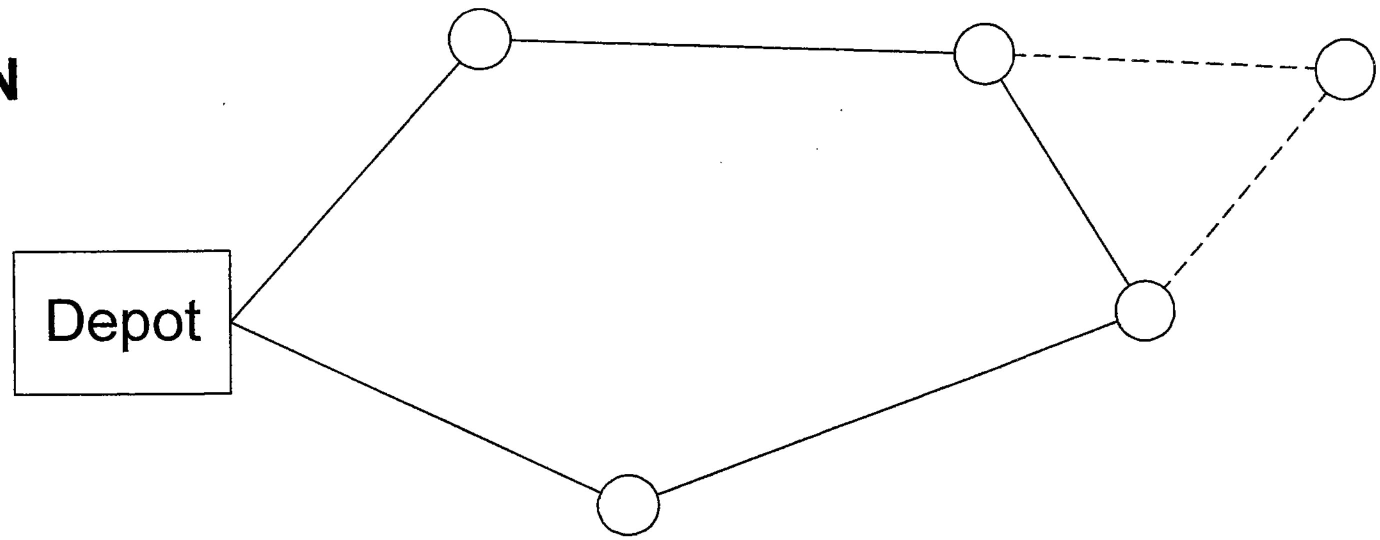
THE VEHICLE ROUTING PROBLEM

CLASSIFICATION OF SOLUTION STRATEGIES IN VEHICLE ROUTING

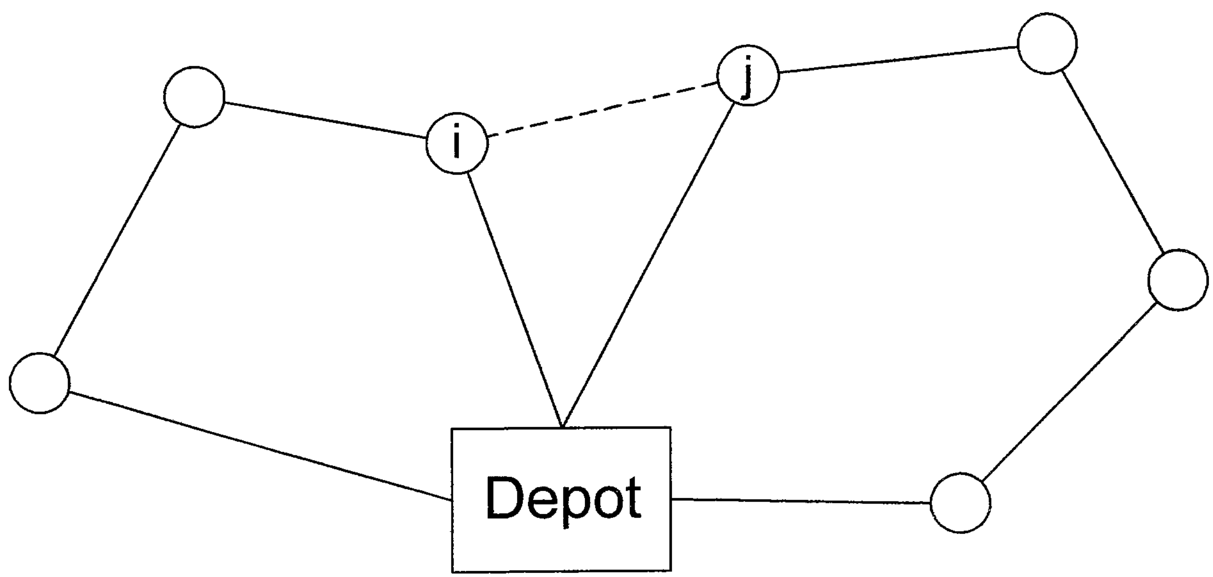
- ♦ Savings or insertion
- ♦ Cluster first — route second
- ♦ Route first — cluster second
- ♦ Improvement or exchange
- ♦ Mathematical programming based
- ♦ Interactive optimization
- ♦ Exact procedures
- ♦ Metaheuristics

