COALITION FORMATION IN MANETS

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ABSTRACT

Wireless ad-hoc networks rely on the cooperation of participating nodes for almost all their functions. However, due to resource constraints, nodes are generally selfish and try to maximize their own benefit when participating in the network. Therefore, it is important to study mechanisms which can be used as incentives to form coalitions inside the network. In this paper, we study coalition formation based on game theory, especially cooperative game theory. First, the dynamics of coalition formation proceeds via pairwise bargaining. We show that the size of the maximum coalition is a decreasing function of the cost for establishing a link. After the coalition formation process reaches the steady state, we are interested in the stability of coalitions. We prove that coalitions are stable in terms of both pairwise stability and coalitional stability.

1. INTRODUCTION

Ad hoc networks rely on the cooperation of participating nodes for almost all their functions, for instance, to route data between source and destination pairs that are outside each other's communication range. However, because nodes are resource constrained, we deal with networks composed of selfish users who are trying to maximize their own benefit from participation in the network. In particular, we assume that each user is in complete control of his network node. In this case, the fundamental user decision is between forwarding or not forwarding data packets sent by other users. Given the constraints (mostly related to battery power) that the user faces, there is a very real cost incurred when choosing to forward. So, all users would like to send their own data packets, but not forward those of other users. Unfortunately, if all users were to do that, the network would collapse. In order to form the necessary infrastructure that makes multi-hop communication achievable, cooperation enforcement mechanisms are needed to cope with such selfish behavior of nodes in ad hoc networks.

One type of mechanism introduces incentives for collaboration. Buttyan and Hubaux (Buttyan and Hubaux, 2003), for instance, propose to use nuglets to reward nodes who choose to forward. In (Buchegger and Boudec, 2003; Marti et al., 2000), trust or reputation systems are used as another mechanism to promote cooperation and circumvent misbehaving nodes. A growing body of literature, a comprehensive overview of which is in (Félegyházi et al., 2006) deals with circumstances under which the cooperation between nodes can be sustained.

The conflict between the benefit from cooperation and the required cost for cooperation naturally leads to game-theoretic studies, where each node strategically decides the degree to which it volunteers its resources for the common good of the network. The players in game theory attempt to maximize an objective function that takes the form of a payoff. Users make choices and each user's payoff depends not only on his own choice, but also on those of the other users. Hence, in the wireless network context, a user's payoff depends not only on whether he decides to cooperate (by transmitting other users' data) or not, but also on whether his neighbors will decide to cooperate. Srinivasan et al. (Srinivasan et al., 2003) address the problem of cooperation among energy constrained nodes and devised behavior strategies of nodes that constitute a Nash equilibrium. In (Johari et al., 2006), there is a link between two nodes if they agree to cooperate. These links are formed through one-to-one bargaining and negotiation.

In this work, we assume that users want to be connected to as many other users as possible, directly (one-hop) or indirectly (multi-hop, through other users). This is the incentive that, according to our scenario, the users have for forwarding packets. In other words, by activating a communication link towards one of their neighbors, they gain by having access to the users with which that neighbor has activated his links, and so on, recursively. The more users that a user has access to, the more desirable it is for his neighbors to activate their link towards him.

We study cooperation based on the notion of coalitions. The concept of users being connected to each other, and -- by getting connected -- acquiring access to all the other users that each of them had so far access to, can be well captured by cooperative game theory (also known as coalitional game theory (Osborne and Rubinstein, 1991)). In cooperative game theory, the central concept is that of coalition formation, i.e., subsets of users that join their forces and decide to act together. Players form coalitions to obtain the optimum payoffs. The key assumption that distinguishes cooperative game theory from non-cooperative game theory is that players can negotiate collectively (Myerson, 1991).

A question that has only relatively recently began to attract attention ((Aumann and Myerson, 1988) is the first work in this area) is the actual way in which the coalition is formed. The cooperative game is usually modeled as a two-period structure. Players must first decide whether or not to join a coalition. This is done by pairwise bargaining, in which both players have to agree to join in a coalition. In our case, this pairwise bargaining involves, for each node, a comparison between the cost for activating the link towards the other node, and the benefit from joining the coalition, which the other player is a member of. The pairwise bargaining is modeled as a non-cooperative game. In the second step, players in the coalition negotiate the payoff allocation. The central problem is to study the payoff allocation scheme and whether the scheme results in a stable solution.

