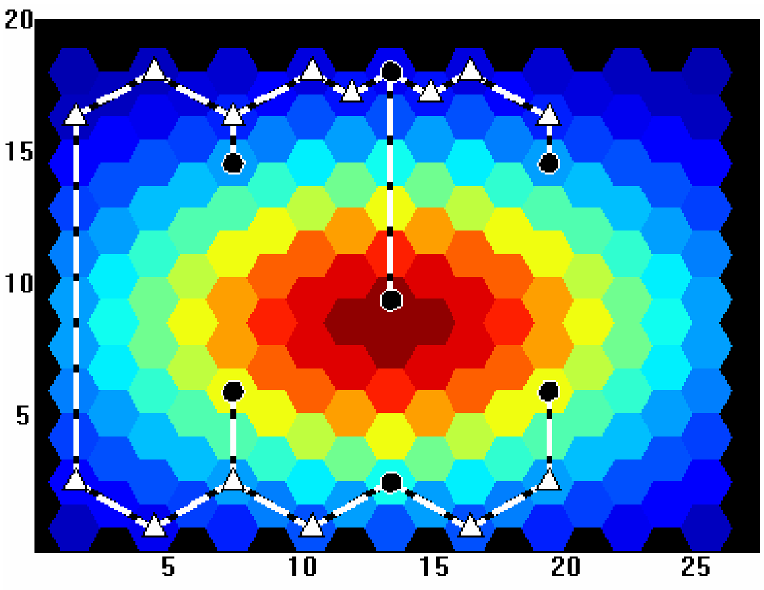
Comparison of Solutions

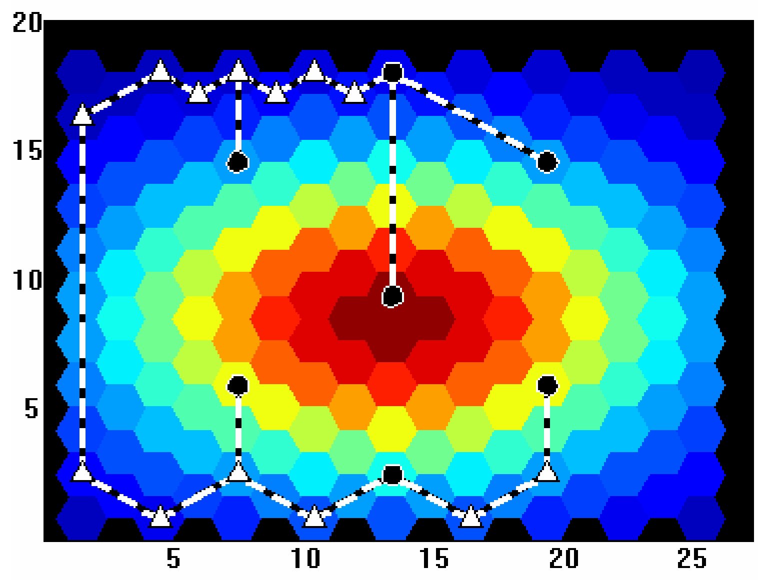
Optimal Solution

Cost = 10.001

Genetic Algorithm Solution

Cost = 10.102





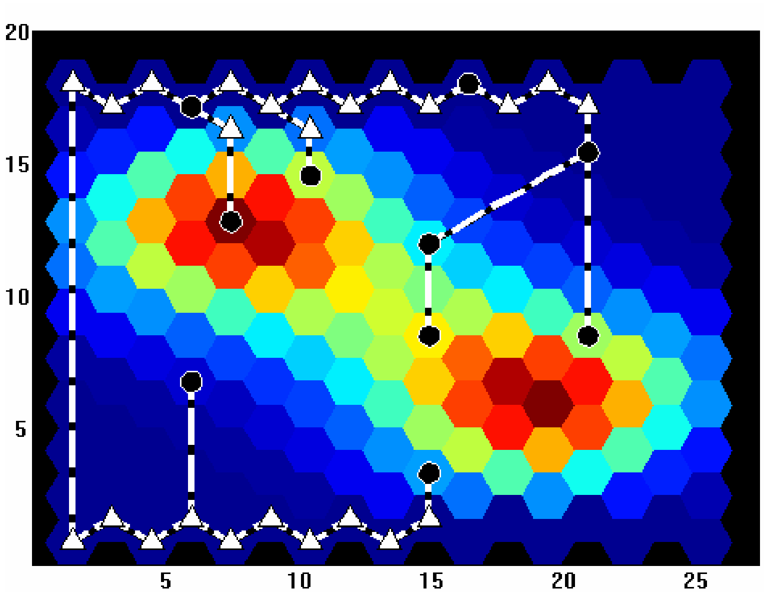
Terminal Nodes

Steiner Nodes

Comparison of Solutions

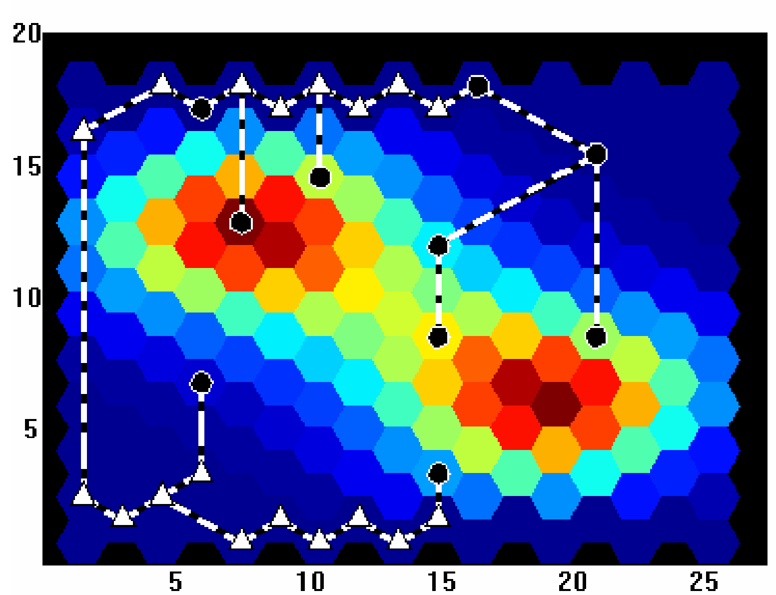
Optimal Solution

Cost = 4.806



Genetic Algorithm Solution

Cost = 4.811



● **Terminal Nodes**

△ **Steiner Nodes**

Two Medium-Size Problems

Problem	Optimal	GA	% Gap	D&C GA	% Gap
1	26.606	29.903	12.4	27.4673	3.2
2	31.3096	34.318	9.6	31.628	1.0

- The GA required 25 minutes per problem
- The optimal algorithm required 12 days per problem
- Divide & conquer GA required 2 minutes per problem
- For the above problems, grid size is 35 by 35 and there are 15 terminal nodes

Conclusions

- There have been many recent applications of heuristic approaches to network design problems
  - Simpler to implement than exact procedures
  - Heuristic approaches are more advanced than before
  - Combining metaheuristics such as GAs with local search is a powerful tool
  - Average processor speed of PCs continues to increase

Conclusions

- We have provided some guidelines, especially, with respect to GAs
- We have described four successful applications of GAs to network design problems
- In the process, we have tried to illustrate the simplicity and flexibility of the GA approach