

# **Algorithms, Logistics, and the New Economy**

by

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Distinguished Scholar-Teacher Lecture  
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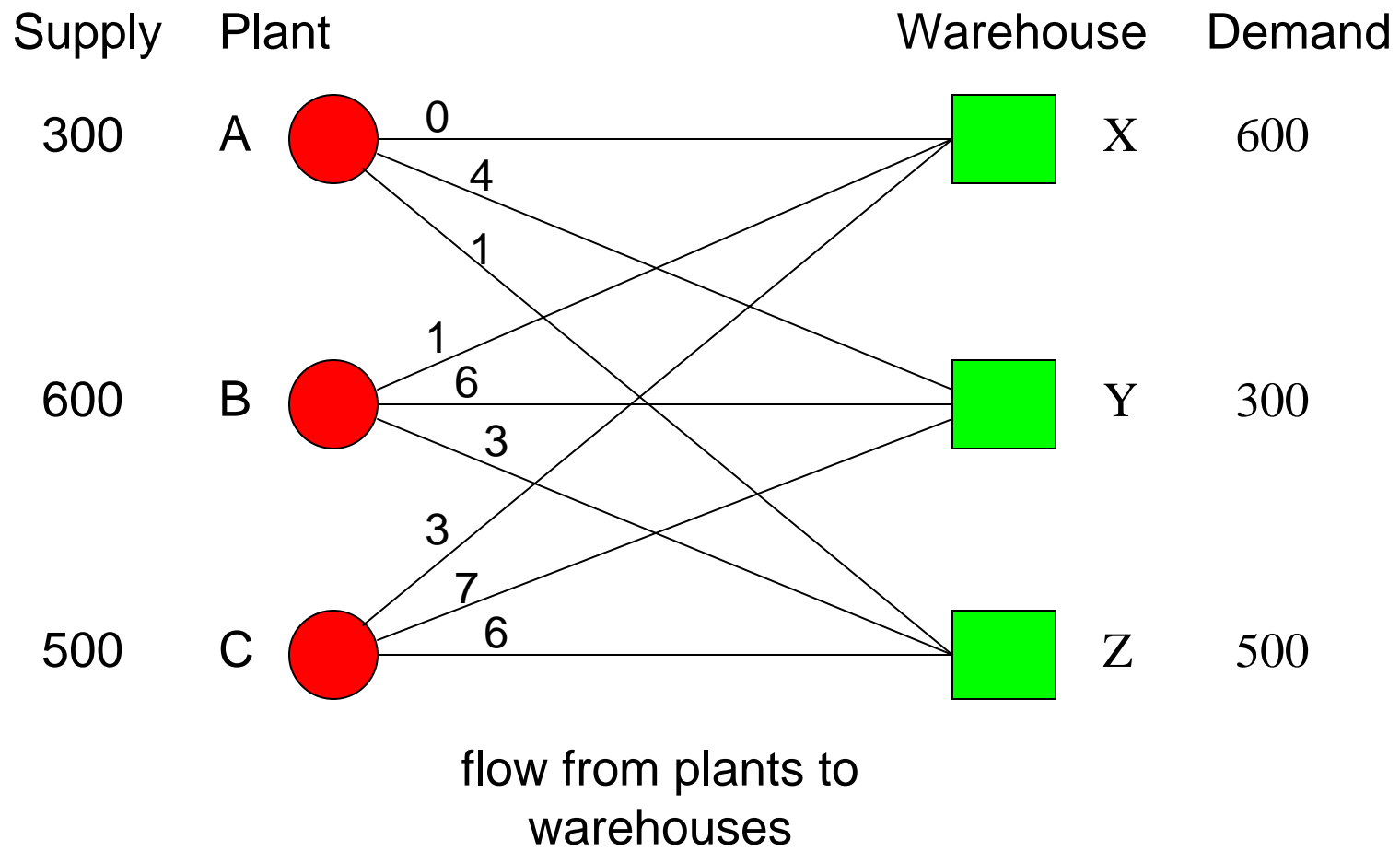
# Focus

- Discuss algorithms and computational effort
- Contrast optimal algorithms with heuristics
- Illustrate several heuristic strategies
- Apply ideas to operational problems in logistics management
- Explore some connections to the New Economy
  - MapQuest
  - RouteSmart
- Conclusions

# The Transportation Problem

- We begin with a simple problem from logistics
- There are  $m$  supply points with items available to be shipped to  $n$  demand points
- Plant  $i$  can ship at most  $S_i$  items, and warehouse  $j$  requires at least  $D_j$  items
- The cost of shipping each unit from plant  $i$  to demand point  $j$  is provided
- The objective is to select a routing plan that minimizes total transportation costs

# A Small Transportation Problem



## A Tabular Representation

### Warehouse

<b>Plant</b>	<b>x</b>	<b>y</b>	<b>z</b>	<b>Supply</b>
<b>A</b>	0	4	1	<b>300</b>
<b>B</b>	1	6	3	<b>600</b>
<b>C</b>	3	7	6	<b>500</b>
<b>Demand</b>	<b>600</b>	<b>300</b>	<b>500</b>	<b>1400</b>

## Ad-Hoc Solution Procedure

1. Avoid the most costly route
2. Fully utilize the cheapest route

### Warehouse

Plant	x		y		z		Supply
<b>A</b>	0	300	4	0	1	0	<del>300</del> <sub>0</sub>
<b>B</b>	1		6		3		<b>600</b>
<b>C</b>	3		7	0	6		<b>500</b>
<b>Demand</b>	<del>600</del> <sub>300</sub>		<b>300</b>		<b>500</b>		<b>1400</b>

## Ad-Hoc Solution Procedure -- continued

### Warehouse

Plant	x		y		z		Supply
<b>A</b>	0	300	4	0	1	0	<del>300</del> <sub>0</sub>
<b>B</b>	1	300	6	300	3	0	<del>600</del> <sub>0</sub>
<b>C</b>	3		7	0	6		<b>500</b>
<b>Demand</b>	<del>600</del> <sub>0</sub>		<del>300</del> <sub>0</sub>		<b>500</b>		<b>1400</b>

## Ad-Hoc Solution Procedure -- continued

### Warehouse

Plant	x		y		z		Supply
<b>A</b>	0	300	4	0	1	0	<del>300</del> 0
<b>B</b>	1	300	6	300	3	0	<del>600</del> 0
<b>C</b>	3	0	7	0	6	500	<del>500</del> 0
<b>Demand</b>	<del>600</del> 0		<del>300</del> 0		<del>500</del> 0		<b>1400</b>

- Total cost =  $300 \times 0 + 300 \times 1 + 300 \times 6 + 500 \times 6 = \$5100$
- This approach is reasonable, but a much better solution exists



# Optimal Solution

## Warehouse

Plant	x		y		z		Supply
<b>A</b>	0	0	4	0	1	300	<b>300</b>
<b>B</b>	1	400	6	0	3	200	<b>600</b>
<b>C</b>	3	200	7	300	6	0	<b>500</b>
<b>Demand</b>	<b>600</b>		<b>300</b>		<b>500</b>		<b>1400</b>

- Total cost =  $300 \times 1 + 400 \times 1 + 200 \times 3 + 200 \times 3 + 300 \times 7 = \$4000$
- Note 1. Nothing is sent along the zero cost route
- Note 2. Most expensive route is used as much as possible
- Note 3. We can obtain this solution using linear programming

## Observations

- We examined a small, toy problem
- We applied common sense and intuition in our ad hoc solution procedure
- The resulting cost was 27.5% above the cheapest (optimal) cost
- Common sense and intuition were not good enough
- In practice, transportation problems are much larger and need to be solved repeatedly
- If we can't use linear programming, we should apply a systematic procedure (a heuristic) that consistently generates feasible, near-optimal solutions
- The minimum-matrix method is one such heuristic

## A Small Diversion

- Linear programming (LP) is one of the fundamental tools of management science
- George Dantzig developed the simplex method in 1947
- Dantzig was born in 1914
- His father, Tobias Dantzig, taught in our Math Dept. for many years
- George wrote his dissertation in mathematical statistics at Berkeley
- He worked in the Pentagon during WWII
- He spent many years as a Professor at Stanford
- He received an Honorary Doctorate from us in 1976
- About the 1975 Nobel Prize in Economics

## The Minimum-Matrix Method

### Warehouse

Plant	x		y		z		Supply
<b>A</b>	0	<i>300</i>	4	<i>0</i>	1	<i>0</i>	<del>300</del> <i>0</i>
<b>B</b>	1		6		3		<b>600</b>
<b>C</b>	3		7		6		<b>500</b>
<b>Demand</b>	<del>600</del> <i>300</i>		<b>300</b>		<b>500</b>		<b>1400</b>

- Find the cheapest route
- Send as much as possible along this route

## The Minimum-Matrix Method -- continued

### Warehouse

Plant	x		y		z		Supply
<b>A</b>	0	<i>300</i>	4	<i>0</i>	1	<i>0</i>	<del>300</del> <i>0</i>
<b>B</b>	1	<i>300</i>	6		3		<del>600</del> <i>300</i>
<b>C</b>	3	<i>0</i>	7		6		<b>500</b>
<b>Demand</b>	<del>600</del> <i>0</i>		<b>300</b>		<b>500</b>		<b>1400</b>

## The Minimum-Matrix Method -- continued

### Warehouse

Plant	x		y		z		Supply
<b>A</b>	0	<i>300</i>	4	<i>0</i>	1	<i>0</i>	<del>300</del> <sub>0</sub>
<b>B</b>	1	<i>300</i>	6	<i>0</i>	3	<i>300</i>	<del>600</del> <sub>0</sub>
<b>C</b>	3	<i>0</i>	7		6		<b>500</b>
<b>Demand</b>		<del>600</del> <sub>0</sub>		<b>300</b>		<del>500</del> <sub>200</sub>	<b>1400</b>

## The Minimum-Matrix Method -- continued

### Warehouse

Plant	x		y		z		Supply
<b>A</b>	0	300	4	0	1	0	<del>300</del> <sub>0</sub>
<b>B</b>	1	300	6	0	3	300	<del>600</del> <sub>0</sub>
<b>C</b>	3	0	7	300	6	200	<del>500</del> <sub>0</sub>
<b>Demand</b>	<del>600</del> <sub>0</sub>		<del>300</del> <sub>0</sub>		<del>500</del> <sub>0</sub>		<b>1400</b>

- Total cost =  $300 \times 0 + 300 \times 1 + 300 \times 3 + 300 \times 7 + 200 \times 6 = \$4500$
- The resulting cost is only 12.5% above the optimal cost