SAVINGS AND INSERTION PROCEDURES

INSERTION

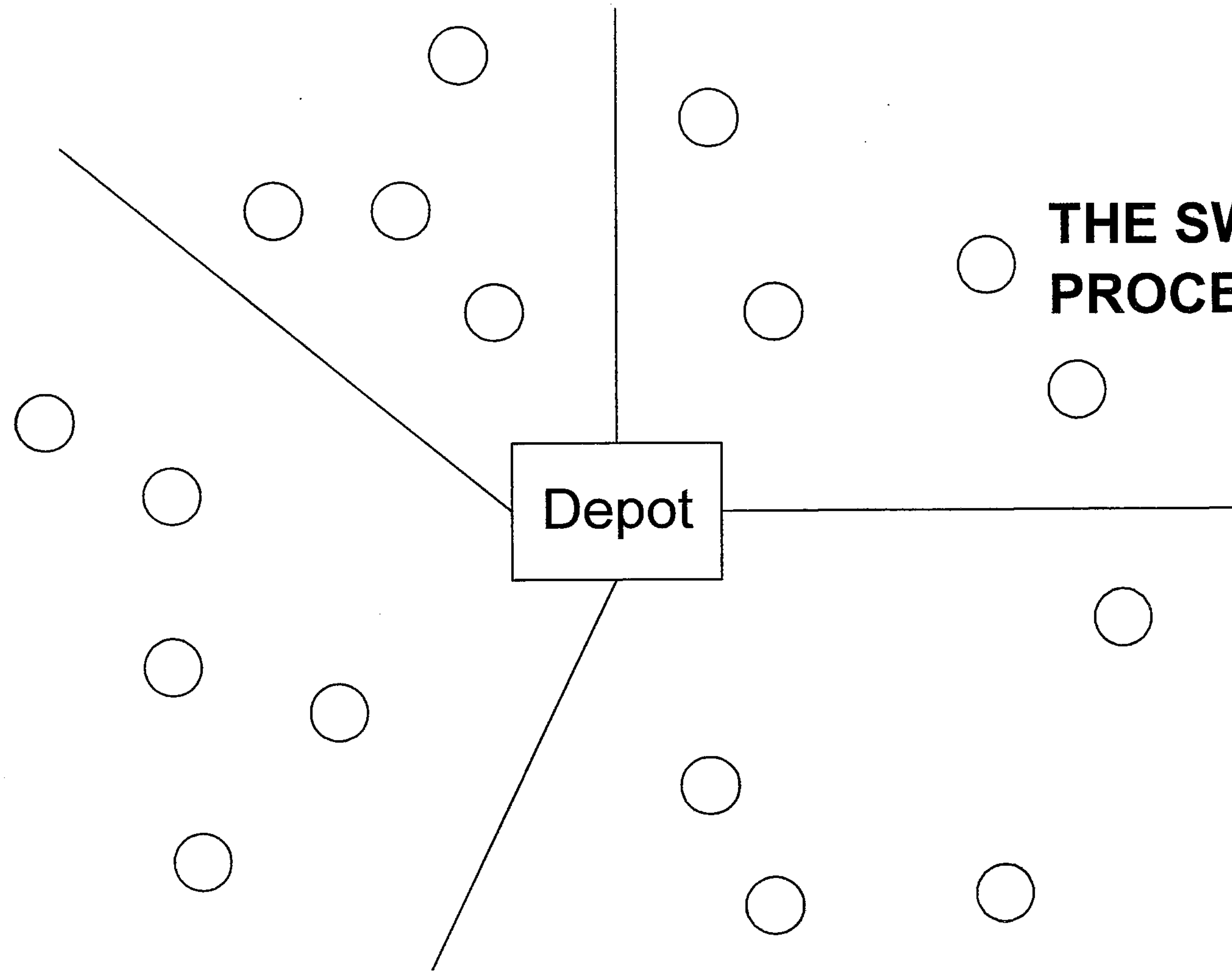


SAVINGS PROCEDURE

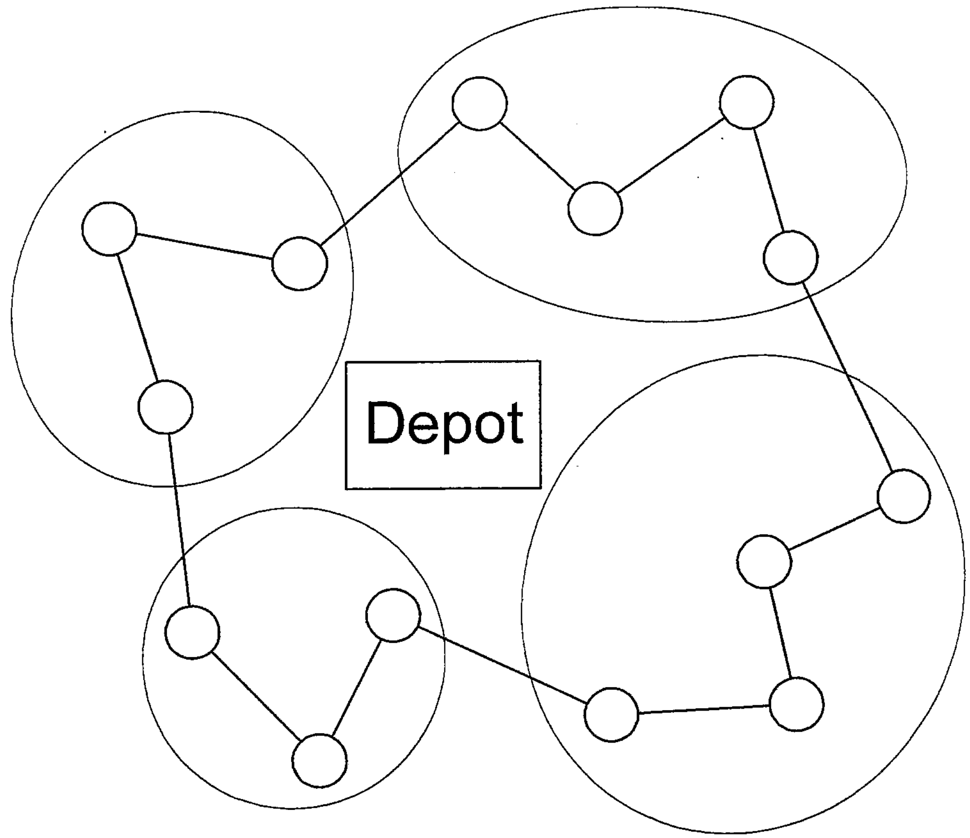


CLUSTER FIRST — ROUTE SECOND

THE SWEEP PROCEDURE



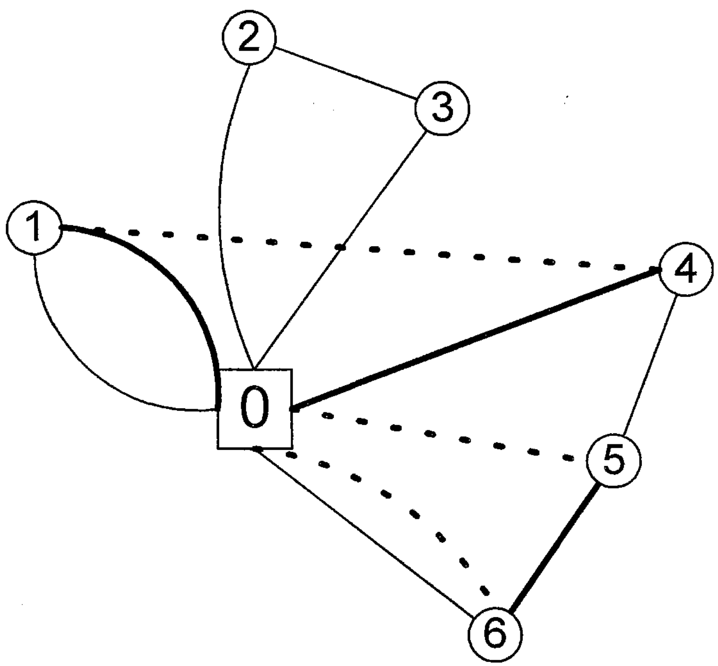
ROUTE FIRST — CLUSTER SECOND



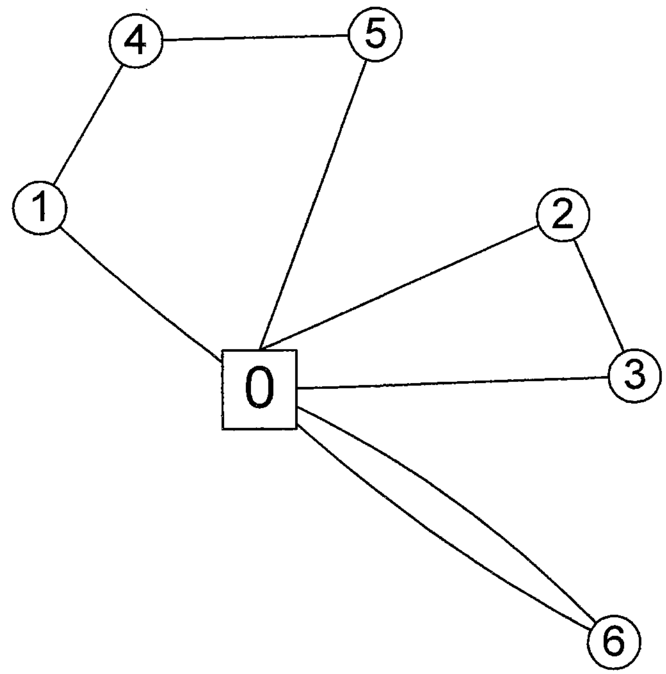
Take capacity and route-length constraints into account while clustering.

IMPROVEMENT PROCEDURES

- Two-opt, OROPT, and three-opt can be easily adapted to handle load and distance constraints



Before three-exchange



After three-exchange

- Metaheuristics can be applied to the VRP as they are applied to the TSP.

SOLVING THE VRP — AN EXAMPLE

- We apply the Clarke-Wright heuristic to the VRP instance below. Note that vehicle capacity is 820.

Customer	Demand
1	486
2	541
3	326
4	293
5	24
6	815
7	296

Distance Matrix

	0	1	2	3	4	5	6
1	19	--					
2	57	51	--				
3	51	10	49	--			
4	49	53	18	50	--		
5	04	25	30	11	68	--	
6	12	80	06	91	62	48	--
7	92	53	47	38	09	94	09

- Node 6 cannot be paired with any other customer.
  So, one route is 0—6—0.

COMPUTATION OF SAVINGS

Savings	Feasible?	Amount of Savings	Rank
1, 2	NO	NA	
1, 3	YES	$c_{01}+c_{03}-c_{13}$ = 19 + 51 - 10 = 60	3
1, 4	YES	$c_{01}+c_{04}-c_{14}$ = 19 + 49 - 53 = 15	8
1, 5	YES	$c_{01}+c_{05}-c_{15}$ = 19 + 4 - 25 = -2	10
