Metaheuristics for the New Millennium

by

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Focus of Paper

- Introduce two new metaheuristics
  - Search space smoothing
  - Demon algorithms
- Discuss several variants of each
- Illustrate performance using the traveling salesman problem
- Explain why the new metaheuristics work so well
- Point out connections to “old” metaheuristics
  - Simulated annealing
  - Genetic algorithms

Search Space Smoothing: Overview

- Developed by Gu and Huang in 1994
- Applied to a small number of small TSP instances
- In smoothing, we first normalize distances so that they fall between 0 and 1
- At each iteration, the original distances are transformed by the smoothing transformation. Two-opt is applied. The degree of transformation decreases from one iteration to the next, until the original distances re-emerge.

Search Space Smoothing Algorithm

Step 1 Let d_{ij} = the distance from city i to city j.

Normalize all distances so that $0 \le d_{ij} \le 1$.

Specify the schedule for the smoothing factor as
$\alpha = 6(\text{begin}),3,2,1(\text{end})$.

Step 2 Generate a random starting tour.

Step 3 Set α equal to the next value in the smoothing schedule and then smooth the distances according to the following function

$$d_{ij}(\alpha) = \begin{cases} \bar{d} + (d_{ij} - \bar{d})^{\alpha}, & d_{ij} \ge \bar{d} \\ \bar{d} - (\bar{d} - d_{ij})^{\alpha}, & d_{ij} < \bar{d} \end{cases}$$

where $\bar{d}$ is the average inter-city distance.

Step 4 Apply a local search heuristic to the TSP with the smoothed distances to produce the current tour.

Step 5 If $\alpha = 1$, stop. The current tour is the final tour. Otherwise, using the current tour, go to Step 3.

Search Space Smoothing: Summary

- Smoothing clearly outperforms two-opt and takes approximately the same amount of time
- Smoothing, like simulated annealing, can accept uphill moves
- Smoothing suggests a new way of classifying heuristics
- Further experimentation with different “smoothing” functions has led to even better results

Other Smoothing Functions (Coy et al., 2000)

- Exponential
- Hyperbolic
- Sigmoidal
- Logarithmic
- Concave
- Convex

Part II: Demon Algorithms

- Review previous work
  - simulated annealing (SA)
  - the demon algorithm
  - preliminary computational work
- Introduce three new demon algorithm variants
- Perform a computational study
- Present conclusions

Simulated Annealing

- Generate an initial tour and set T (temperature)
- Repeat until stopping condition:
  - Generate a new tour and calculate ΔE (change in energy)
  - If $\Delta E \leq 0$, accept new tour
  - Else, if $rand(0,1) < exp(-\Delta E/T)$, accept new tour
  - Else, reject new tour
  - Implement annealing schedule ($T=a*T$)
- The choice of T and a are essential

Demon Algorithms

- Wood and Downs developed several demon algorithms for solving the TSP
- In DA, the demon acts as a creditor
- The demon begins with credit = $D > 0$
- Consider an arc exchange
- If $\Delta E < D$, accept new tour and $D = D - \Delta E$
- Arc exchanges with $\Delta E < 0$ build credit
- Arc exchanges with $\Delta E > 0$ reduce credit

Demon Algorithms (continued)

- To encourage minimization, Wood and Downs propose two techniques
  - Impose an upper bound on the demon value, restricting the demon value after energy decreasing moves
  - Anneal the demon value
- Wood and Downs also propose a random component
  - The demon value is a normal random variable centered around the demon mean value
  - All changes in tour length impact the demon mean value

Demon Algorithms (continued)

- This leads to four algorithms (Wood and Downs)
  - Bounded demon algorithm (BD)
  - Randomized bounded demon algorithm (RBD)
  - Annealed demon algorithm (AD)
  - Randomized annealed demon algorithm (RAD)

New Demon Algorithms

- Two new techniques come to mind (Pepper et al.)
    - Annealed bounded demon algorithm (ABD)
    - Randomized annealed bounded demon algorithm (RABD)
- The idea is to impose a bound on the demon value (or demon mean value) and anneal that bound in ABD and RABD
- For RAD and RABD, anneal both the bound on the demon mean and the standard deviation. This leads to two additional algorithms, ADH and ABDH

Computational Study

- Eleven algorithms in all
- We selected 29 instances from TSPLIB
- The instances range in size from 105 to 1,432 nodes
- The instances have different structures
- Each algorithm was applied 25 times to each instance from a randomized greedy start
- Best and average performance and running time statistics were gathered