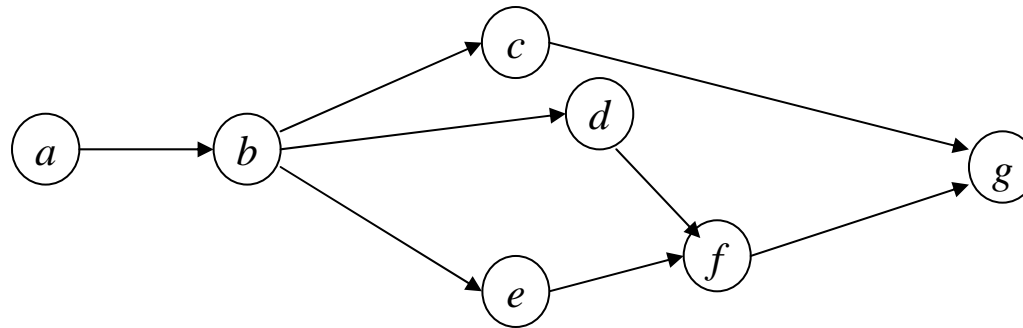
# Logistics Management

- Logistic management involves the cost-effective flow and storage of goods, services, and personnel from origin to destination according to a set of pre-specified requirements
- The transportation problem is a simple example of logistics management
- The maximum flow, shortest path, and traveling salesman problem are others



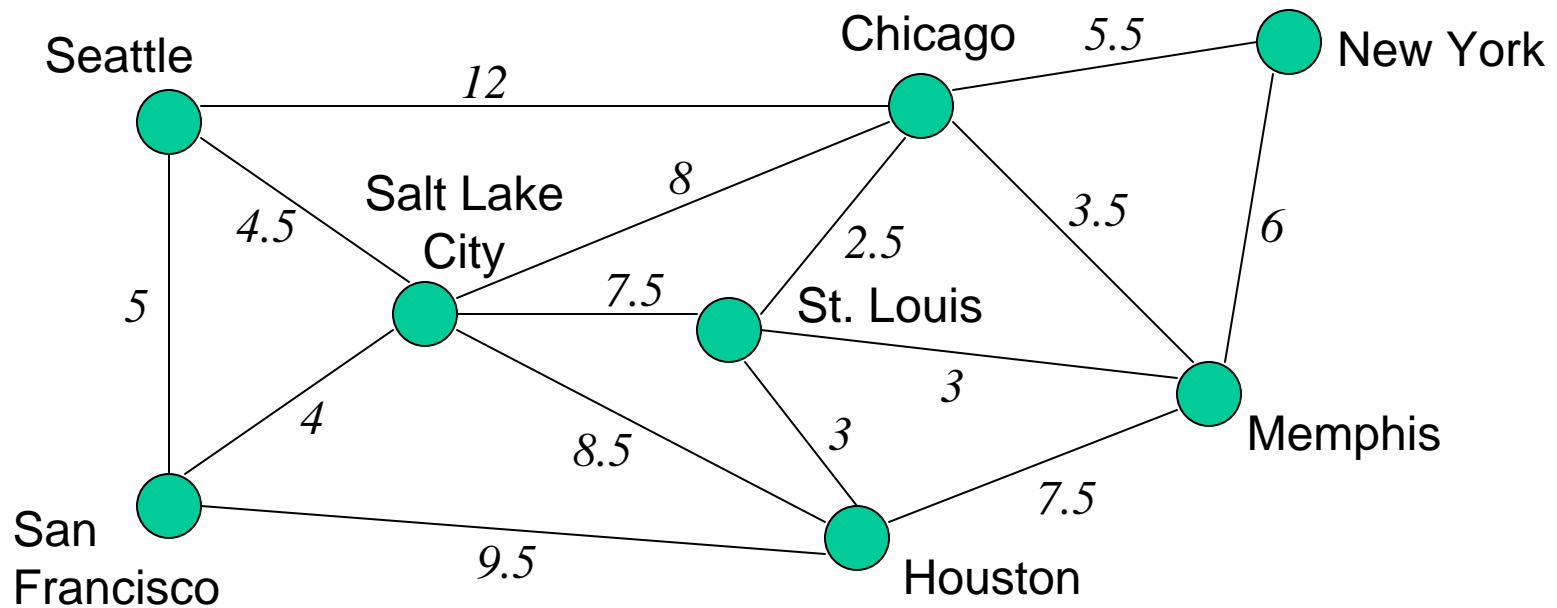
## Maximum Flow in a Railway Network

- A large number of commuters travel from **a** to **g** every day
- Three routes are shown below



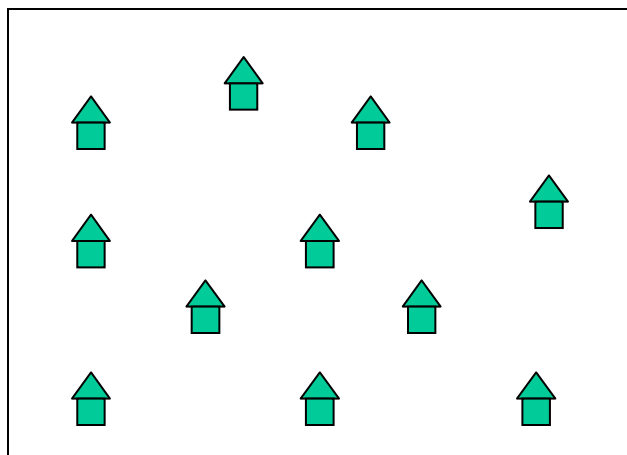
- Each section of the track (each link) has a capacity (number of trains per hour) based on safety considerations
- Suppose each capacity is 10
- How many trains per hour can reach **g**?
- How can the number be increased by changing the capacity of a single link?

# Find the Most Economical Package - Delivery Route from San Francisco to New York

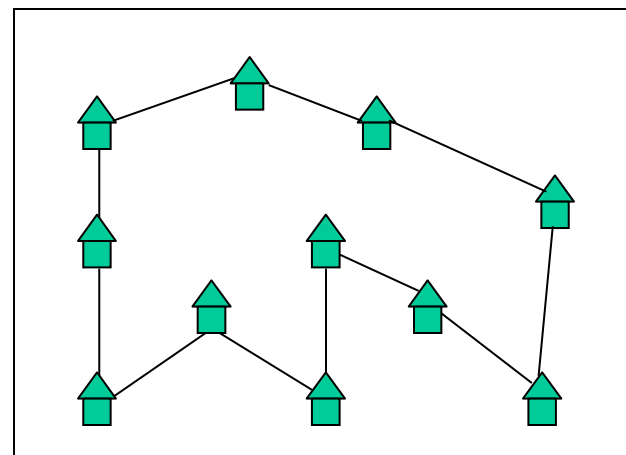


# The Traveling Salesman Problem

- Imagine a suburban college campus with 140 separate buildings scattered over 800 acres of land
- To promote safety, an experienced security guard must inspect each building every evening
- The goal is to sequence the 140 buildings so that the total time (travel time plus inspection time) is minimized
- This is an example of the well-known TSP



*Original problem*



*Possible solution*

# Analysis of Algorithms

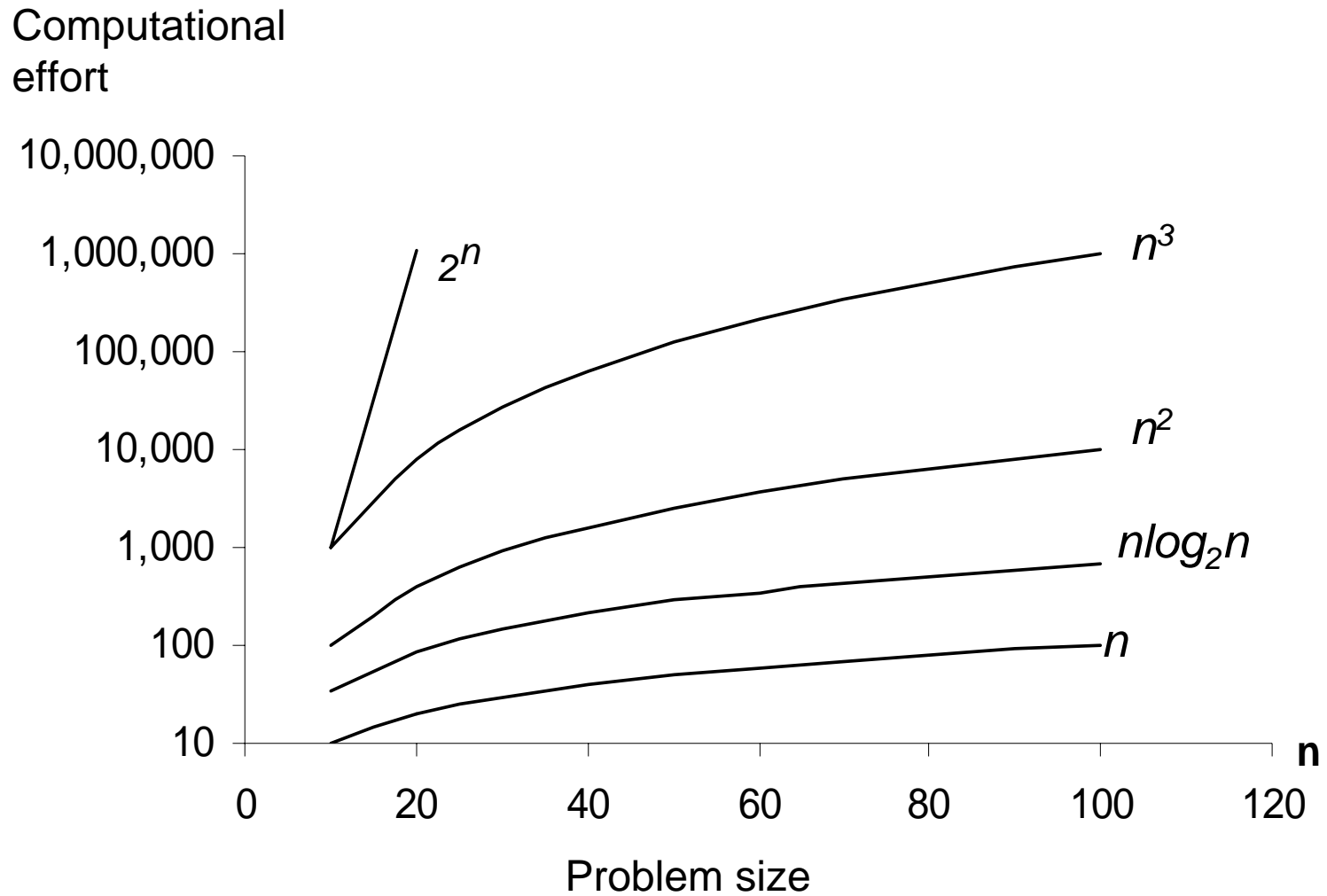
## ■ Definitions

- Algorithm – method for solving a class of problems on a computer
- Optimal algorithm – verifiable optimal solution
- Heuristic algorithm – feasible solution

## ■ Performance Measures

- Number of basic computations / Running time
- Computational effort
  - Problem size
  - Player one
  - Player two

# Computational Effort as a Function of Problem Size



# Good vs. Bad Algorithms

## ■ Terminology

- Researchers have emphasized the importance of finding polynomial time algorithms, by referring to all such polynomial algorithms as inherently good
- Algorithms that are not polynomially bounded, are labeled inherently bad

## ■ Good Optimal Algorithms Exist for these Problems

- Transportation problem
- Maximum flow problem
- Shortest path problem
- Linear programming