1, 7	YES	$c_{01}+c_{07}-c_{17}$ = 19 + 92 - 53 = 58	4
2, 3	NO	NA	
2, 4	NO	NA	

COMPUTATION OF SAVINGS

Savings	Feasible?	Amount of Savings	Rank
2, 5	YES	$c_{02}+c_{05}-c_{25}$ = 57 + 4 - 30 = 31	7
2, 7	NO	NA	
3, 4	YES	$c_{03}+c_{04}-c_{34}$ = 51 + 49 - 50 = 50	5
3, 5	YES	$c_{03}+c_{05}-c_{35}$ = 51 + 4 - 11 = 44	6
3, 7	YES	$c_{03}+c_{07}-c_{37}$ = 51 + 92 - 38 = 105	2
4, 5	YES	$c_{04}+c_{05}-c_{45}$ = 49 + 4 - 68 = -15	11
4, 7	YES	$c_{04}+c_{07}-c_{47}$ = 49 + 92 - 9 = 132	1
5, 7	YES	$c_{05}+c_{07}-c_{57}$ = 4 + 92 - 94 = 2	9

FORMATION OF ROUTES

- Link nodes 4 and 7. The load is 589.
- Link nodes 1 and 3. The load is 812.
- Link nodes 2 and 5. The load is 565.
- No other pair of nodes can be linked, due to vehicle capacity. Total distance =345.

The VRP Solution

Route	Load	Distance
0-6-0	815	24
0-4-7-0	589	150
0-1-3-0	812	80
0-2-5-0	565	91

A REAL-WORLD APPLICATION: Route Planning for Coast Guard Ships

- U. S. Coast Guard maintains waterway navigation aids such as buoys.
- Buoys serve many roles
  - mark channels,
  - mark isolated rocks,
  - mark mine fields,
  - mark cables,
  - collect oceanographic and meteorological data.
- 40,000 buoys must be inspected and serviced at least once/year.
- For example, there are approximately 250 buoys in the Long Island Sound alone.