Preliminary Computational Results & Observations

- Simulated annealing was best overall
- RABD and ABD are nearly competitive with SA
- The intuition behind the hybrids makes sense, but parameter setting becomes more difficult
- The normal distribution can be replaced by “easier” distributions
- Smarter DA variants may exist

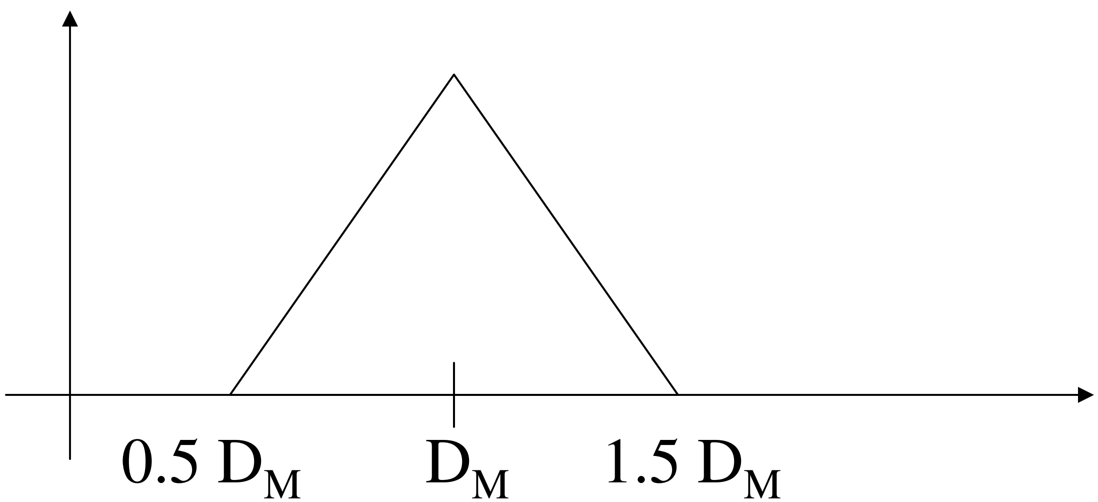
Parameter Settings

- We selected three representative test instances
    - For each algorithm, a GA determines a set of parameter values (parameter vector) that works well on these instances

- Resulting parameter vector is applied to all 29 instances

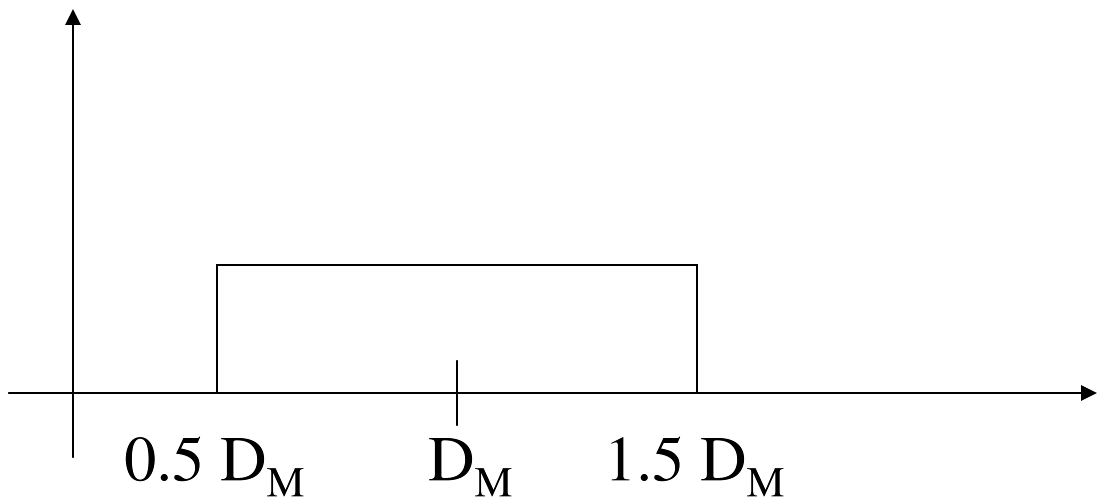
New Variant #1: Triangular Demon Algorithm

- Instead of sampling from a normal distribution, the demon value is sampled from the p.d.f. below



