An Experimental Design-Based Method for Finding Effective Parameter Values for Heuristic Methods

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Introduction

- Substantial effort has been devoted to tailoring general-purpose metaheuristics to solve difficult combinatorial optimization problems.
  - Most of the efforts to develop effective metaheuristics have been computationally burdensome and very time consuming (e.g., solving the VRP with tabu search).
- All of the metaheuristics that have been used to solve the VRP contain several parameters.
  - For example, the network flow-based tabu search heuristic of Xu and Kelly has 32 parameters whose values have to be set.

- Determining appropriate values for parameters found in VRP metaheuristics is not an easy task.
  - Procedures used to set a parameter's value have ranged from ad hoc trial-and-error methods to sophisticated sensitivity analysis.
  - Reported results are often from multiple passes of the heuristic with different parameter values used on each pass.
  - Time to select good parameter values can be substantial.

Objectives

- Propose a procedure based on statistical design of experiments that can be used to find effective parameter values in a structured way.
- Demonstrate how our procedure works
  - Select a small set of problems from a set of 34 VRPs.
  - Perform our procedure on a small set of problems to find high-quality parameter values for two new VRP heuristics.
  - Set the parameter values of the heuristics using experimental design.
  - Solve the entire set of problems.

Steps in the Procedure

1. Select a subset of problems to analyze (analysis set) from the entire set of problems.

2. Set the starting value of each parameter, the range over which each parameter will be varied, and the amount to change each parameter.

