Jitter Analysis of CBR Streams in Multimedia Networks *

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***Abstract*— The performance of real time applications such as video and voice streams relies on packet delay jitter. Large delay jitter causes buffer overflow or underflow at the receiver end and the user encounters interrupts. The delay jitter is mainly due to the perturbation of background traffic in the bottleneck router. Fulton and Li [12] provide an analytical approximation for the first-order and second-order statistics of delay jitter. However, their analysis is based on a Markovian model of the background traffic, which is not quite suitable for Internet traffic and requires lots of computational effort. We propose an efficient method to predict the jitter variance of a CBR (constant-bit rate) connection based on the wavelet model of the background traffic. The wavelet analysis extracts the statistical properties of background traffic and the analysis result can be used to predict an upper bound for the jitter variance of the CBR connection.**

***Index Terms*— Delay Jitter, Multifractal, Wavelet.**

I. Introduction

REAL time applications such as voice and video streams have become exceedingly popular in recent years. The performance of real time connections is very sensitive to the connection quality such as the packet jitter in the network. The traffic behavior of a real time application is usually quite smooth and stable. After entering the network, the real time packet stream is multiplexed and shares the link bandwidth with the background traffic. Hence, the packet jitter is mainly due to the perturbation of the background traffic in the bottleneck router. Most jitter analysis methods [12] [4] [21] are based on Markovian models of the background traffic. However, recent studies [25] [24] [20] [14] on Internet traffic have shown that the aggregate background traffic driven by TCP is long range dependent and self-similar. Wavelet analysis [29] [13] also demonstrates that background traffic is monofractal (self-similar) at large time scales and multifractal at small time scales. The traditional Markovian traffic model is unable to capture this multifractal behavior well and is not a proper model for performance analysis [19] [20]. In order to predict the jitter more efficiently and accurately, we applied wavelet analysis to characterize the background traffic and propose an upper bound for the jitter variance of a constant-bit rate connection.

The arrangement of this paper is as follows. In the next section, we briefly introduce the wavelet analysis for traffic and show the multifractal behavior of a real traffic trace. In section 3, the background traffic is characterized by the Logscale diagram. An approximation of queue length distribution is derived from properties of wavelets and the Logscale diagram. The upper bound of CBR jitter variance is developed in section 4. Simulation and analysis results are demonstrated in section 5. Section 6 contains conclusions and suggestions for future work.

II. Wavelet Analysis of the Background Traffic

The wavelet technique is a multi-resolution analysis tool widely used in signal processing and data analysis [7] [5]. It has remarkable advantages in analyzing stochastic processes with long range dependence [1] [23] [2] [30] [16] [15] [3]. For instance, wavelet analysis can eliminate the effect of deterministic trends hidden in random processes if the wavelet function is chosen properly. Given the scaling function ϕ_0 and the mother wavelet ψ_0, the discrete wavelet transform of the continuous time process $X(t)$ is defined as follows:

Definition (Discrete Wavelet Transform) [23]: *Given the scaling function ϕ_0 and the mother wavelet ψ_0, the approximation coefficients $a_{j,k}$ and detail coefficients $d_{j,k}$ of the discrete wavelet transform of the process $X(t)$*

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are defined as

$$a_{j,k} := \int_{-\infty}^{\infty} X(t)\phi_{j,k}(t)dt \quad (1)$$

$$d_{j,k} := \int_{-\infty}^{\infty} X(t)\psi_{j,k}(t)dt, \quad (2)$$

where

$$\phi_{j,k}(t) := 2^{-j/2}\phi_0(2^{-j}t - k) \quad (3)$$

$$\psi_{j,k}(t) := 2^{-j/2}\psi_0(2^{-j}t - k). \quad (4)$$

It can be shown that the $\phi_{j,k}$ and $\psi_{j,k}$ form an orthonormal basis. $X(t)$ has the following representation

$$X(t) = \sum_k a_{0,k}\phi_{0,k}(t) + \sum_{j=0}^{\infty}\sum_k d_{j,k}\psi_{j,k}(t). \quad (5)$$

For the discrete time process X_i, $i = 0, 1, 2, ...$, the discrete wavelet transform can be implemented by the fast pyramidal algorithm [26]. To understand the behavior of the traffic X_i, we are more interested in the detail process of the discrete wavelet transform $d_{j,k}$. It is well known that the Internet traffic is long range dependent. Many studies [8] [17] [19] [6] have shown that this long range dependence property plays an important role in network performance.