## Good vs. Bad Algorithms -- continued

### ■ Good Optimal Algorithms Don't Exist for these Problems

- Traveling salesman problem (TSP)
- Complex logistics problems (e.g., vehicle routing, vehicle fleet management, delivery with time-windows, pickup and delivery systems, integration of inventory and transportation)

### ■ Why Focus on Heuristic Algorithms?

- For the above problems, optimal algorithms are not practical
- Efficient, near-optimal heuristics are needed to solve real-world problems
- The key is to find fast, high-quality heuristic algorithms

## Some Heuristic Algorithm Strategies

- Aggregate / disaggregate
- Divide and conquer
- Allow uphill moves for minimization problems



# The Elastic Net Algorithm

[Durbin and Willshaw 87]

- Find the shortest TSP tour through n points
- Algorithm
  - Introduce a rubber band as a small circle around the center of gravity of the points
  - Stretch the rubber band towards the points by minimizing an energy function of the form:

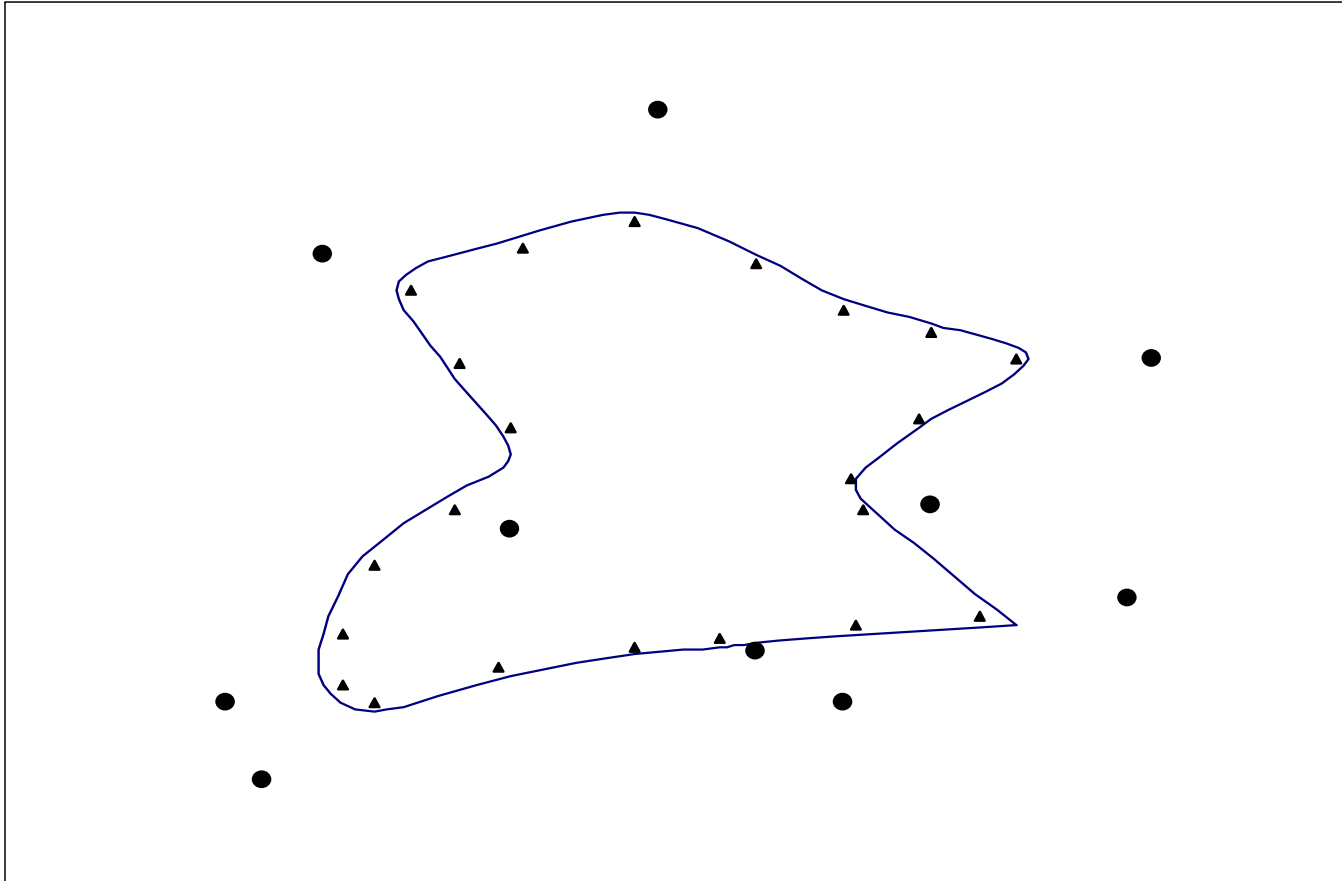
$$E = -\alpha K \sum_{i=1}^n \ln \sum_{j=1}^m e^{-\|x_i - y_j\|/2K^2} + \beta \sum_{j=1}^m \|y_j - y_{j+1}\|$$

where  $K \rightarrow 0$  and  $\|a - b\| = (a_1 - b_1)^2 + (a_2 - b_2)^2$

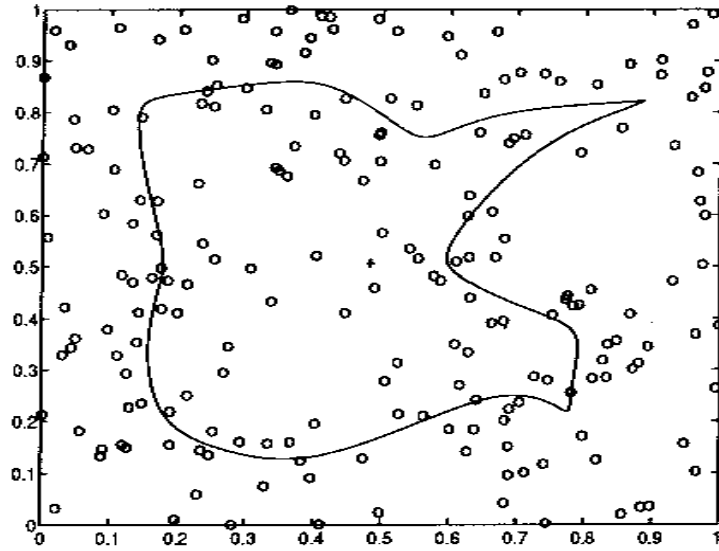
- The rubber band is stretched over time, using ideas from calculus, until the rubber band and the points coincide

## Key Observations

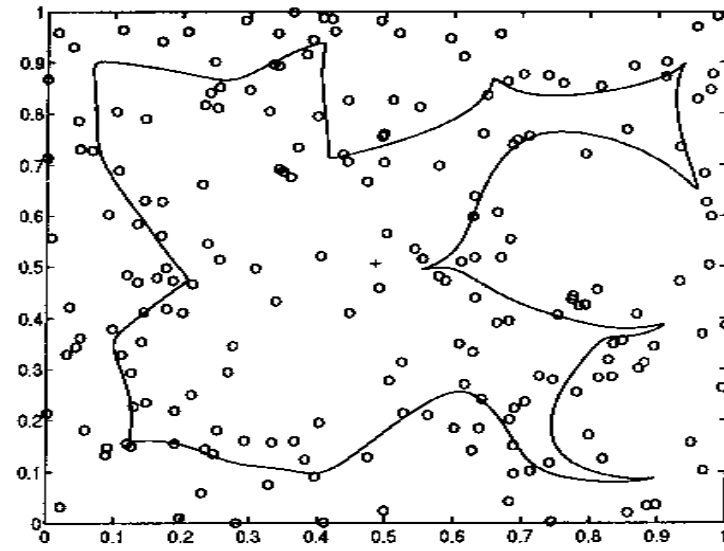
- Explain the two terms of the energy function
- Procedure is iterative
- At each iteration, the most burdensome step involves  $n \times m$  exponentiations
- To speed up the algorithm, we must address this bottleneck