The rest of the paper is organized as follows: In Section 2, we describe the mathematical framework within which we deal with the concepts just discussed. The terminology we use in the paper is defined. In Section 3, we state and prove some properties of our model. We also evaluate our approach using simulations. Section 4 analyzes, from the point of view of cooperative game theory, the stability of the solutions (coalitions) that our model finds. Section 5 concludes the paper.

2. PROBLEM FORMULATION

The model we use for our scenario is a complete undirected graph $G=(V,E)$ with weighted edges, where $w(i,j)$ is the weight of edge $(i,j) \in E$. In general, however, we allow for a different weight $w(j,i) \neq w(i,j)$ for the same edge. The motivation will soon be clear. Each node corresponds to a user, and the weight $w(i,j)$ represents the cost that user i needs to pay in order to *activate* the edge towards j. This cost is in general different from the cost that user j needs to pay, hence $w(j,i) \neq w(i,j)$. If they both decide to activate their edge, then the edge is said to be *active*. Note that all possible topologies can be described in our model, since we can model non-existent edges with edges of infinite weight. The reason is that a node will never choose to activate an edge if he has to pay an infinite cost to do so, which means that that edge is as good as non-existent.

The easier way to link the link activation cost to something specific is to equate it with one-hop transmission energy (or power). So, the cost for user i to activate his communication link to user j is equal to the transmission energy (or power) necessary for i to send data to j. But the cost can also be something like the inverse of the expectation that j will forward i's data: the higher the expectation, the lower the cost. Or it can be the degree of trust between users i and j: the more i trusts j, the lower the cost of establishing the link.

When both i and j activate the edge towards each other, they join in a *coalition*. A coalition is a subset of nodes that is connected in the subgraph induced by the active edges (i.e., the graph $G'=(V,E')$, where E' is the set of active edges). In other words, two users are in the same coalition, if and only if there exists a path of active edges between them. The users in a coalition are the coalition's *members*. If two members of separate coalitions join, then the two coalitions *merge* into one. The *size* of a coalition is the number of its members. A single isolated user can also be said to be in a trivial coalition of size 1.

We have described a user's incurred cost of activating an edge. The *incentive* for user i to activate the edge (i,j) is the size of the coalition that user j is a member of. So, the larger j's coalition is, the greater the incentive for i to activate the edge, provided that they are in separate coalitions. If, on the other hand, i and j are in the same coalition, they will only pay the costs E_{ij}, E_{ji}, but they will not gain anything, since they are already part of the same coalition. In Section 3, we will use this observation to show that coalitions can only have a tree topology (i.e., no cycles formed exclusively with nodes of a coalition).

Putting both the cost and the incentive together, we create a game on the graph G. The players of the game are the users, and the strategies available to user i are "Activate edge (i,j)" and "Not activate edge (i,j)" for each neighboring user j. If the edge (i,j) becomes active, then both users pay the costs and reap the benefits (each his own). Otherwise, neither one receives or pays anything. The game on each edge is shown in Fig. 1.

		User j: A	User j: NA
User i	A	$N_j - E_{ij}, N_i - E_{ji}$	$0, 0$
	NA	$0, 0$	$0, 0$

Fig. 1 The game between i and j for the activation of the edge (i,j), when they are in different coalitions. Only if they both decide to activate, will the edge become active. *A*: Activate, *NA*: Not Activate, N_i, N_j: sizes of i's and j's

coalitions, E_{ij}, E_{ij}: costs for edge activation for *i* and *j*, respectively

We can see from Fig. 1 that an edge (*i*,*j*) will become active if and only if the following *edge activation condition* holds (N_i, N_j: sizes of *i*'s and *j*'s coalitions):

$$N_j \geq E_{ij} \wedge N_i \geq E_{ji}. \quad (1)$$

Note, however, that the values N_i can change after coalitions are formed. In particular, if users *i* and *j* are in different coalitions, the activation of edge (*i*,*j*) will change both N_i, N_j to $N_i + N_j$. This, in turn, may make the edge activation condition true for edges for which it used to be false, and thus enable more coalitions to be formed.

This leads us to observe that the dynamics of the game can be separated in rounds of successive coalition expansions. That is, in the first round, some edges will be activated based on the initial *N*-values, which will increase the appropriate *N*-values for the next round. In the second round, we will examine the new *N*-values to see if new edges can potentially become active, and so on. When no further edge activations are possible, the game has reached an equilibrium.