Route Planning for Coast Guard Ships-continued

- The Coast Guard would like to
  - schedule ships conveniently using an automated procedure so as to minimize a measure of overall resource utilization,
  - use a route planning system to help determine which ships to acquire in order to meet anticipated, future needs.
- Constraints encountered in practice
  - Different home bases for ships.
  - Ships of different types visit buoys from different home bases.
  - Different types of ships have different capabilities.
  - Travel routes are ship-dependent.
  - Different buoys may require different ships.

Route Planning for Coast Guard Ships—continued

- Constraints encountered in practice —continued
  - Emergencies may require ships to be reassigned or rescheduled on short notice.
  - Numerous resource limitations on ships and bases.
    - distance
    - fuel
    - time
    - daylight
    - storage space
    - etc.
- Complicating Issues
  - Multiple ships
  - Deep and shallow water
  - Overnight trips
  - Dynamic scheduling — real-time adjustments

Route Planning for Coast Guard Ships—continued

- Complicating issues — continued
  - ♦ Impact of weather on cruising speed
  - ♦ Strategic planning
    - What would happen if additional ships were assigned to a given base?
    - What would happen if a different mix of ships was assigned to a given base?
    - What would happen if a different set of buoys was assigned to a given base?

EXAMPLES OF MORE COMPLICATED VRP'S

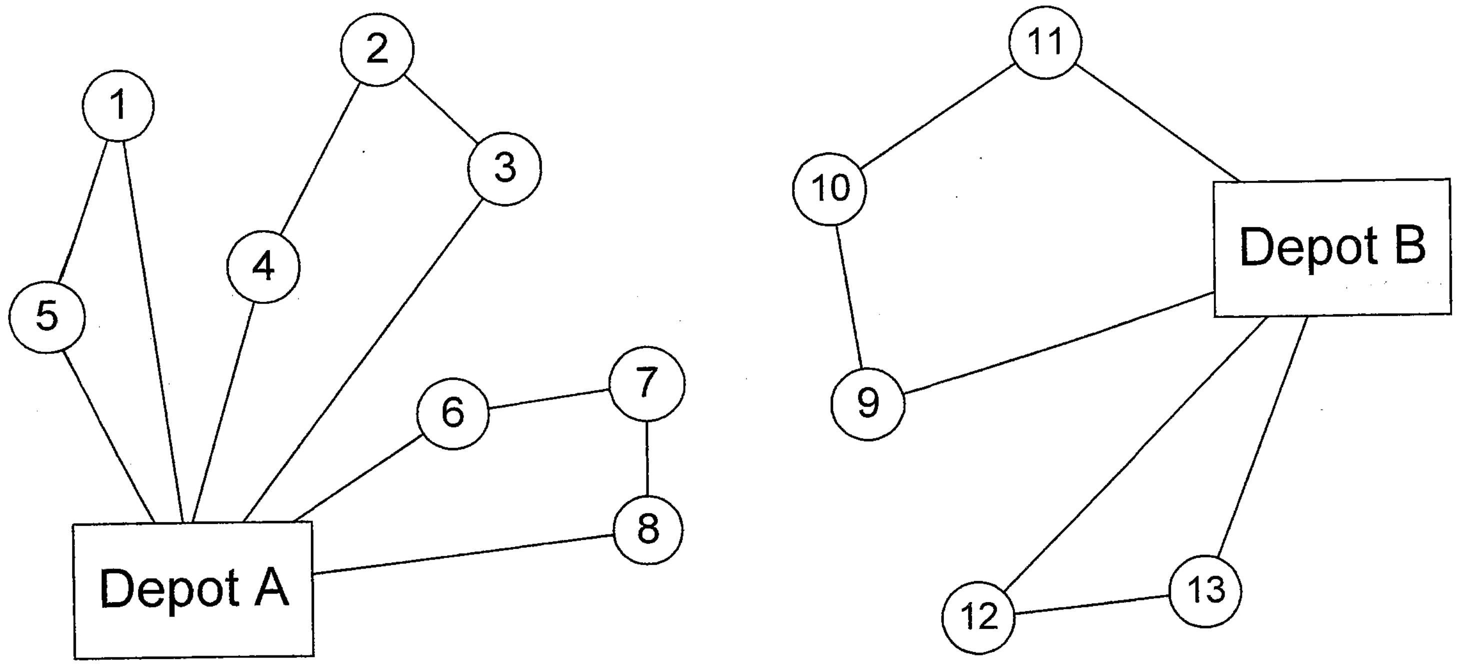
- The multi-depot vehicle routing problem (MDVRP)
- The period vehicle routing problem (PVRP)
- The vehicle routing problem with backhauls

THE MULTI-DEPOT VEHICLE ROUTING PROBLEM

PROBLEM DESCRIPTION

- More than one depot.
- Known customer locations and demands.
- Homogeneous fleet of vehicles.
- Capacity constraints.
- Route-length constraints.
- Each vehicle departs from and returns to same depot.
- Each customer is visited once.
- Customer demand must be fully satisfied.
- We seek to minimize total distance traveled by the fleet.

ILLUSTRATION OF THE MDVRP



Route 1: Depot A - 1 - 5 - Depot A

Route 2: Depot A - 3 - 2 - 4 - Depot A

Route 3: Depot A - 6 - 7 - 8 - Depot A

Route 4: Depot B - 9 - 10 - 11 - Depot B

Route 5: Depot B - 12 - 13 - Depot B

THE PERIOD ROUTING PROBLEM

Problem Description

- Design a set of routes for a fleet of vehicles over all days of a planning period in which the customers may not all require delivery on every day in the period.

- Ten customers serviced by a fleet of three vehicles over a five-day work week.
  - vehicle capacity is 30 units.
  - customers have known demands and delivery combinations.