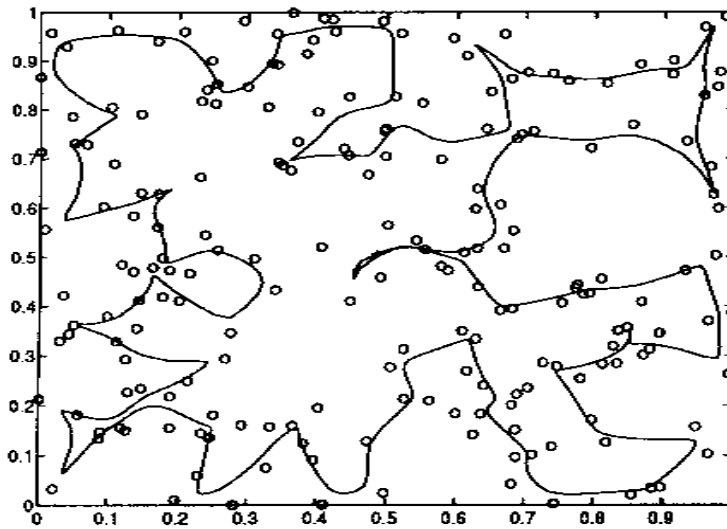
The Concept of Rubber Band Points



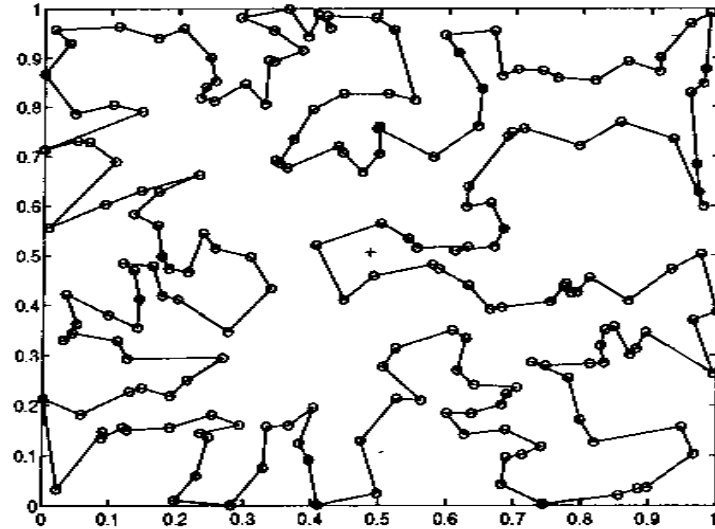
Iteration = 200



Iteration = 400



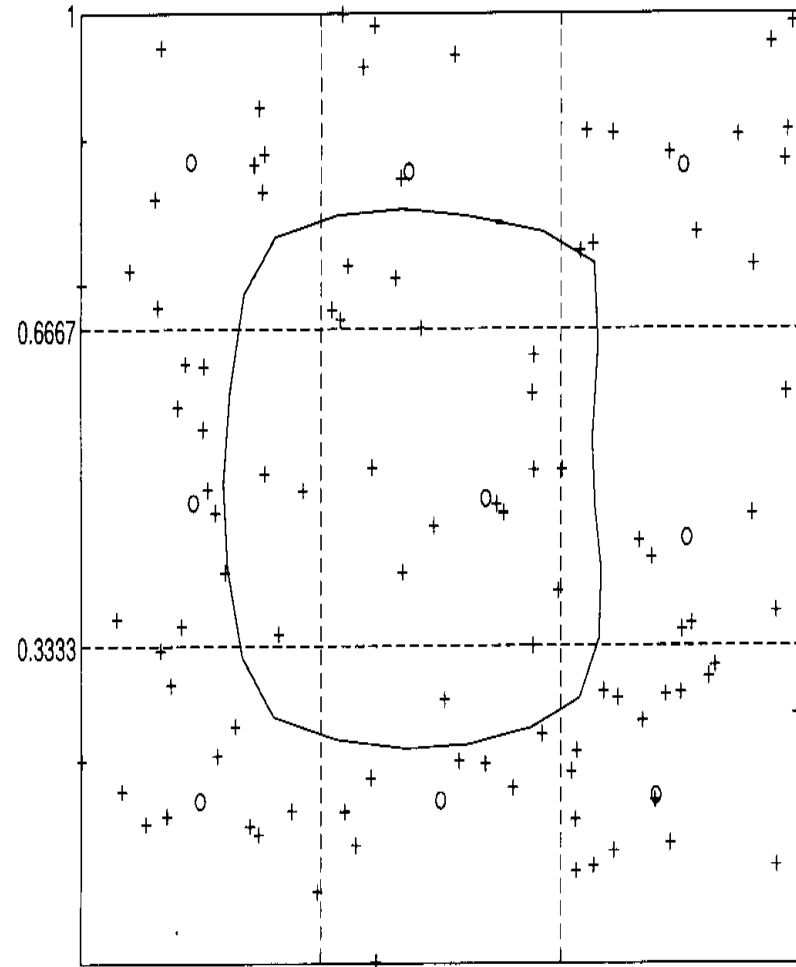
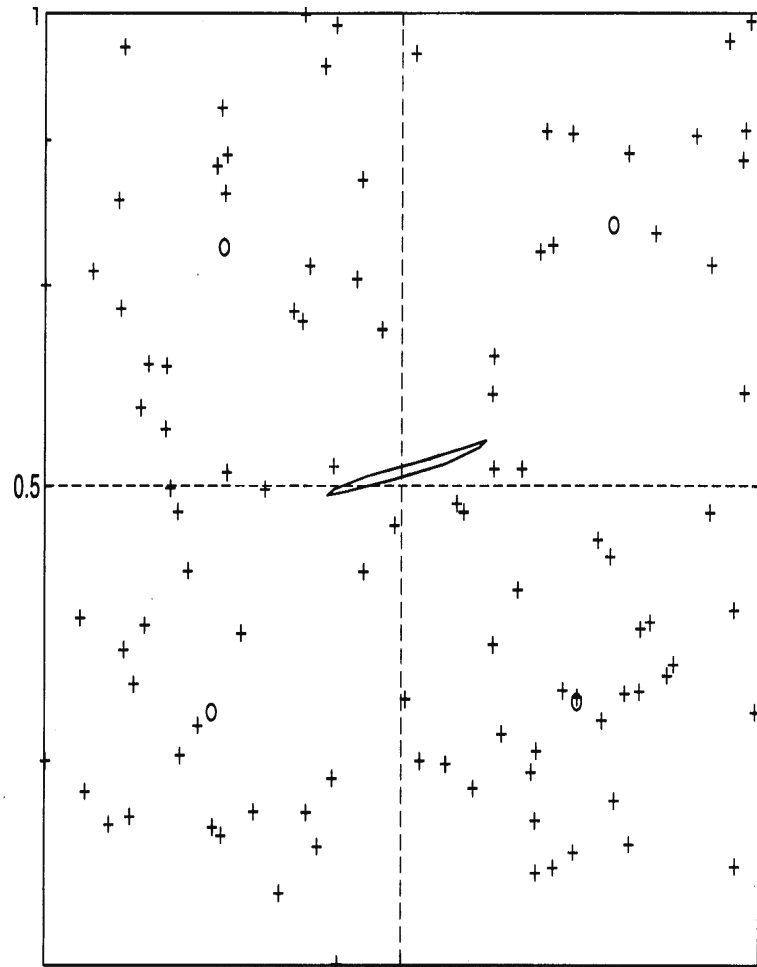
Iteration = 800

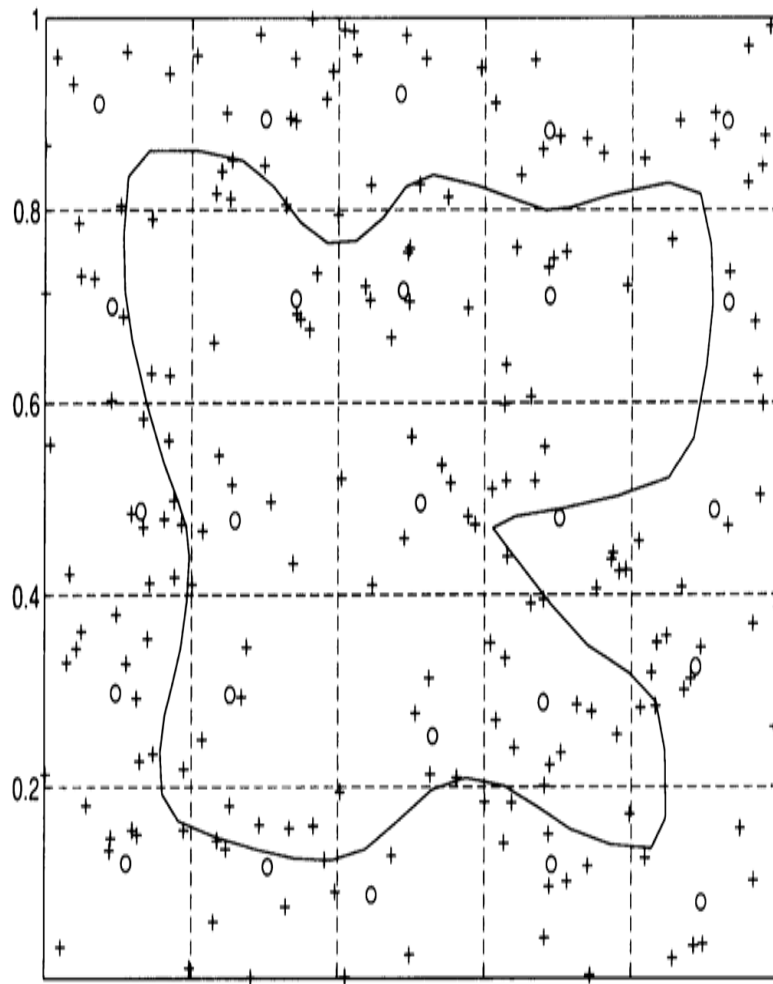
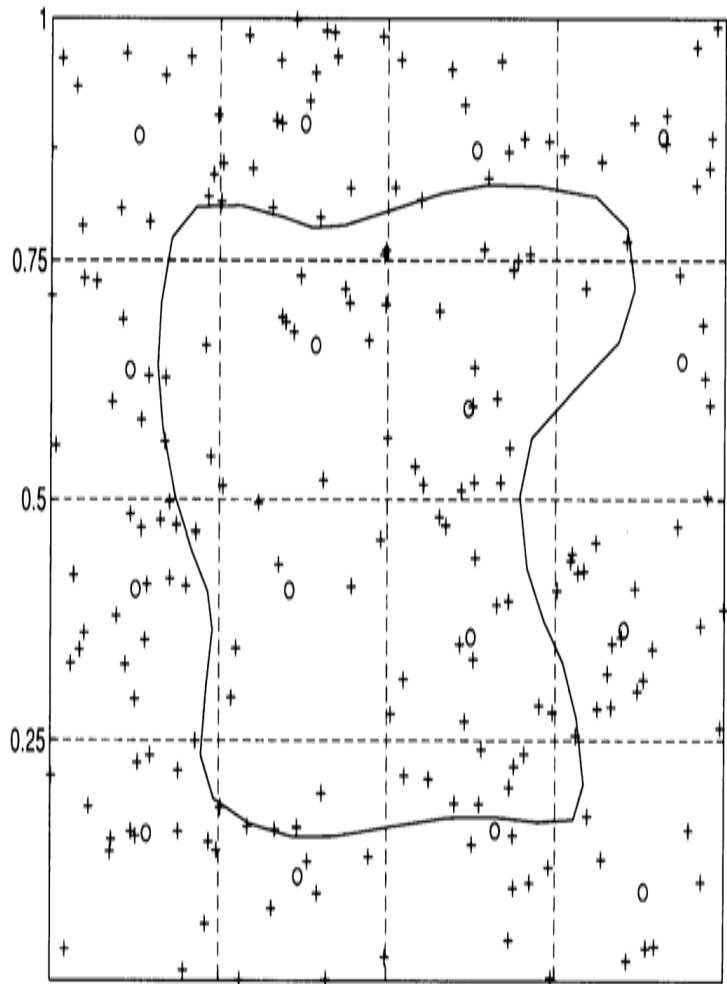