To simplify the analysis, in Section 3 we consider that the weights E_{ij} are either equal to ∞ (i.e., the corresponding edges do not exist) or to E_i for all neighbors *j*, that is, the cost for user *i* to activate an edge depends only on *i*. Therefore, in the following sections we refer to the *E* weights as node weights, as opposed to edge weights. The edge activation condition in Eqn. (1) becomes

$$N_j \geq E_i \wedge N_i \geq E_j. \quad (2)$$

3. DYNAMIC FORMATION OF COALITIONS

3.1 General Properties

In this section we derive some of the general properties of our model. Despite the fact that a lot depend on the actual topology and the weights (node weights from now on), we can still reach some interesting results.

In general, the weights can take any non-negative real value. But we now show that when it comes to deciding which edges will become active, allowing non-integer values does not make a difference.

Proposition 1: A weight E_i can be replaced with its ceiling $\lceil E_i \rceil$, with no changes in the game dynamics.

Proof: This is rather obvious if we consider the activation condition (2), and notice that the *N* values can only be integers. More formally,

$$N \in \mathbf{N}, E \in \mathbf{R} \Rightarrow (N \geq E \Leftrightarrow N \geq \lceil E \rceil) \quad (3)$$

■

The following proposition deals with the question: Can an active edge stop being active in later rounds? We should note that we do not allow edges to "break" by accident, as could be the case in a real wireless network, where communication links may fail. We only consider purposeful deactivation by a user, in the case where the gain no longer outweighs the cost of activation.

Proposition 2 (monotonicity): If an edge becomes active, then it is never deactivated in the future.

Proof: By induction on the rounds of coalition forming. In the first round, an edge (*i*,*j*) is activated iff E_i=E_j=1, which means that the values N_i and N_j are increased. In the second round, the number of potential edge activations is increased, since the N_i values are increased. Since the N_i values are monotonically non-decreasing, the activation conditions for the already activated edges cannot become false.

■

We now attempt to characterize the coalitions that can form. In particular, given only the first round results, we can already say some things about coalitions that can never form.

Definition 1 (Subset Distance): The distance between two subsets of nodes V_1, $V_2 \subseteq V$ is the shortest distance between any two nodes $i \in V_1$, $j \in V_2$. Formally,

$$d(V_1, V_2) = \min_{i \in V_1, j \in V_2} d(i, j).$$

Definition 2 (Coalition Boundary): The boundary ∂C of coalition $C \subseteq V$ is the set of nodes outside the coalition that have at least one neighbor inside the coalition. Formally,

$$\partial C = \{1 \in V \setminus C \mid (i, j) \in E \text{ for some } j \in C\}.$$

If we know the boundaries of coalitions formed in the first round (i.e. the coalitions consisting exclusively of nodes *i* with E_i=1), then we can determine what coalitions will never form. In particular, the following proposition holds.

Proposition 3: If a coalition *C* is more than 3 hops away from the nearest coalition, then *C* will not join any other coalition.

Proof: If there are 3 nodes (i.e., 4 hops) separating *C* from the nearest first round coalition, then the 2 nodes that are on the boundaries of the two coalitions have weight necessarily larger than 1. The node in the middle is not initially and will never be a member of any non-trivial coalition, since the adjacent nodes will never want to activate their edges towards him. So, *C* will at most expand to include its boundary, but not more than that.

■

As a corollary, only first round coalitions whose boundaries intersect (i.e., the set intersection of their boundaries is non-empty) or touch (i.e., there exists an edge from a node of one boundary to a node of the other boundary) can eventually join each other directly (i.e., not through other coalitions). For example, two coalitions whose boundaries intersect will join directly, if and only if at least one intersection node has weight less than or equal to both coalition sizes.

So, in effect, the nodes with *E*-values larger than 1 create a barrier for the expansion of the coalition. If that barrier is more than 2 nodes deep, i.e., the distance between two first round coalitions is more than 3, then it can never be overcome. The next proposition is one we have already hinted at in a previous section.

Proposition 4: All coalitions formed are trees.

Proof: Since the coalition value is a function of the number of its members, and not of its structure, any extra links between nodes that are already connected only cost, and do not add to the value.