3. Find good parameter values for each problem in the analysis set using design of experiments.

4. Combine the values obtained in Step 3 to obtain high-quality parameter values.

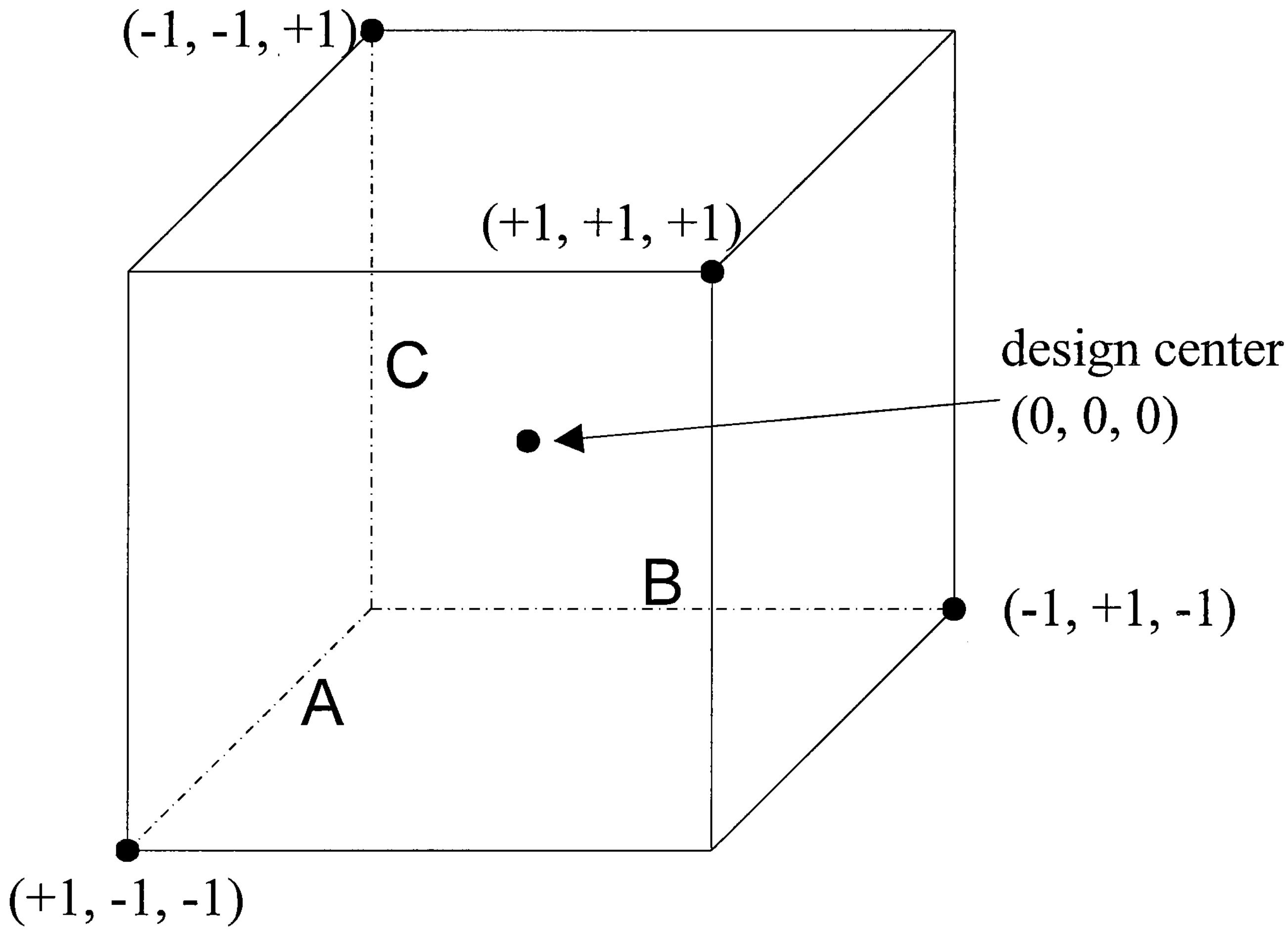
Step 3 Details

- Conduct a two-level fractional factorial experiment.
- Fit a linear regression model to the data generated by the factorial experiment.
- Find the path of steepest descent.
- Test performance of the heuristic at steps along the path of steepest descent.
- Stop when additional steps do not improve the objective function value.
- Save the parameter values associated with the minimum objective function value.

Factorial Experimental Design

- Two-level factorial design
  - Each parameter is tested at a low level and a high level.
    - The low and high levels are usually denoted as -1 and +1, respectively.
    - Tests are performed at design center - Δ and design center + Δ.
    - Some *a priori* knowledge of how the process performs under different conditions is necessary to determine a design center and Δ.
- In practice, it is usually necessary to use a fractional factorial design.
  - Full factorial designs are usually too time consuming for a large number of parameters.

Fractional Factorial Design (2^{2-1})



Computational Test

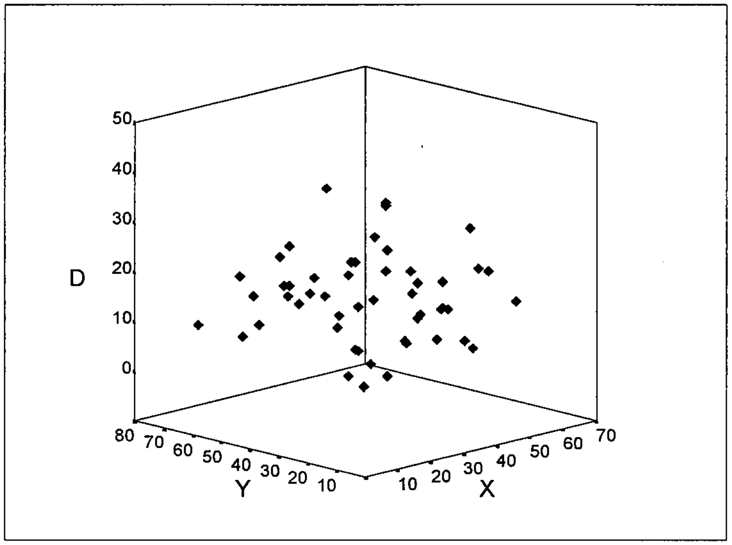
- Our test consists of 34 VRPs.
    - 14 problems from Christofides, Mingozzi, and Toth.
    - 20 Large-scale VRPs that range in size from 240 to 483 cities.
- Two heuristics based on Lagrangean relaxation.
    - LT uses two-opt as the local search during the Lagrangean-relaxation phase.
    - LS uses an iterative data smoothing procedure (sequential smoothing) and two-opt during the Lagrangean-relaxation phase.
- Two experiments for each heuristic.
    - Capacity-constrained problems (six parameters).
    - Capacity and distance-constrained problems (eight parameters).