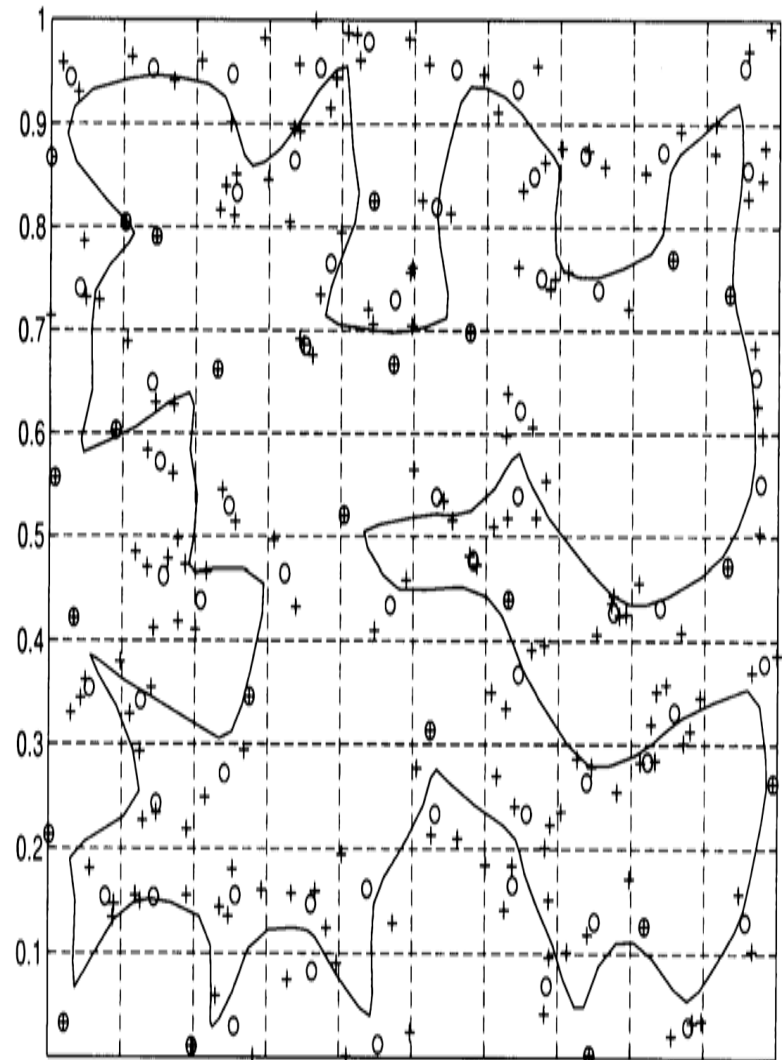
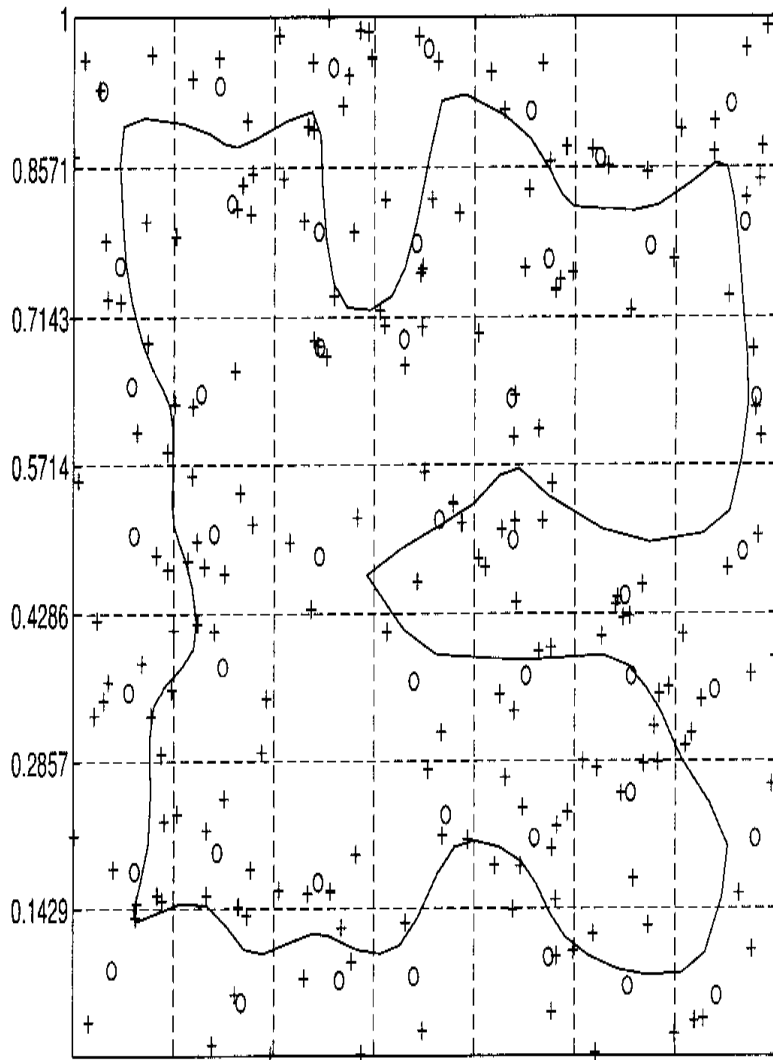
Iteration = 1000 (final)

## Aggregation / Disaggregation Strategy

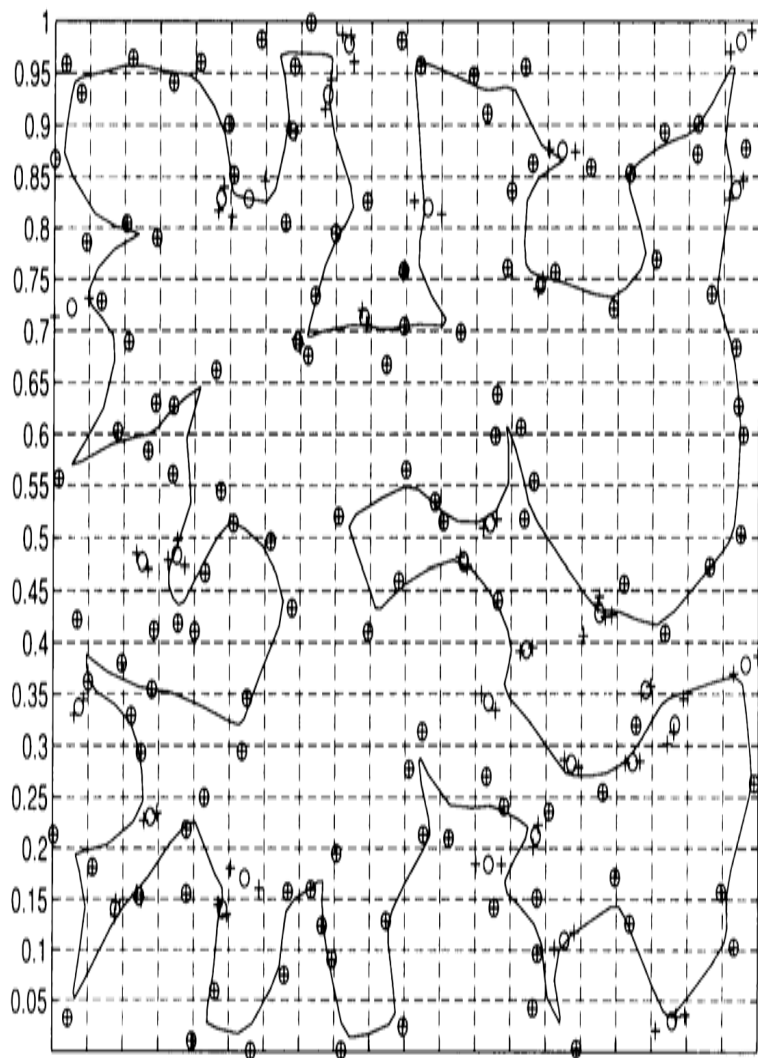
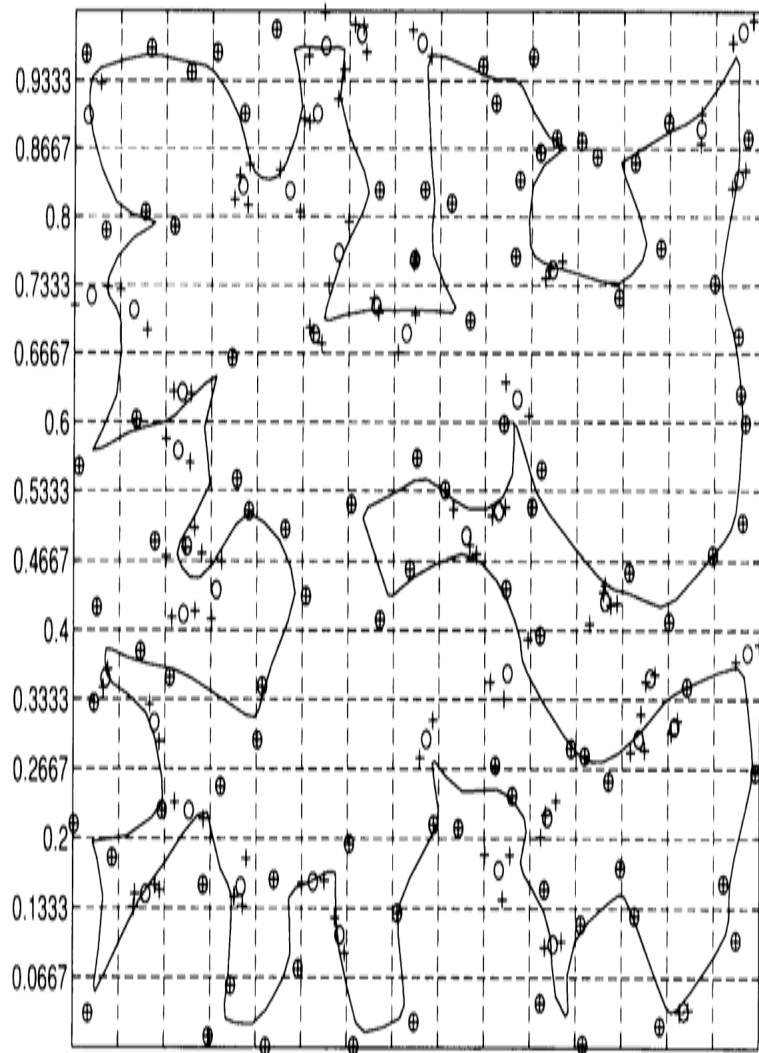
- Reduce both  $n$  and  $m$  by aggregating the points
- Slowly disaggregate
- Define center of gravity