■

From Proposition 2, we see that, once formed, a link will never be destroyed. This, however, crucially depends on the payoff that each node receives by joining the coalition. The way our non-cooperative game is described, each member of the coalition receives a payoff equal to the size of the coalition. That is, the total value of a coalition is equal to the square of its size, which is exactly the network effect.

$$v(C) = |C|^2 \tag{4}$$

3.2 Experimental Evaluation

For a specific topology and a specific choice of *E*-weights, the coalitions that will form are deterministically defined. We are interested in the size of the largest coalition. The most desirable outcome would be if the global coalition would form, but this does not always happen.

In our simulations, we consider 100 players in a 10×10 square grid topology. Each player is connected to his N, S, W, and E neighbors, except, of course, the players on the sides of the square. The node weights are chosen according to an exponential distribution with parameter λ. Since, as we have seen in proposition 1, only the ceiling of each weight matters, the distribution of the rounded up weights is as follows:

$$\Pr\{E_i = k\} = \Pr\{k-1 < E_i \le K\} = \int_{k-1}^{k} \lambda e^{-\lambda x} dx = e^{-\lambda(k-1)} - e^{-\lambda k}. \tag{5}$$

The moment generating function for this random variable is

$$\Phi_{E_i}(s) = (e^{\lambda} - 1)\frac{e^{s-\lambda}}{1 - e^{s-\lambda}}, \text{ for } s < \lambda.$$

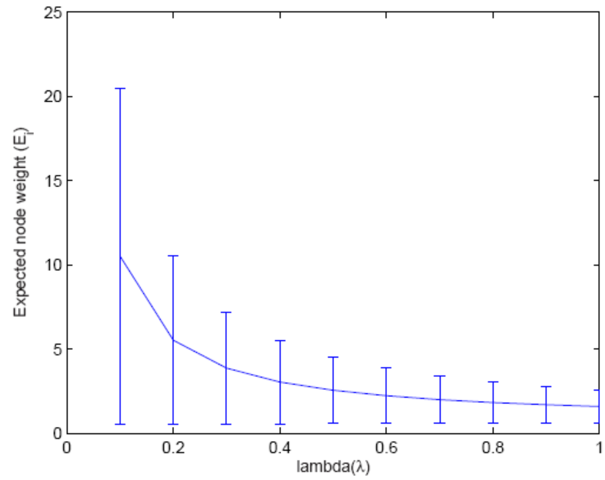

Fig. 2 The expected value $e^{-\lambda} / (e^{-\lambda} - 1)$ of the node weights as a function of λ. The errorbars depict one standard deviation $e^{-\lambda/2} / (e^{-\lambda} - 1)$ on each side of the average.

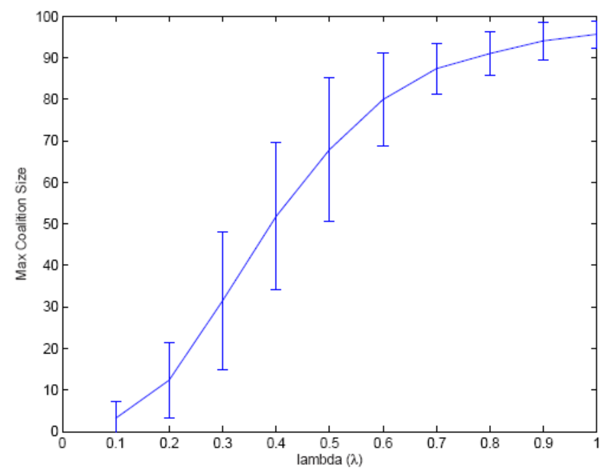

Fig. 3 Maximum coalition size as a function of the parameter λ. Edge weights are λ-exponentially distributed. Averages of 100 simulations are plotted, along with one standard deviation on each side of the average. Averages and standard deviations are in Table 1.

So we can compute the expected value for this discretized exponential distribution: $e^{-\lambda} / (e^{-\lambda} - 1)$ and the standard deviation: $e^{-(\lambda/2)} / (e^{-\lambda} - 1)$, both of which are shown in Fig. 2. In Fig. 3, we show that as the parameter λ increases, which corresponds to decrease of the expected node weight, the maximum coalition size increases.

Table 1 Results for averages and standard deviations. Data are shown in Fig. 3.