Selecting Problems for the Analysis Set

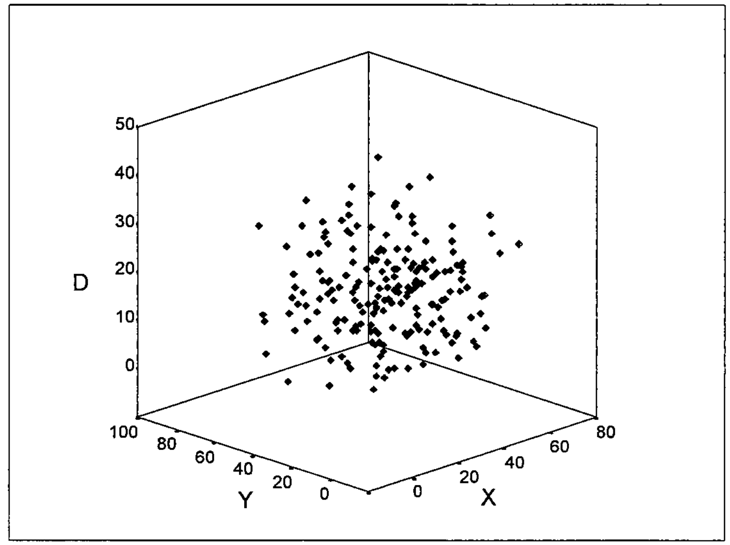
- Select the problems so that most of the structural differences found in the problem set are represented in the analysis set.

- Since the time it takes to perform the procedure is directly related to the size of the analysis set, keep the set small.

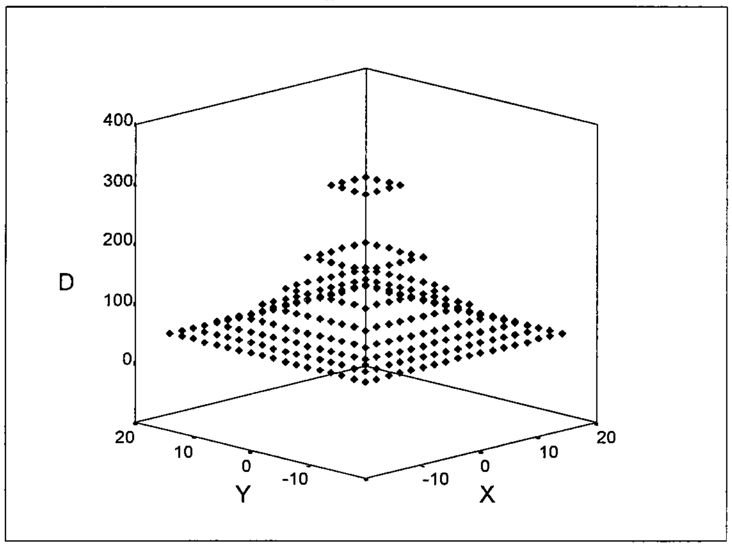
Analysis Set for the Capacity-constrained Experiments



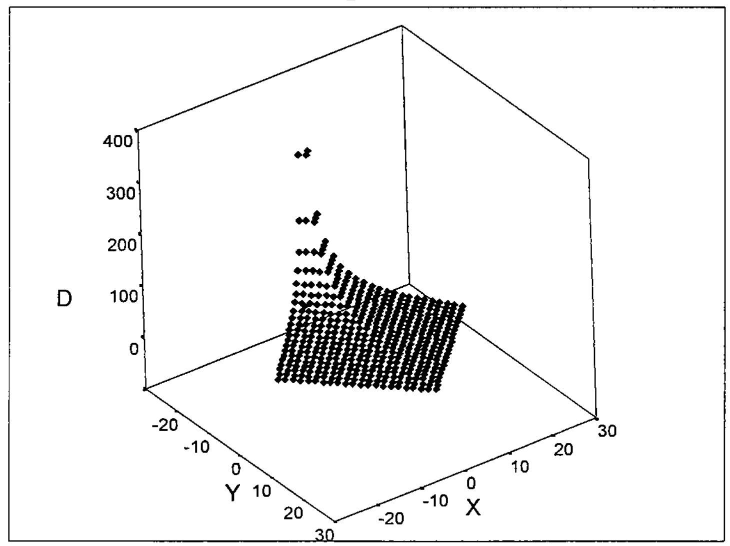
Problem 1. N = 50; depot located at (40, 40)