New Variant #2: Uniform Demon Algorithm

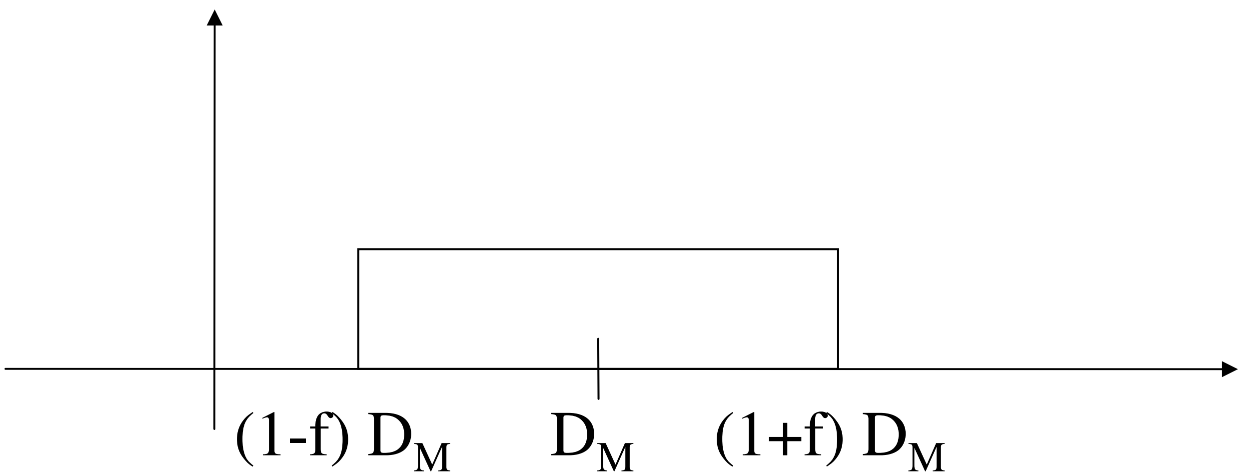
- Instead of sampling from a normal distribution, the demon value is sampled from the p.d.f. below



New Variant #3: Annealed Uniform Demon Algorithm

- Instead of sampling from a normal distribution, the demon value is sampled from the p.d.f. below

- *f* is set to 0.5 initially and is annealed over time



Advantages of New Variants

- Only two parameters need to be set (initial demon value and annealing schedule) – same as for SA

- The mean and standard deviation are annealed at the same time

- Sampling is easier in these three cases than sampling from a normal distribution

Experimental Design

- We selected 36 symmetric, Euclidean instances from TSPLIB
- The instances range in size from 105 to 1,432 nodes
- For each algorithm, parameters were set using a GA-based procedure on a small subset of the instances

Setting Parameters

- Single-stage genetic algorithm

- Fitness of parameter vector v is

$$F(v) = 100\sqrt{\frac{1}{m}\sum_{i=1}^{m}((D(v,i)\,/\,B(i)) - 1)^2}$$

where *m* is the number of test problems in the subset, D(v,*i*) is the tour length generated by vector v on test problem *i*, and B(*i*) is the optimal solution to problem *i*

- The fitness is the root mean square of percent above optimal

Experimental Design (continued)

- Starting tour
  - greedy heuristic
  - savings heuristic
- Tour improvement using 2-opt
- Termination condition: 50 iterations of no improvement after going below the initial tour length or a maximum of 500 iterations

Experimental Design (continued)

- Each algorithm was run 25 times for each of the 36 instances
- Averaged results are presented
- All runs were carried out on a Sun Ultra 10 workstation on a Solaris 7 platform
- The six best algorithms are compared

Experimental Results

Algorithm	Greedy Tour			Savings Tour		
	Average % above optimal	Standard deviation	Running time (hours)	Average % above optimal	Standard deviation	Running time (hours)
Simulated Annealing	2.85	1.19	17.45	2.84	1.01	10.68
Annealed Bounded	2.85	1.34	22.38	2.38	0.70	12.26
Randomized Annealed Bounded	3.54	1.53	13.19	3.12	0.89	6.06
Uniform	2.86	1.54	24.47	2.67	0.82	11.56
Annealed Uniform	2.74	1.28	18.90	2.65	0.80	7.85
Triangular	3.14	1.41	20.90	2.51	0.74	8.96

Part II: Conclusions

- With a greedy start, Annealed Uniform is best
- When the savings heuristic is used at the start
  - Annealed Bounded is best
  - Triangular and Annealed Uniform also perform well and beat SA
- Demon algorithms are sensitive to the starting conditions
- Using the savings heuristic significantly reduces computation times
- Demon algorithms can be applied to other combinatorial optimization problems

Final Comments

- Smoothing and demon algorithms are widely applicable
- They are simple and elegant
- There are few parameters to tune
- They involve approximately 20 lines of code