λ	0.1	0.2	0.3	0.4	0.5
Avg	3.2	12.4	31.5	51.81	67.9
Std Dev	3.9	9.1	16.6	17.7	17.4
λ	0.6	0.7	0.8	0.9	1.0
Avg	79.9	87.4	91.0	94.1	95.7
Std Dev	11.1	6.1	5.3	4.5	3.2

For a toy example of how coalitions are formed, consider the following 5×5 grid, where the node weights are shown (exponential with parameter 0.3, so the expected node weight is 3.9, with standard deviation 3.3):

$$A = \begin{pmatrix} 2 & 3 & 3 & 4 & 3 \\ 5 & 2 & 1 & 3 & 2 \\ 6 & 4 & 1 & 2 & 3 \\ 2 & 1 & 10 & 2 & 10 \\ 3 & 4 & 1 & 14 & 1 \end{pmatrix}$$

In the first round, the players at A_{23} and A_{33} will join, but no other coalition will be formed. In the second round, because there exists a coalition of size 2, some expansions are possible: A_{22} will join with A_{23}, and A_{34} will join with A_{33}. In the beginning of round 3, there exists a coalition of size 4 comprising A_{22}, A_{23}, A_{33}, A_{34}. So, nodes A_{13}, A_{24}, A_{32} will join, for a size 7 coalition. There are no more changes after that. The result is shown below, where the coalition members are circled.

$$A = \begin{pmatrix} 2 & 3 & \textcircled{3} & 4 & 3 \\ 5 & \textcircled{2} & \textcircled{1} & \textcircled{3} & 2 \\ 6 & \textcircled{4} & \textcircled{1} & \textcircled{2} & 3 \\ 2 & 1 & 10 & 2 & 10 \\ 3 & 4 & 1 & 14 & 1 \end{pmatrix}$$

Note that, with the exception of A_{43}=10, all the other nodes that are adjacent to the coalition are willing to join in. However, none of the ``outer'' nodes of the coalition will let them join, so to speak. All the outer nodes have weights more than 1, so they would only join with a coalition of size greater than 1. We can see that these “outer” nodes, which are the boundary ∂C of the coalition C of 1-weighted nodes formed in the first round, form an obstacle to the further expansion of the coalition.

However, if the average weight is lower, a larger coalition will form. For λ=0.5, the expected node weight is 2.5, with standard deviation 2.0. In the following example grid, 13 1s appear as opposed to 5 in the previous case.

$$B = \begin{pmatrix} 1 & 1 & 1 & 2 & 2 \\ 4 & 1 & 3 & 1 & 1 \\ 2 & 1 & 3 & 2 & 1 \\ 1 & 4 & 1 & 1 & 5 \\ 4 & 1 & 1 & 2 & 2 \end{pmatrix}$$

The final coalition will include the circled nodes, which in this case is almost every node, as the reader can verify.

$$B = \begin{pmatrix} \textcircled{1} & \textcircled{1} & \textcircled{1} & \textcircled{2} & \textcircled{2} \\ \textcircled{4} & \textcircled{1} & \textcircled{3} & \textcircled{1} & \textcircled{1} \\ \textcircled{2} & \textcircled{1} & \textcircled{3} & \textcircled{2} & \textcircled{1} \\ 1 & \textcircled{4} & \textcircled{1} & \textcircled{1} & \textcircled{5} \\ \textcircled{4} & \textcircled{1} & \textcircled{1} & \textcircled{2} & 2 \end{pmatrix}$$

4. STABILITY OF COALITIONS

We study cooperation based on the notion of *coalitions*. The concept of users being connected to each other, and -- by getting connected -- acquiring access to all the other users that each of them had so far access to, can be well captured by cooperative game theory (also known as coalitional game theory). In cooperative game theory, the central concept is that of coalition formation, i.e., subsets of users that join their forces and decide to act together. Players form coalitions to obtain the optimum payoffs. The key assumption that distinguishes cooperative game theory from non-cooperative game theory is that players can negotiate collectively (Myerson, 1991). We believe that the cooperative game model fits better to the practical scenarios, where agents naturally form coalitions, such as soldiers in the same group. Furthermore, the cooperative games are constrained, in the sense that there is communication constraining the collaboration, so they are called *network-restricted cooperative games* (Myerson, 1977).