Problem 5. N = 199; depot located at (40, 40)



Problem 28. N = 320; depot located at (0, 0)



Problem 26. N = 483; depot located at (0, -21)

A Half-fraction Factorial Design (2^{6-1}) for the Capacity-constrained Problems

	Modified Lagrangean Relaxation Parameters					
Run	A	B	C	D	E	F
1	-1	-1	-1	+1	+1	-1
2	+1	-1	-1	+1	+1	+1
3	-1	+1	-1	+1	+1	+1
4	+1	+1	-1	+1	+1	-1
5	-1	-1	+1	+1	+1	+1
6	+1	-1	+1	+1	+1	-1
7	-1	+1	+1	+1	+1	-1
26	+1	-1	-1	-1	-1	+1
27	-1	+1	-1	-1	-1	+1
28	+1	+1	-1	-1	-1	-1
29	-1	-1	+1	-1	-1	+1
30	+1	-1	+1	-1	-1	-1
31	-1	+1	+1	-1	-1	-1
32	+1	+1	+1	-1	-1	+1
33	0	0	0	0	0	0

Each parameter value on a given run is determined by multiplying Δ by the experimental design coefficient and adding the result to the parameter value at the design center.

For example, on run 1 the value for parameter A is

$C_A + (-1)*\Delta_A$.