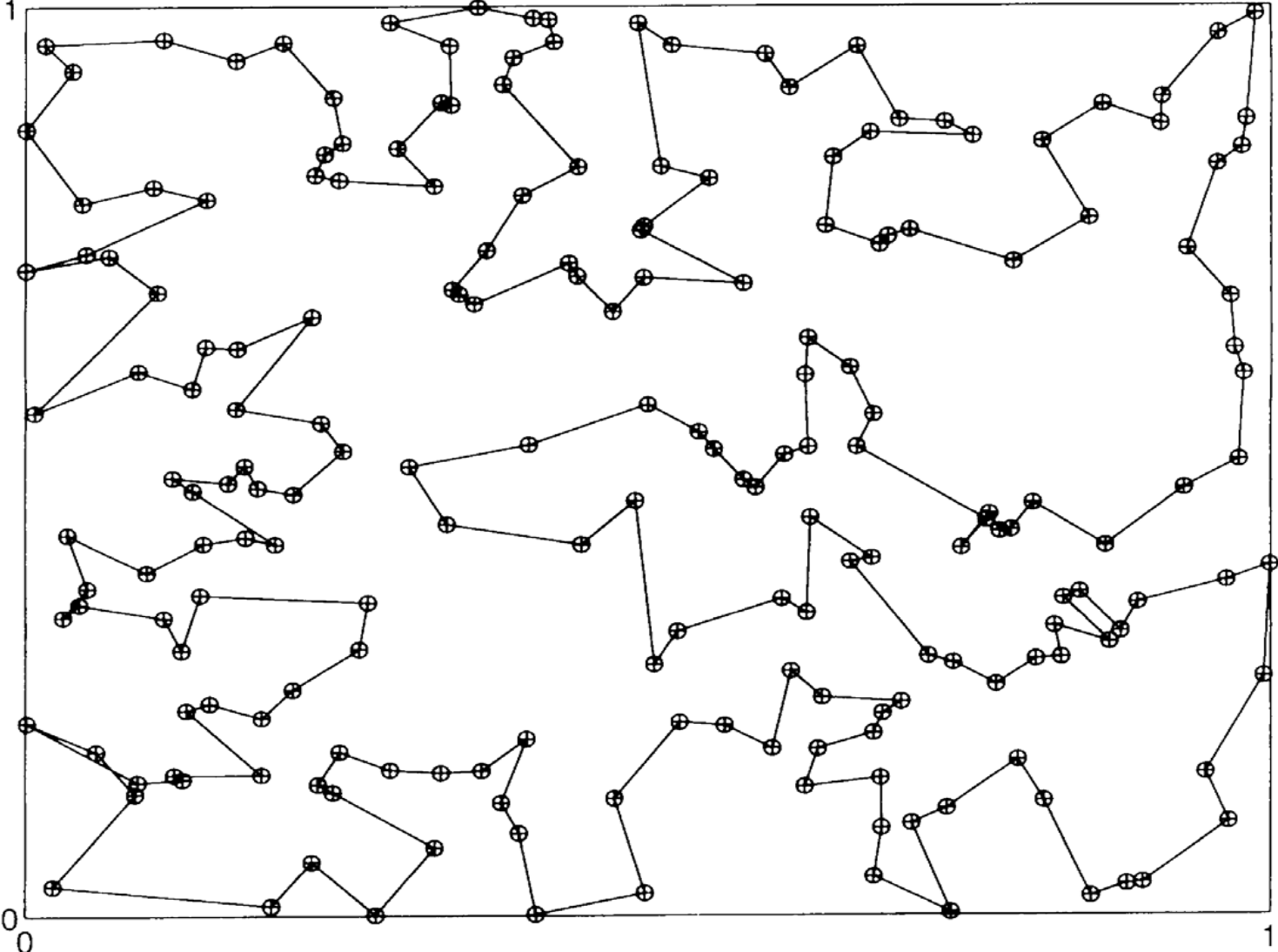








Final EN Tour Solution to TSP



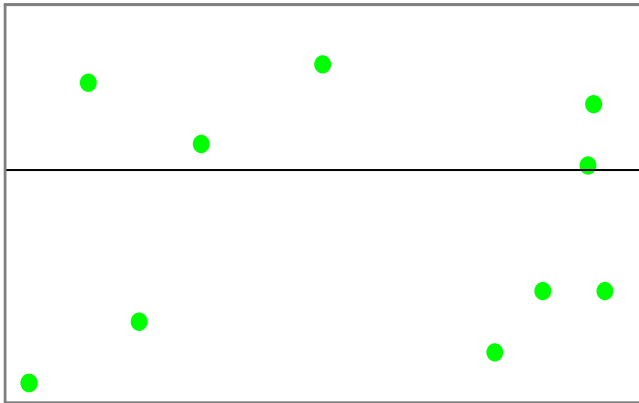
## Where Does the Speedup Come From?

<u>Iteration Number</u>			<u>Original Elastic Net</u>			<u>Accelerated Elastic Net</u>		
			n	m	n x m	n	m	n x m
1	-	50	500	1250	625,000	4	12	48
51	-	100	500	1250	625,000	9	27	243
101	-	150	500	1250	625,000	16	48	768
151	-	200	500	1250	625,000	25	75	1875
201	-	250	500	1250	625,000	36	108	3888
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951	-	1000	500	1250	625,000	500	1250	625,000

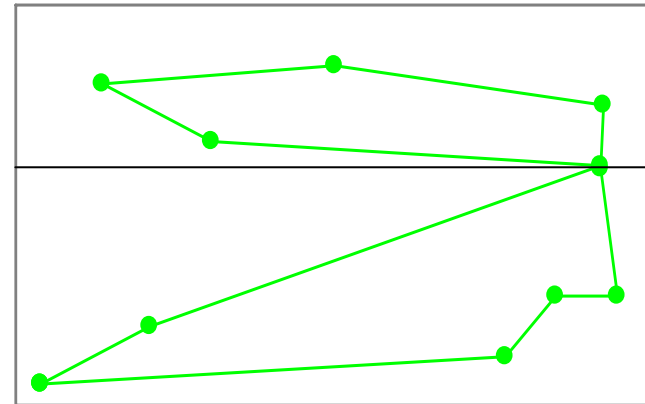
## Practical Observations

- The approximate speedup for 500 points is 16.4
- The approximate speedup for 1000 points is 17.2
- Impact: 10 minutes vs. nearly 3 hours
- The accelerated elastic net is faster and yields higher quality tours

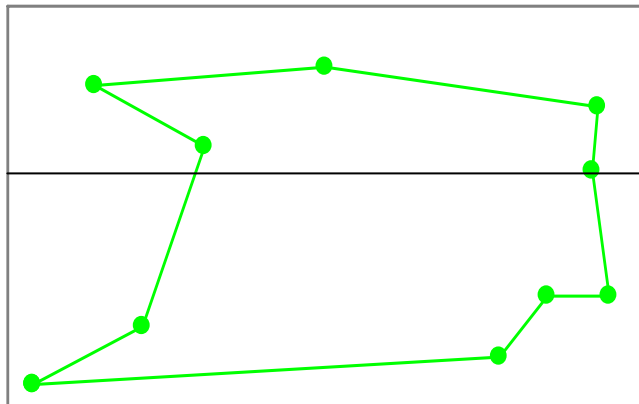
# Divide and Conquer



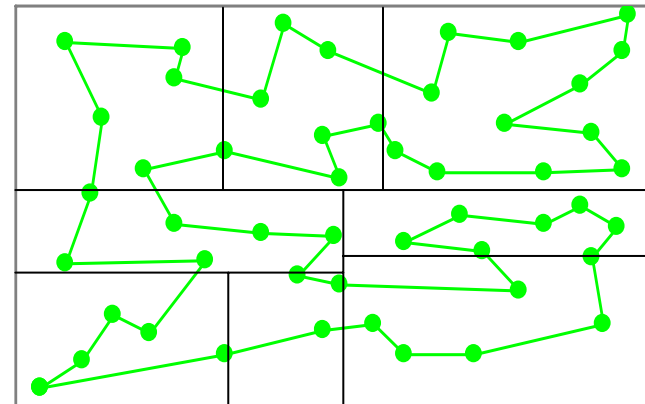
a. Horizontal bisection



b. Subproblems solved

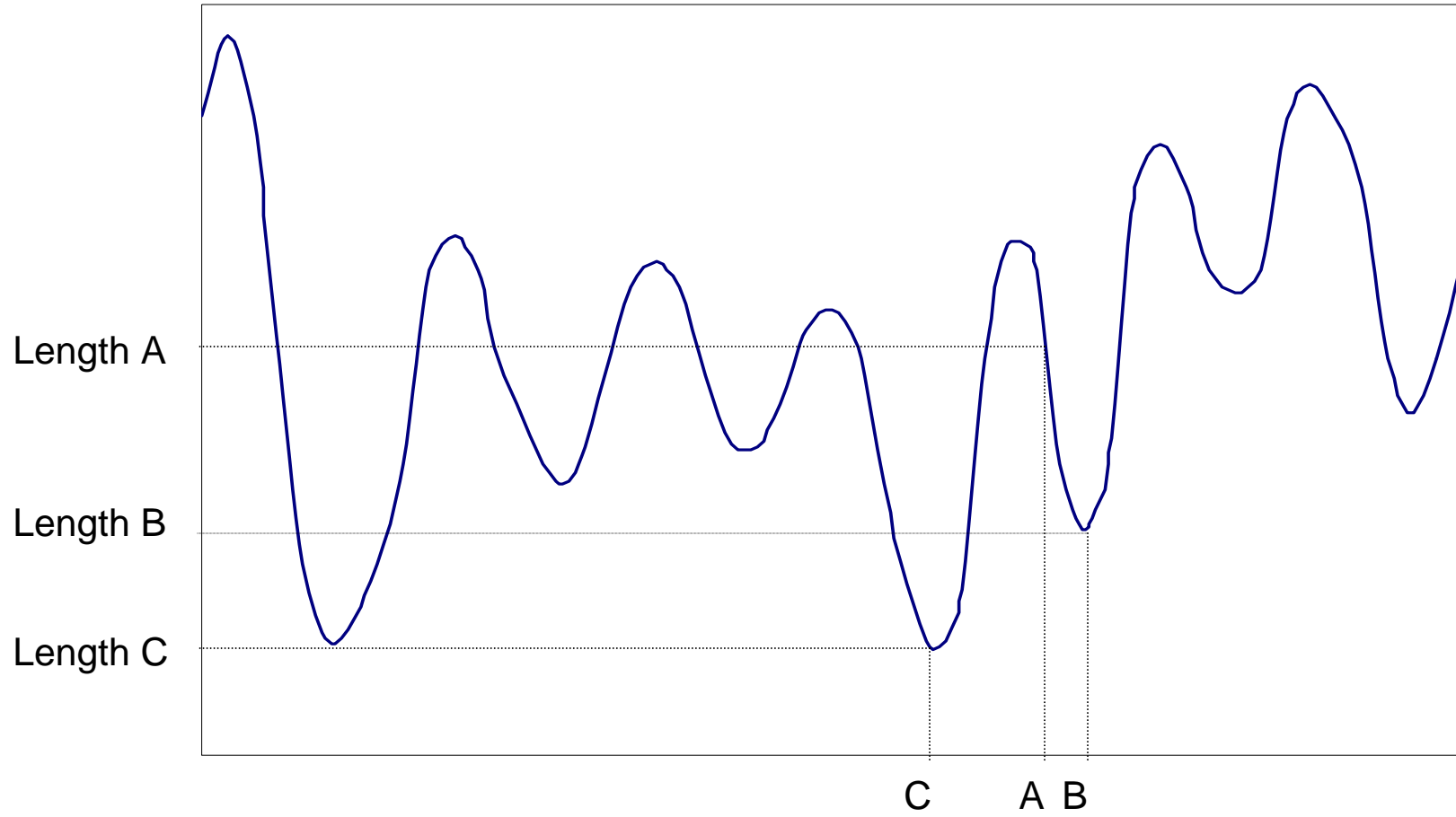


c. Patched solution



d. Recursive divide and conquer

# Allow Uphill Moves



# Geographic Information Systems

- A GIS is a computer-based tool for the manipulation and spatial analysis of geographic data
- The field is about 30 years old
- Annual sales of GIS software exceed \$1 billion
- The growth rate in sales has been approx. 25% per year
- Three GIS-based products of interest: MapQuest, Microsoft MapPoint 2001, and RouteSmart

## **RouteSmart Technologies, Inc.**

- Company designs and sells vehicle routing and scheduling software
- Small Md. Firm, based in Columbia, founded in 1980
- Owned by a large NY civil engineering company
- I've been associated with the firm from the beginning
- Currently run by Larry Levy -- my newspaper boy in 1978 & 1979
- RouteSmart clients include the NY Times, The WSJ, and the San Francisco Chronicle
- RouteSmart has major installations in the newspaper, utility, sanitation, and small package delivery industries
- Larry Levy graduated (B.S. & Ph.D.) from our Business School in the 1980s