A question that has only relatively recently began to attract attention (Slikker and Nouweland, 2001; Dutta and Jackson, 2003) is the actual way in which the coalition is formed. The cooperative game is usually modeled as a *two-phase structure*. Players must first decide whether or not to join a coalition. This is done by pairwise bargaining, in which *both* neighboring nodes have to agree to join in a coalition. In our case, this pairwise bargaining is modeled as the iterated game in Sec. 2. In the second step, players in the coalition negotiate the payoff allocation. The central problem is to study the payoff allocation scheme and whether the scheme results in a stable solution. One of the payoff solutions is the Myerson value (Myerson, 1977) for the network-restricted cooperative game, which is equivalent to the Shapley value in cooperative games.

Having formed coalitions in the network, we are interested in studying the stability of such coalitions. Two concepts of stability are considered: pairwise stability and coalitional stability, where the latter one is stronger. These two concepts of stability are defined in followings.

- **Weak stability** (or pairwise stability): there is no single node that can gain more by deviating from the current strategy, i.e., activating an inactive edge or destroying an active edge.

- **Strong stability** (or coalitional stability): once a coalition is formed, for any subset of nodes in this coalition, it cannot gain more by forming a separate coalition which only consists of this subset.

We first study the weak stability of coalitions formed via the dynamic game defined in the previous section. The definition of weak stability is equivalent to the Nash equilibrium in non-cooperative games. We have the following proposition.

Proposition 5 (Weak stability): When the dynamic of coalition formation reaches the steady state, every pair of neighboring nodes is in the Nash equilibrium of the pairwise game defined in Fig. 1, and the formed coalition is weekly stable.

Proof: Following the same reasoning for the monotonicity of coalition formation in Proposition 2, no node gains by deactivating any of its active edge. On the other hand, since no new active edges can introduce higher payoff in the steady state, no node gains by activating one of its links.

■

Strong stability allows that a node is able to interact and negotiate with any other node in the same coalition, which belongs to the *cooperative* game theory.

In cooperative games, a coalition is a subset of nodes that are accessible with each other. Among all coalitions, there are so-called *maximal* coalitions which are not subsets of any other coalition, i.e., if S is a maximum coalition, then $\forall i \in S, j \notin S$, i and j are disconnected with each other. In this paper, all coalitions are maximal coalitions, so we omit *maximal* from now on. An important concept in cooperative games is the characteristic function, which is the summation of the payoffs from all cooperative pairs in the coalition as follows

$$v(S) = \sum_{i,j\in} x_{ij}, \tag{6}$$

where x_{ij} is the payoff that node i gains in the game with node j. Notice that $\forall i, v(\{i\}) = 0$. We denote the cooperative game defined from the characteristic function $v(S)$ as $\Gamma = (N, v)$.

In our game, some nodes are not directly connected with each other due to the limit of wireless communication range and activation of edges, therefore the game we consider has to take the communication constraints into consideration. Myerson (Myerson, 1977) was the first to introduce a new game associated with communication constraints, the *network-restricted game*, which incorporates both the possible gains from cooperation as modeled by the cooperative game and the restrictions on communication reflected by the communication network. We define the network-restricted game as (N, v^G) which is associated with a cooperative game (N,v) and a network G. In particular, given the pairwise payoff defined in Fig. 1 and the graph G'=(V, E') induced by the active edges, we have

$$x_{ij} = \begin{cases} N_j^{(i)} - E_{i,} & \text{if } (i,j) \in E' \\ 0, & \text{otherwise} \end{cases}. \tag{7}$$

As we have discussed, the coalition S must be a tree. By removing link (i,j), the tree is divided into two subtrees. Then $N_j^{(i)}$ is the number of nodes in the subtree which includes j. Therefore, the characteristic function of the network-restricted game is defined as

$$v^{G'}(s) = \sum_{\substack{i,j\in S \\ (i,j)\in E'}} N_j^{(i)} - E_i. \tag{8}$$

Suppose at the end of the coalition formation process in Section 3, R is one of the maximal coalitions. If we consider R as a network, we have a network-constrained cooperative game $\Gamma^{G'}(R) = (R, v^{G'})$. Because of monotonicity, all nodes in R have only joined the coalition R throughout the formation procedure. Therefore, it is legitimate to limit our consideration to nodes in R. From the definition of $v^{G'}$, we have the following fact:

Proposition 6: $\Gamma^{G'}(R) = (R, v^{G'})$ is a superadditive game.