TEN CUSTOMER EXAMPLE

Customer	Allowable Combination	Daily Demand	Number of Days Per Week
1	M Tu W Th F	5	5
2	M Tu W Th F	4	5
3	MWF, MThF, TuThF	7	3
4	MWF, MThF, TuThF	8	3
5	TuF, MTh	10	2
6	TuF, MTh	9	2
7	M Tu W Th F	12	1
8	M Tu W Th F	8	1
9	M Tu W Th F	11	1
10	M Tu W Th F	6	1

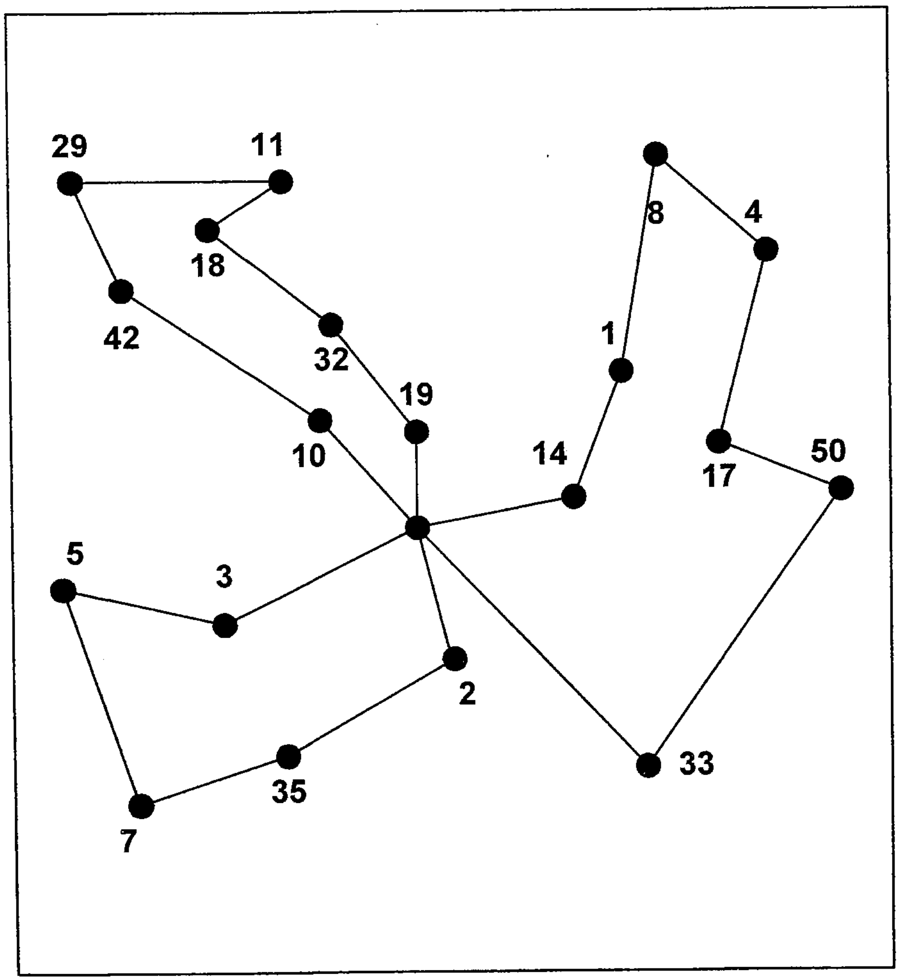
THE PERIOD ROUTING PROBLEM – CONTINUED

Problem Definition

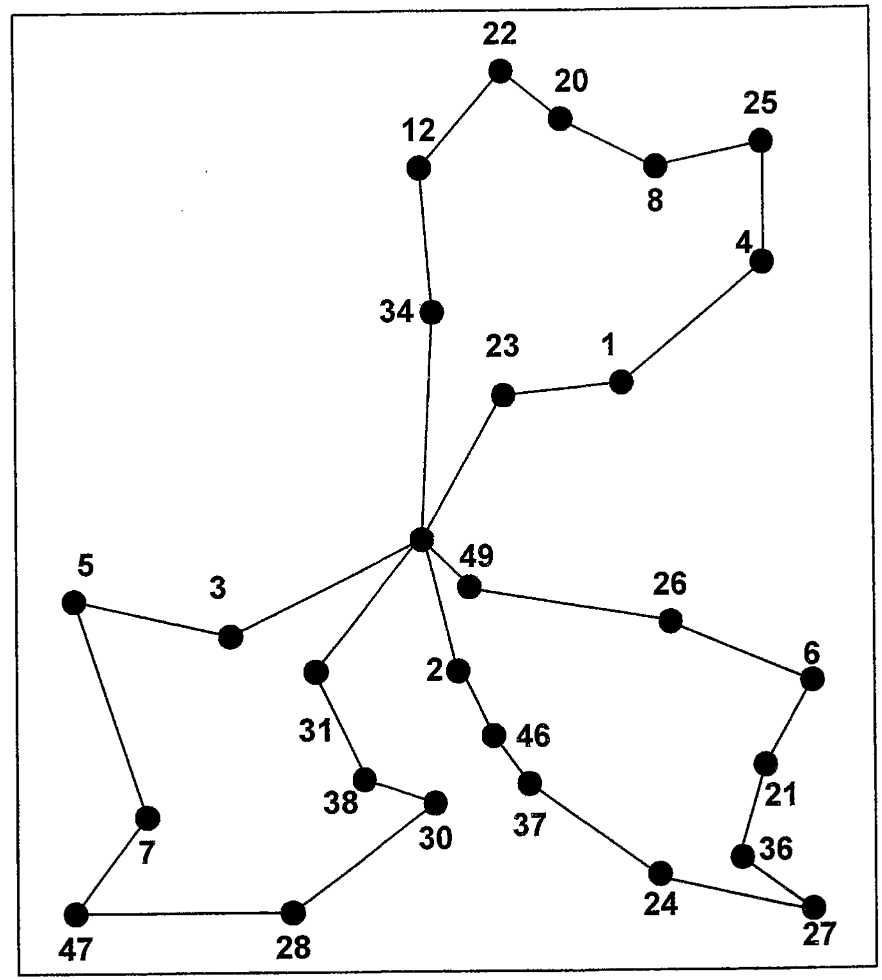
- Given the daily demand for each customer, the location of each customer, and the allowable delivery combinations, design routes for each day of the planning period.