# The Size of the Small Package Delivery Industry

## ■ FedEx

- 5 million shipments / business day
- 55,000 vehicles in the ground fleet

## ■ UPS

- 13.1 million packages daily
- 150,000 vehicles in the ground fleet

## ■ USPS

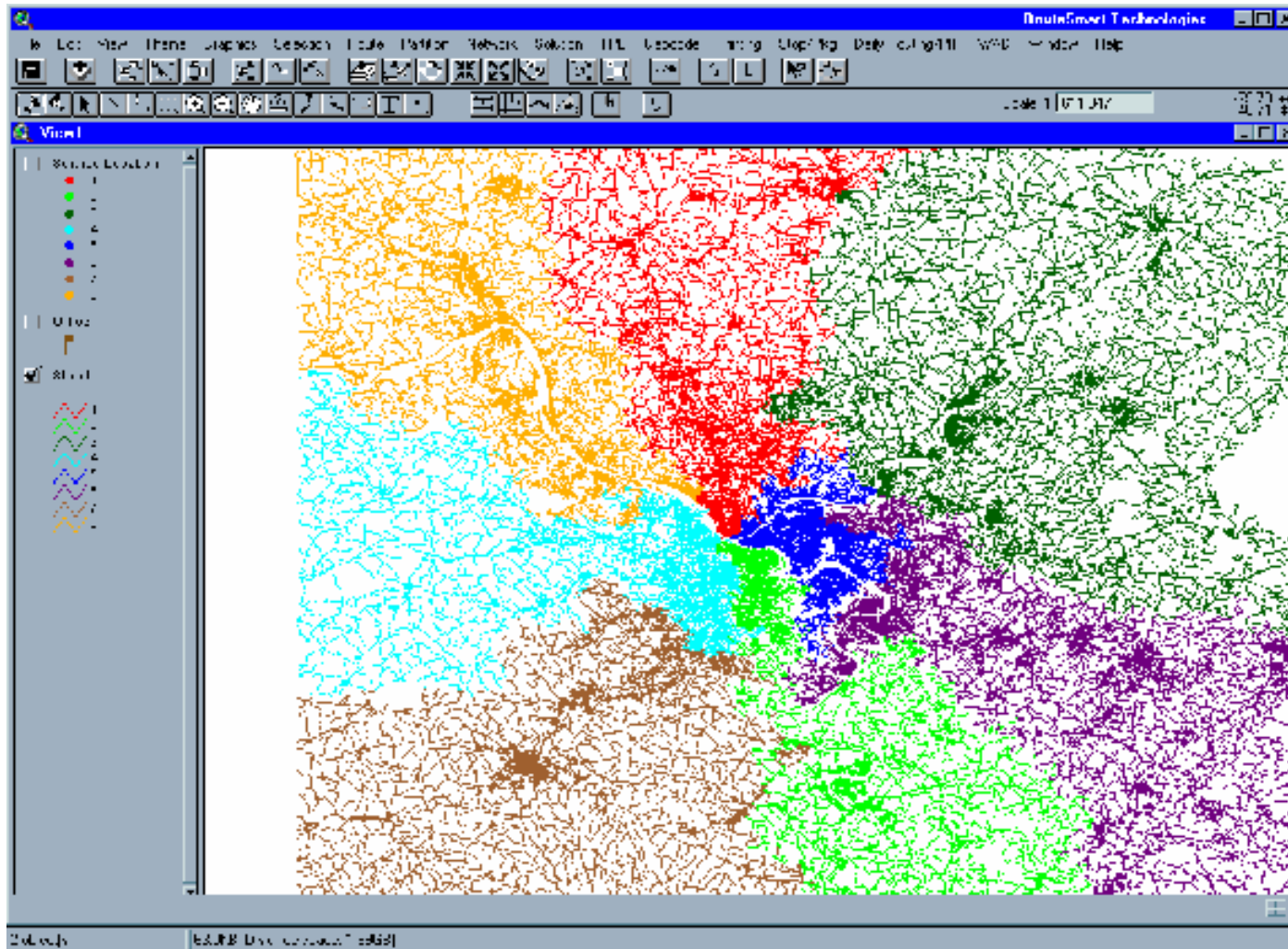
- More than 200 billion pieces of mail over 302 delivery days
- 237,000 routes, 6 days a week

## **Business-to-Consumer Ecommerce**

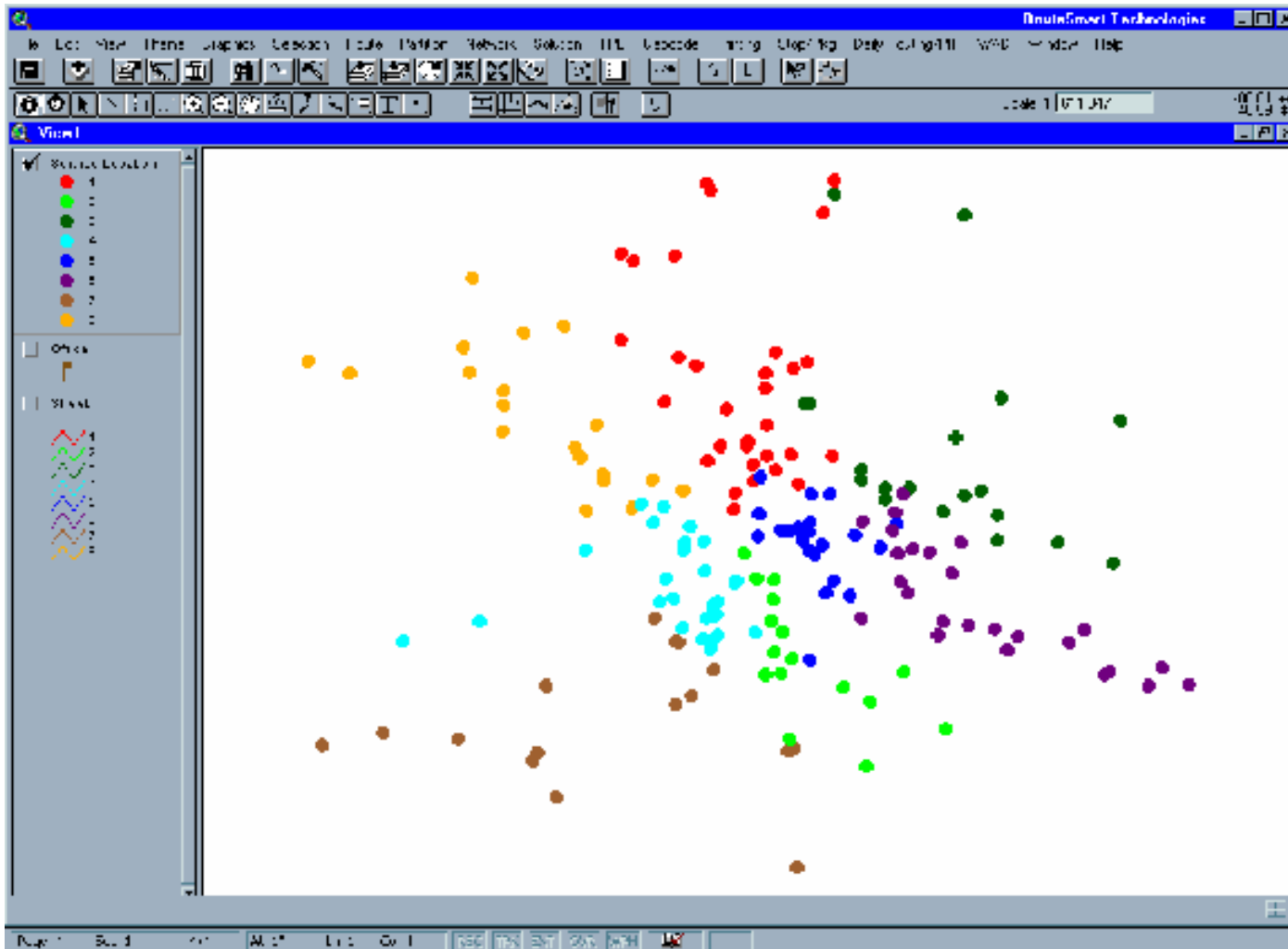
- By 2004, home delivery revenues from B2C ecommerce alone are expected to exceed \$125 billion
- Competition among FedEx, UPS, and the USPS is intense
- With express mail, air transportation is key
- FedEx took the early lead and UPS had to play catch-up
- With B2C ecommerce, ground transportation is key
- UPS has the advantage in fleet size and FedEx must play catch-up

## **FedEx Home Delivery and RouteSmart**

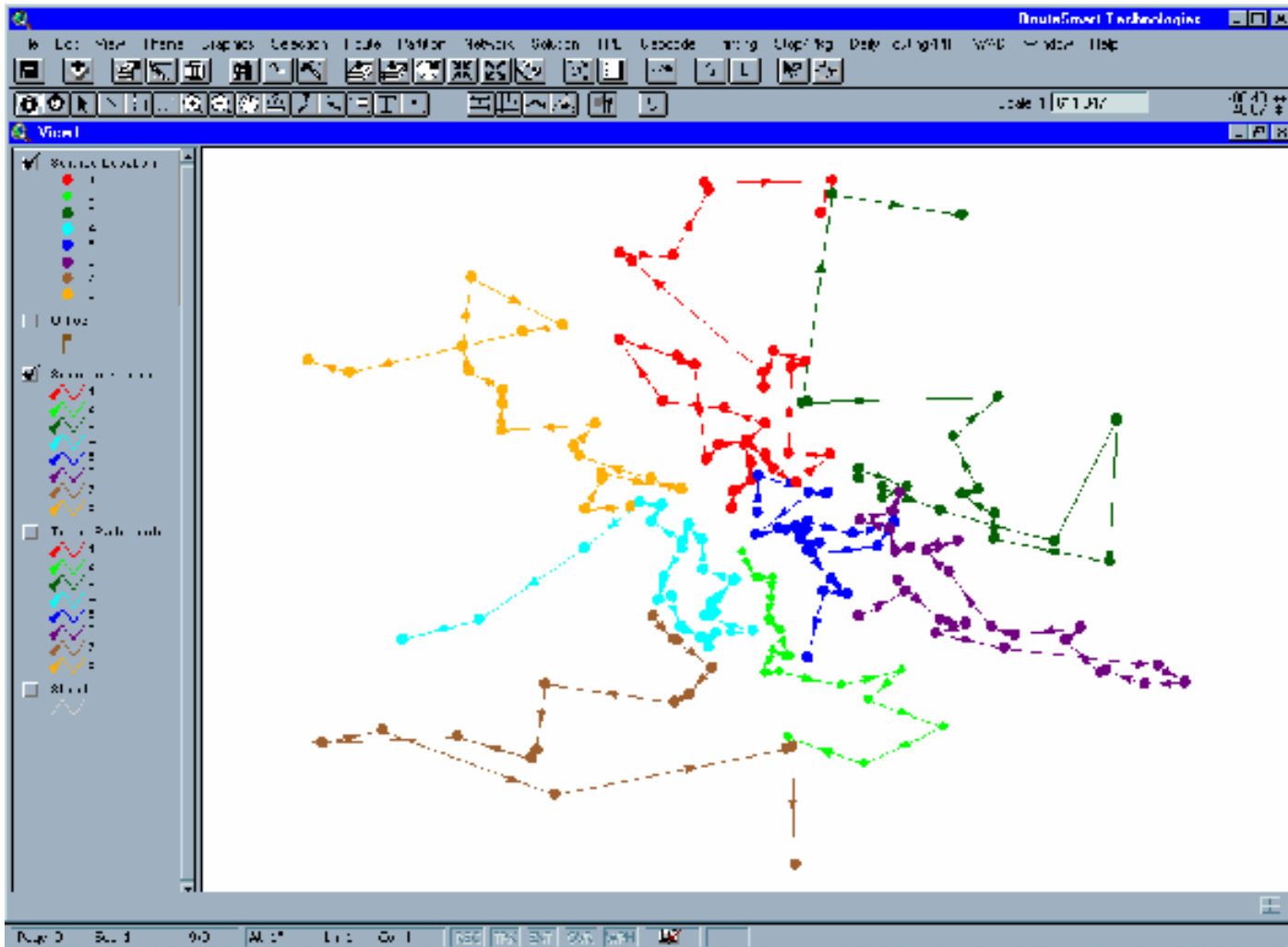
- FedEx Ground (formerly the RPS Group) focuses on B2B
- Opened Home Delivery in March 2000
- 68 terminals in U.S. currently
- 150 additional terminals to open in 2001
- 20,000 packages / business day (growing steadily)
- Focus on responding to B2C ecommerce
- All packages are routed for delivery using RouteSmart