Proof: Suppose S and T are two disjoint sets in R ($S \cap T = \varnothing$) and $i \in S$ and $j \in T$, then

$$v^{G'}(S \cup T) = \begin{cases} v(S) + v(T) + \\ (N_j^{(i)} - E_i + N_i^{(j)} - E_j) & \text{if } \exists (i,j) \in E' \\ v(S) + v(T) & \text{otherwise} \end{cases} \tag{9}$$

Therefore, $v^{G'}(S \cup T) \geq v(S) + v(T)$. The inequality holds because if $(i,j) \in E'$, the link satisfies the activation condition, i.e. $N_j^{(i)} - E_i > 0$ and $N_i^{(j)} - E_j > 0$.

■

Having constrained the game into the coalition R, the strong stability of R means that the *core* in the game $\Gamma^{G'}(R) = (R, v^{G'})$ is non-empty. The core is defined as a set of payoff allocations in which nodes in R could not get better payoffs if they form separated coalitions than form the grand coalition R, which is formally defined as the set of all n-vectors $\boldsymbol{x}$ satisfying the linear inequality:

$$x(S) \geq v(S) \quad \forall \mathrm{S} \subset \mathrm{N}, \tag{10}$$

$$x(N) = v(N), \tag{11}$$

where $x(S) = \sum_{i \in S} x_i$ for all $S \subseteq N$. If Γ is a game, we will denote its core by $C(\Gamma)$. It is known that the core is possibly empty. Therefore, it is necessary to discuss existence of the core for the game Γ. We first give the definition of a family of very common games: convex games. The convexity of a game can be defined in terms of the marginal contribution of each player. $d_i : 2^N \to \mathbf{R}$ of *v* with respect to player *i* is

$$d_i(S) = \begin{cases} v(S \bigcup \{i\}) - v(S) & \text{if } i \notin S \\ v(S) - v(S \setminus \{i\}) & \text{if } i \in S \end{cases}.$$

A game is said to be *convex*, if for each $i \in N$, $d_i(S) \le d_i(T)$ holds for any coalitions $S \subseteq T$.

Proposition 7: $\Gamma^{G'}(R) = (R, v^{G'})$ is a convex game.

Proof: For $\Gamma^{G'}(R)$ and $S \subseteq R$,

$$d_i(S) = \begin{cases} N_S - E_i & \text{if } i \in S \\ N_S + 1 - E_i & \text{if } i \notin S \end{cases}, \tag{12}$$

where N_S is the number of nodes in *S*. Take two sets *S* and *T*, where $S \subseteq T$,

$$d_i(T) - d_i(S) = \begin{cases} N_T - N_S - 1, & \text{if } i \notin S, i \in T \\ N_T - N_S, & \text{otherwise} \end{cases}. \tag{13}$$

According to Eqn. (13), $d_i(T) - d_i(S)$ is always greater or equal to 0. Therefore, the game is convex.

■

Because the core of a convex game is nonempty (Forgo et al., 1999), we have $C\left(\Gamma^{G'}(R)\right) \ne \emptyset$

Proposition 8: $\Gamma^{G'}(R)$ has a nonempty core, and the payoff allocation scheme $x_i = \sum_{j \in N_i} x_{ij}$ is in the core.

Thus, given the payoff x_i for each node $i \in R$, nodes in coalition *R* cannot get higher gain if some of them form a separate coalition. In other words, the coalition *R* is strongly stable.

5. CONCLUSION

In this paper, we study coalition formation in wireless ad-hoc networks, where nodes are selfish due to resource constraints. In such networks, most of the functions (routing, mobility management, and security) must rely on cooperation between nodes, because most nodes are not directly connected to each other. In this paper, we focused on the dynamics of coalition formation and stability of coalitions.

Nodes cooperate with the other nodes in the same coalition. We modeled the interactions between nodes as a cooperative game, where nodes form a coalition to maximize their payoffs. The cooperative game is divided into two steps. In the first step, a node bargains with each one of his neighbors about activating (or not) their common edge. There is a cost associated with the activation of each edge. The accompanying benefit is that nodes have access to more users. In the second step, nodes in one coalition negotiate about their payoff allocations. If the coalition is stable, nodes are satisfied with their payoffs and participate in the functions within the coalition. We show that the coalitions that are formed from the dynamics of the first step are stable.

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