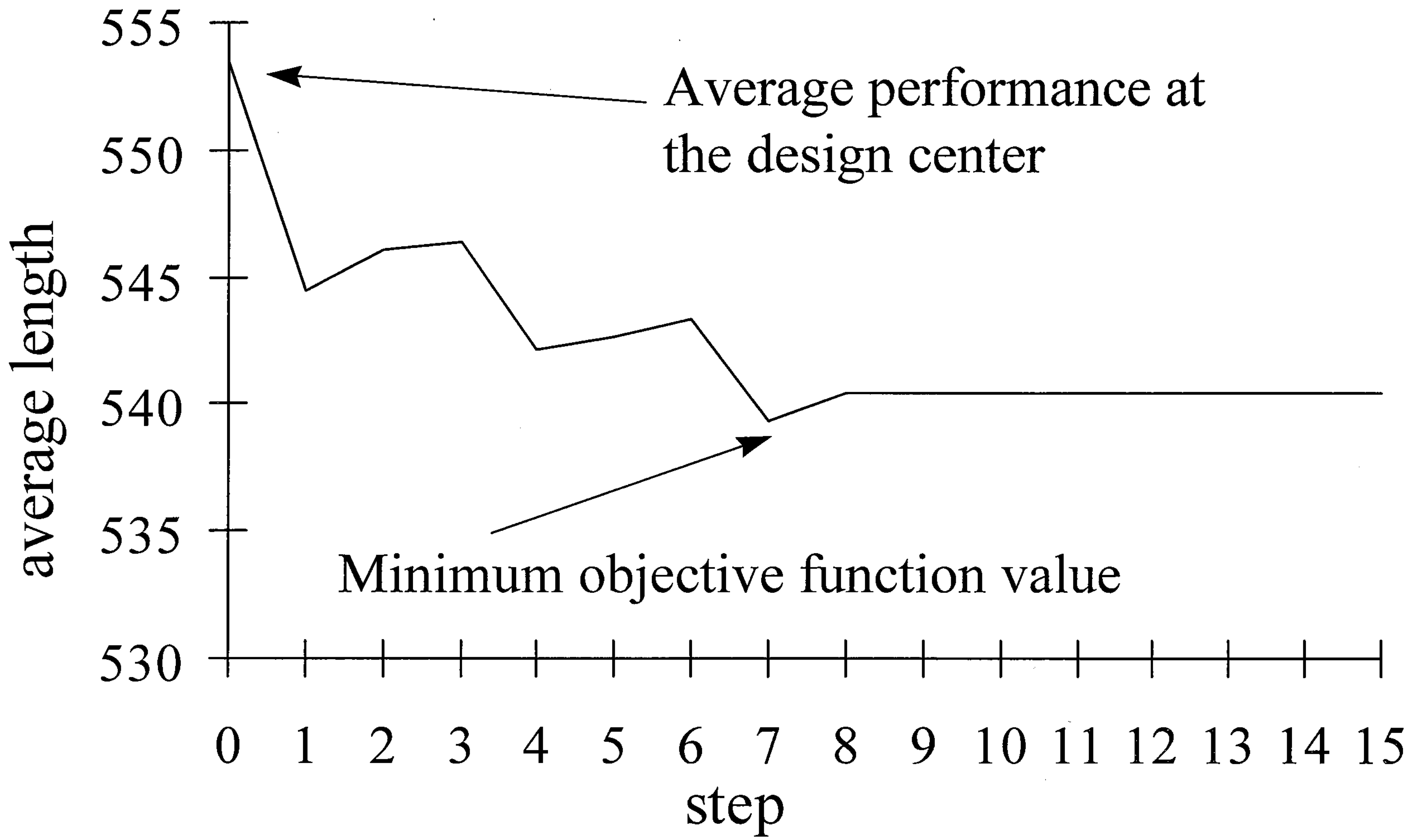
Fractional Factorial Experiment

- For our experiments, we performed five trials for each set of parameter values associated with a row in the experimental design.
  - Starting solutions were generated randomly.
  - The same five solutions were used for each row of the experimental design.
- Total route length from each of the five trials was averaged together to evaluate the heuristic for a set of parameter values.

Response Surface Optimization

- Fit a linear regression model to the data generated by the factorial experiment and find the path of steepest descent on the response surface.
- Adjust the parameters to make small steps down the path of steepest descent.
- Evaluate the performance of the heuristic after each step by running the heuristic with the current parameter values on the same five initial solutions used earlier.
- Stop when it is clear that additional steps would not improve the objective function value.

Response Surface Optimization on Experiment 1, Problem 1



Combining the Parameter Values

Experiment 1 (LT)						
Prob.	A	B	C	D	E	F
1	0.1068	1.2500	0.5000	0.1500	1.7500	11.2500
5	0.0750	1.0176	0.0135	0.1500	1.7500	12.9495
28	0.1757	1.0250	0.5000	0.1500	1.7500	6.0000
26	0.0425	1.0277	0.5000	0.1500	1.7500	15.0000
Avg.	0.1000	1.0801	0.3784	0.1500	1.7500	11.2999

Computational Results

Problem set	LT	LS	TS	RTR
CMT	1.88	**1.38**	1.72	2.72
Large-scale	4.47	**2.89**	2.91	3.26
Overall	3.41	**2.27**	2.44	3.04
Total proc. time (hrs.)	18.1	23.5	903.9	15.6

Results are the average percentage above the best-known solution and total processing time in hours.

LT — Lagrangean relaxed two-opt

LS — Lagrangean relaxed sequential smoothing

TS — Tabu search heuristic of Xu and Kelly, 1996

RTR — record-to-record travel heuristic of Golden et al., 1998

Conclusions

- In this paper, we propose a single-pass procedure based on statistical design of experiments that systematically selects effective parameter values.
- In a computational study of heuristics for the VRP, our procedure worked well.
  - The procedure appeared to make reasonable choices.
  - Even though we restricted each of our heuristics to a single choice of parameter values, they were still able to generate high-quality solutions.