- The PVRP is more complex that the VRP.
  - In the VRP, the set of customers to be serviced each day is known and fixed in advance and routes are then designed one day at a time. Each day is independent.
  - In the PVRP, the days interact.

ILLUSTRATION OF THE PVRP

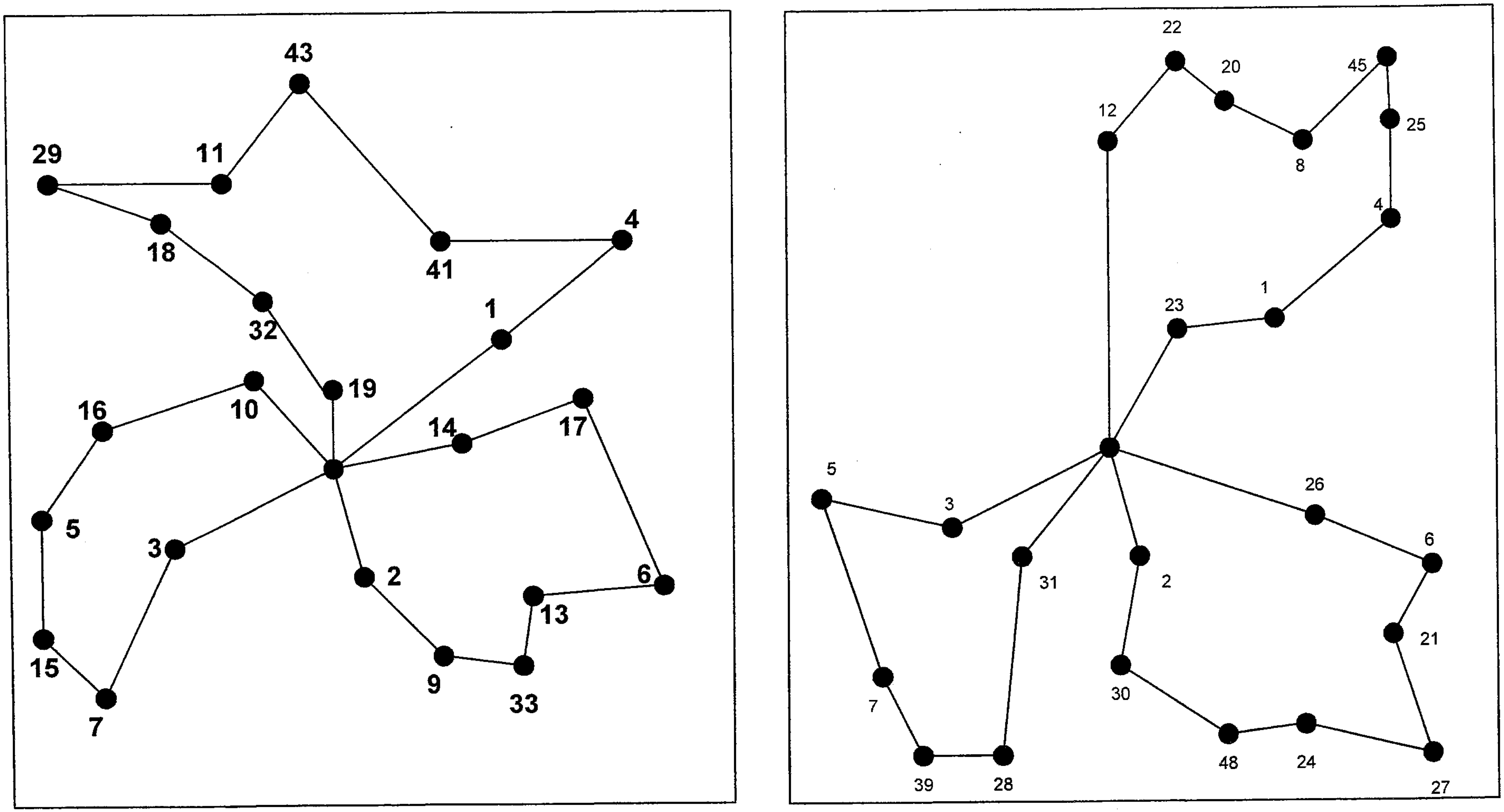


Monday



Tuesday

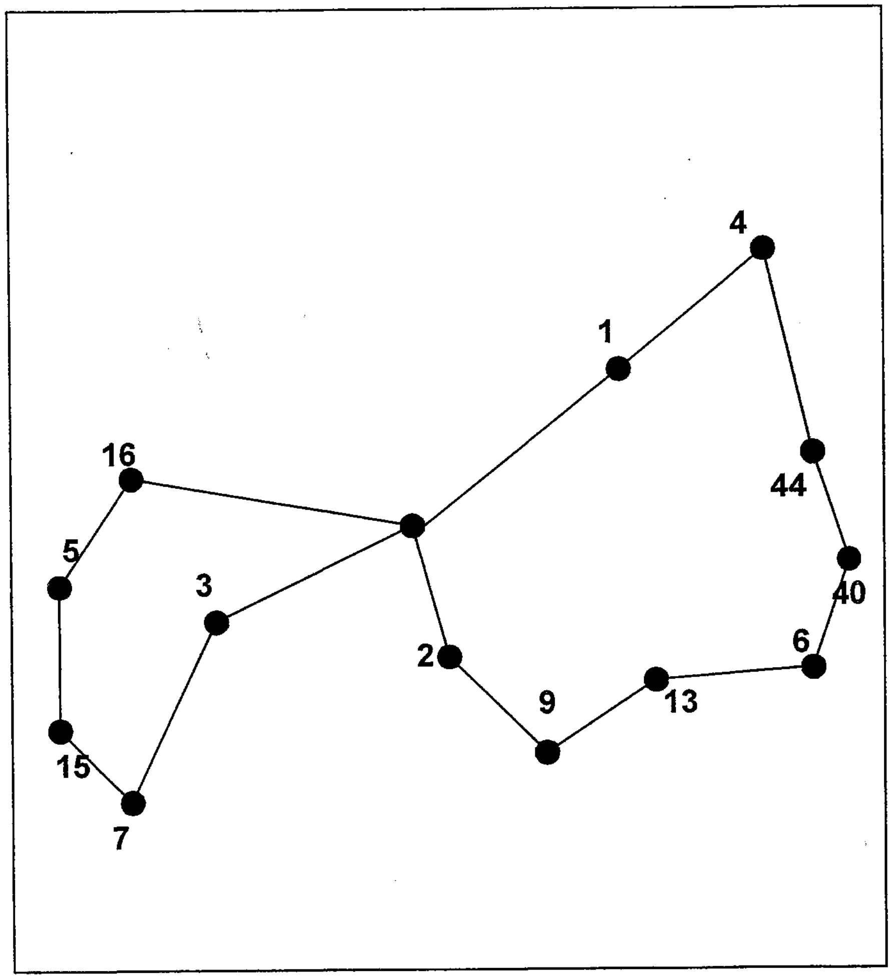
ILLUSTRATION OF THE PVRP



Wednesday

Thursday

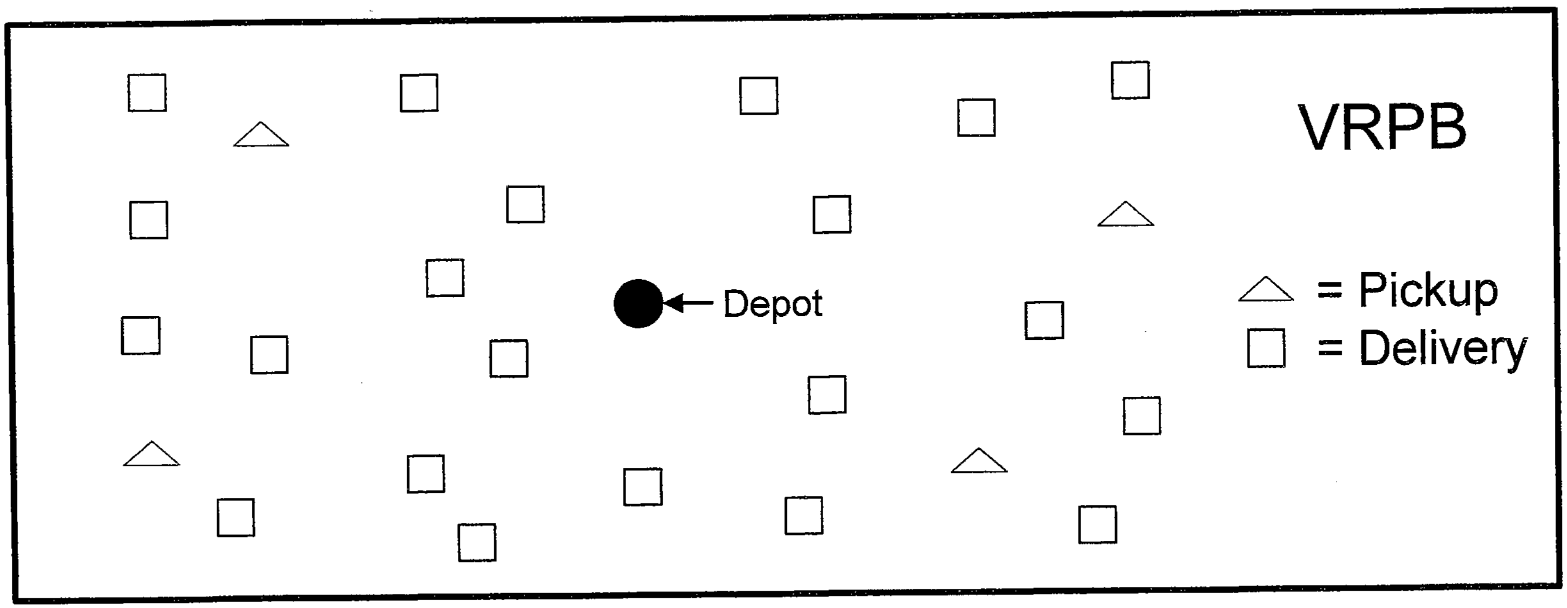
ILLUSTRATION OF THE PVRP



Friday

THE VEHICLE ROUTING PROBLEM WITH BACKHAULS (VRPB)

- Problem statement
  - Find a set of vehicle routes that services the delivery and backhaul (pickup) customers such that vehicle capacity is not violated and total distance traveled is minimized.



THE VEHICLE ROUTING PROBLEM WITH BACKHAULS (VRPB) — CONTINUED

- Problem statement —continued
  - Such a customer mix occurs in many industries (e. g., the grocery industry).

    supermarkets — delivery points

    poultry processors, fruit vendors — backhauls

- Backhauls are serviced near the end of a route.
  - deliveries are high-priority stops
  - small number of backhauls
  - difficult to rearrange on-board load in rear-loaded vehicles

Integrated Logistics Models

- Coordinating with production and inventory
- Integrating vehicle routing and purchasing decisions
  - Propane gas distribution example
  - Four-level supply chain system