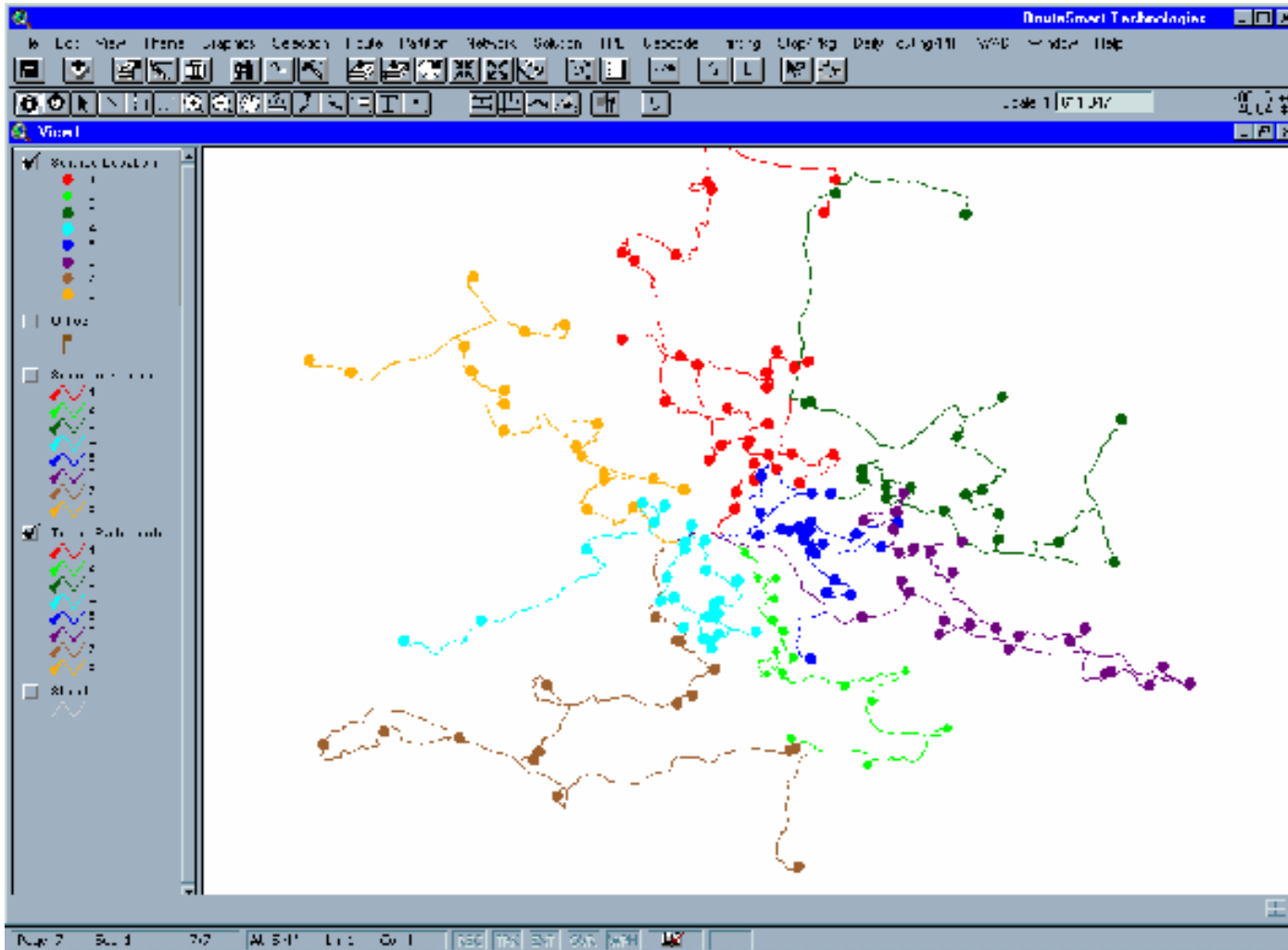
**Streets Pre-assigned to Routes**



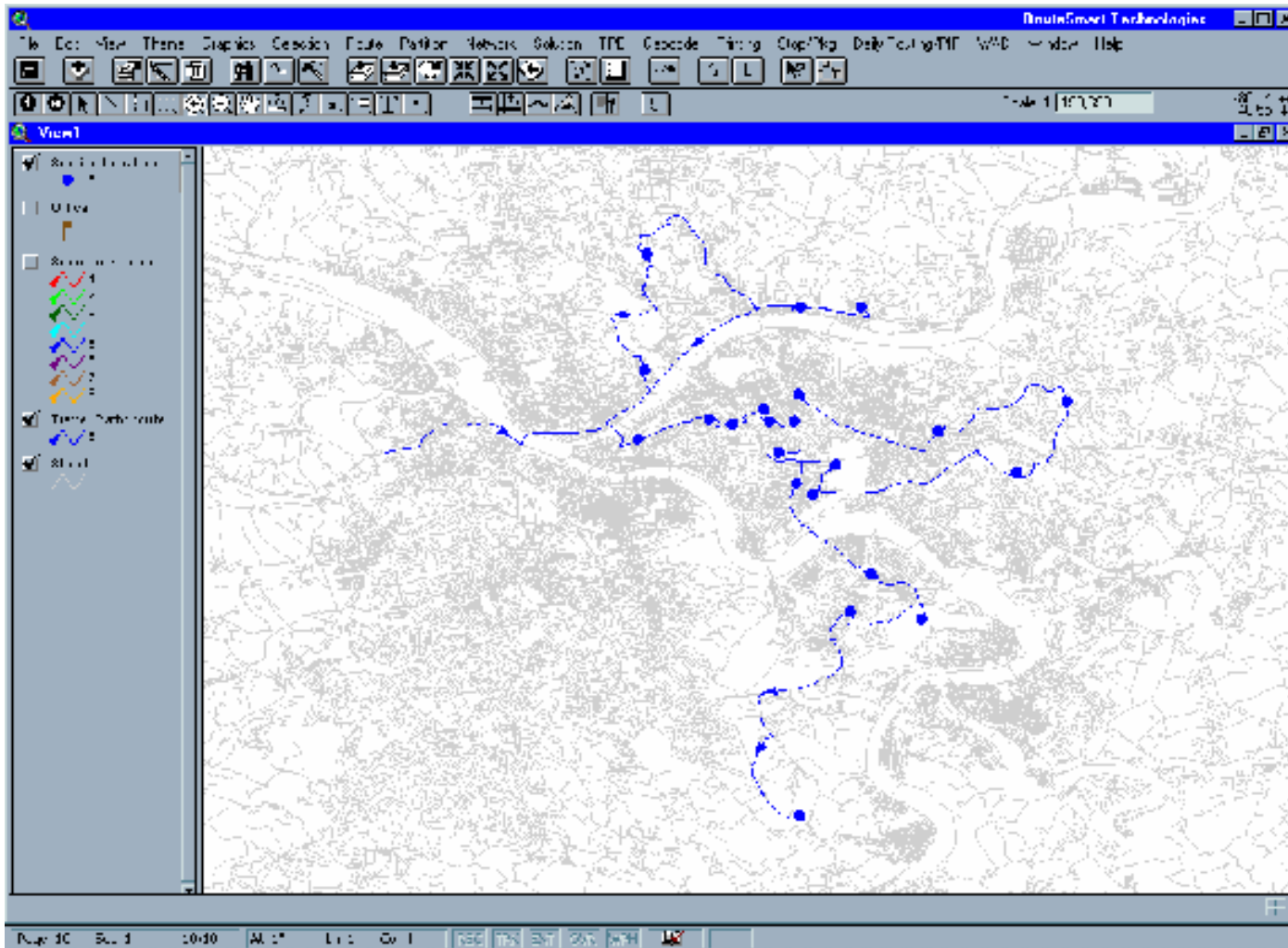
**Stops with Streets Suppressed**



**Sequenced Stops as Crow Flies (Streets Suppressed)**



## Sequenced Stops over the Street Network (Streets Suppressed)



**Detailed Display of a Single Travel Path (Gray Streets)**



## Conclusions

- Logistics and distribution management has often been taken for granted
- This has changed in the last decade or so
- We observed that even “simple” logistics problems can be difficult to solve
- Common sense and intuition are not good enough
- Systematic procedures (e.g., heuristics) often need to be designed and applied
- We discussed several strategies for designing heuristics for solving the TSP
- These strategies can be used to tackle important real-world problems (e.g., the FedEx Home Delivery problem)
- A wide variety of more complex logistics and distribution management problems can be approached in the same way

## Recommended Reading

- Coy, Golden, Runger, & Wasil, “ See the Forest before the Trees: Fine-tuned Learning and its Application to the TSP, ” IEEE Trans. on Systems, Man, and Cybernetics (Part A), 28 (4), 454-464 (1998)
- Michalewicz & Fogel, How to Solve It: Modern Heuristics, Springer (2000)
- Rivett, The Craft of Decision Modelling, Wiley (1994)
- Vakhutinsky & Golden, “ A Hierarchical Strategy for Solving TSPs Using Elastic Nets, ” Journal of Heuristics, 1(1), 67-76 (1995)

## Epilogue: The 1975 Nobel Prize in Economics

- It was given for work in LP, but not to Dantzig
- Instead, it was awarded to Koopmans (Yale) and Kantorovich (USSR)
- Prize was \$240K – to be split
- Many were outraged at Dantzig's omission
- Koopmans considered declining the Prize
- In the end, he accepted the Prize and immediately donated \$40K to a Think Tank in Austria with which Dantzig was affiliated
- He was left with \$80K (one-third of \$240K) which was the most he felt he deserved