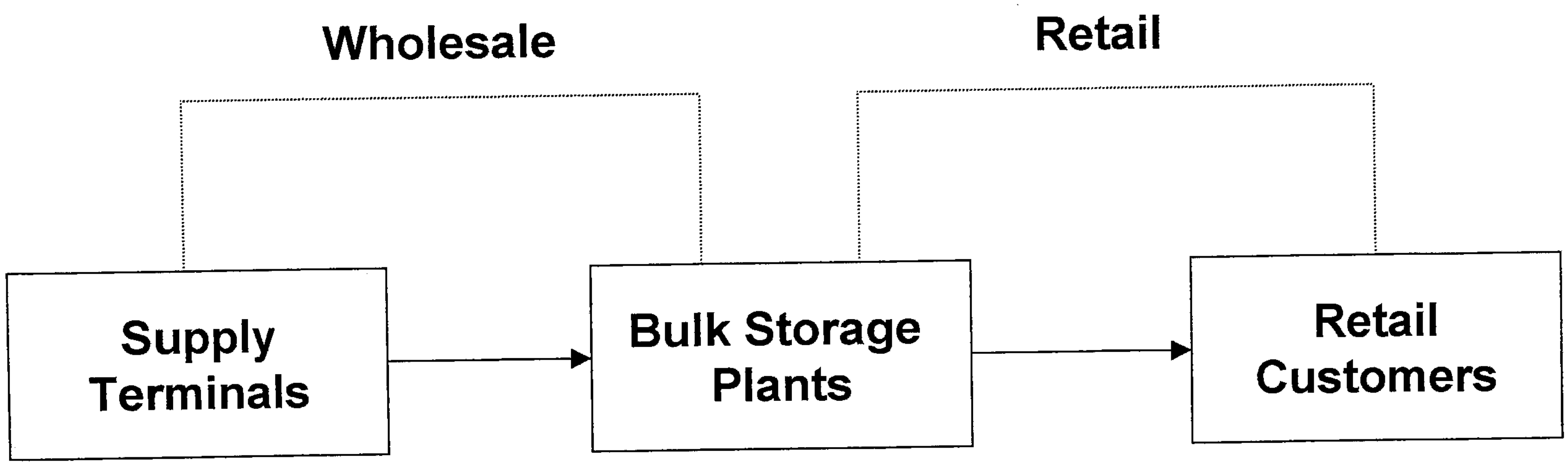
    propane producers

    regional supply terminals

    distributor-owned storage plants

    customers

Propane Gas Logistics



- Chiang and Russell (1998) focus on the wholesale portion of the supply chain

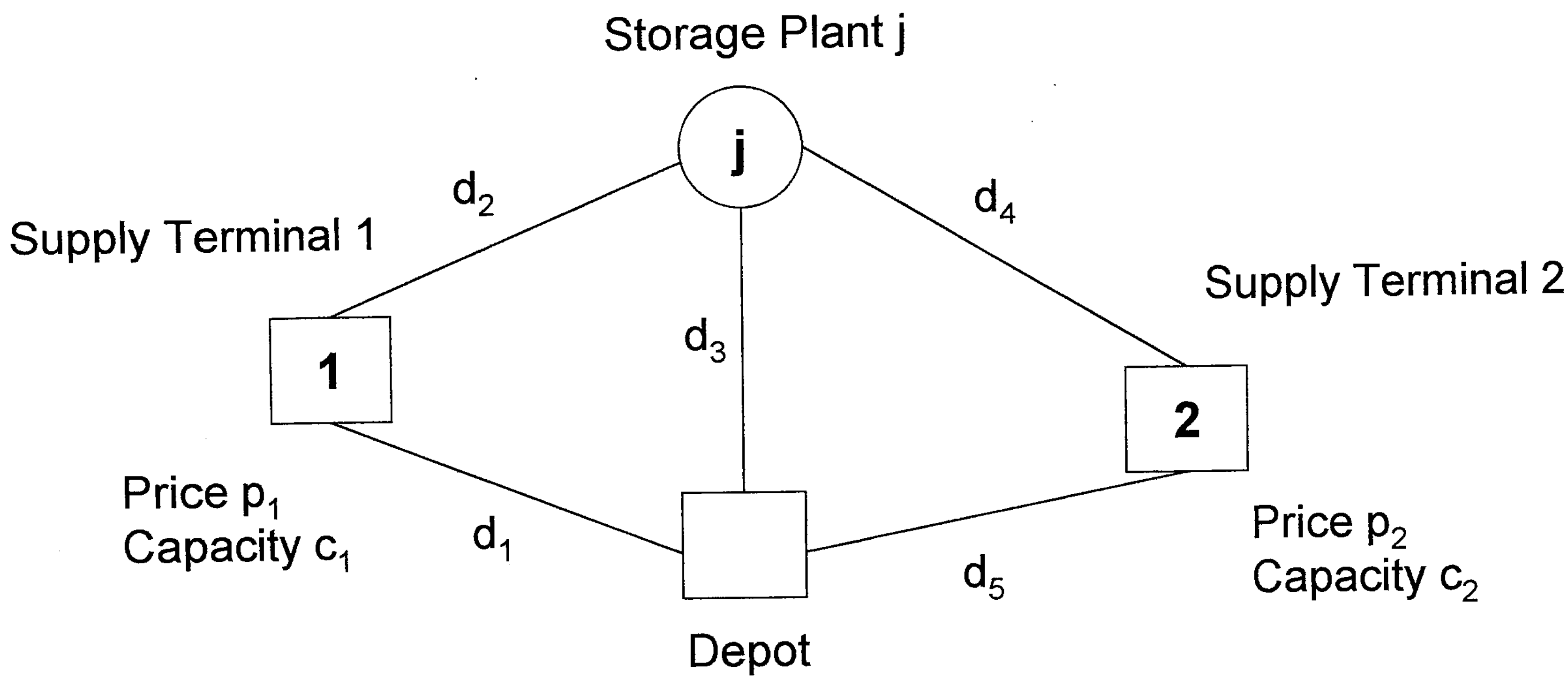
Problem Characteristics

- Inventory levels at storage plants are known
- Time windows
- Tankers are routed around the clock
- DOT regulations limit drivers
- Gas prices vary from one supply terminal to the next
- Queuing delays are possible at supply terminals
- All deliveries are full loads
- On average, two storage plants are visited per route

Key Decisions

- Identification of the storage plants to be visited on a given day
- Determination of which depot services the storage plants
- Determination of which terminal supplies the storage plant given price and capacity constraints
- Construction of efficient routes involving depot, terminal(s), and storage plant(s)

Typical Cost Structure



Assigning j to 1 costs: \$ 9000 p_1 + \$ 1.50 $(d_1 + d_2 + d_3)$

Assigning j to 2 costs: \$ 9000 p_2 + \$ 1.50 $(d_3 + d_4 + d_5)$

Weekly Results on Sample Data

Approach	Tankers used	Miles driven	Gas cost	Total cost
Gasco	56	19,808	$ 532,826	$ 562,536
Least Distance	44	14,803	$ 544,695	$ 565,818
Least Cost	58	23,155	$ 500,282	$ 535,013

Main Trends

- Better evaluation of methods
- New algorithms
- More realistic vehicle routing problems
- Real-time applications
- Integrated logistics models