Definition (Long Range Dependence) [18]: *A stationary finite-variance process X_i displays long range dependence with parameter α if its spectral density $S(\omega)$ satisfies*

$$S(\omega) \sim C_f|\omega|^{-\alpha} \;\; as\; \omega \to 0, \quad (6)$$

where $0 < \alpha < 1$ and C_f is a positive constant. It also implies that the autocovariance function $r(k) := E[(X_i - EX_i)(X_{i+k} - EX_i)]$ satisfies

$$r(k) \sim C_r k^{\alpha-1} \;\; as\; k \to \infty, \quad (7)$$

where $C_r = C_f 2\Gamma(1-\alpha)sin(\pi\alpha/2)$, and Γ denotes the Gamma function.

The mother wavelet $\psi_0(t)$ is usually a bandpass function between ω_1 and ω_2 in the frequency domain. Note that the detail coefficient $d_{j,k}$ is the output process of the corresponding bandpass filter. The square of the detail process $d_{j,k}^2$ roughly measures the amount of energy around the time $t = 2^j k\Delta$ and the frequency $2^{-j}\omega_0$, where Δ is the unit time interval and $\omega_0 := \frac{\omega_1+\omega_2}{2}$.

Proposition [18]: *If a stationary finite-variance process X_i has long range dependence with parameter α, then the corresponding detail coefficients $d_{j,k}$ have the following property:*

$$log_2 E[d_{j,\cdot}^2] \approx j\alpha + log_2 C(\alpha, \psi_0). \quad (8)$$

Note that $C(\alpha, \psi_0)$ is independent of the variable j.

This property suggests that the parameter α can be estimated by the slope of the $log_2 E[d_{j,\cdot}^2]$ $v.s.$ j plot. This plot is named the *Logscale* diagram. One advantage of wavelet analysis is that even when the original process X_i has long range dependence, its wavelet transform $d_{j,k}$ still has short range dependence if the number of vanishing moments N of the mother wavelet $\psi_0(t)$ is chosen large enough ($N > \alpha/2$).

Definition [28]: *The number of vanishing moments N of the mother wavelet $\psi_0(t)$ is defined as:*

$$\int t^k\psi_0(t) \equiv 0, \quad k = 0, 1, 2, ..., N-1. \quad (9)$$

Proposition [9] [18]: *If the number of vanishing moments $N > \alpha/2$, then $d_{j,k}$ is stationary and no longer exhibits long range dependence but only short range dependence.*

$$E[d_{j,k}d_{j',k'}] \approx |2^j k - 2^{j'}k'|^{\alpha-1-2N} \;\; as\; |2^j k - 2^{j'}k'| \to \infty,$$

where $j \neq j'$ and $k \neq k'$. This implies the higher N, the smaller the correlation.

Figure 1 is the Logscale diagram of a real traffic trace. Veitch and Arby [27] developed an asymptotically unbiased and efficient joint estimator for the parameter α and $C(\alpha, \psi_0)$. They also provided a closed-form expression for the covariance matrix of the estimator and showed its accuracy. The Logscale diagram not only demonstrates the long range dependence property of the traffic but also extracts the second order statistics at every time scale. In this paper, we characterize the traffic in terms of the traffic mean rate and the Logscale diagram. With the Logscale diagram, we are able to predict the overflow probability and the jitter variance at the bottleneck router.

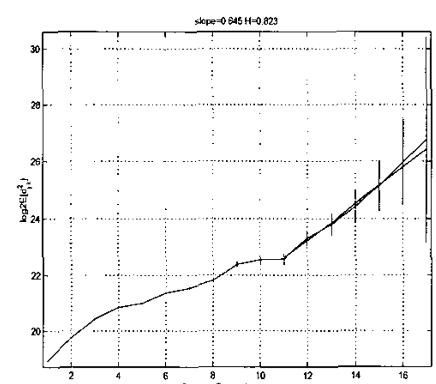

Fig. 1. Logscale diagram of a real trace

III. Traffic Model and Overflow Probability

The wavelet analysis has many advantages in parameter estimation and traffic analysis. The background traffic in this paper is characterized by the Logscale diagram and mean rate m. An upper bound of the overflow probability is provided by using the properties of wavelets.

Lemma *Let $a_{j,k}$ and $d_{j,k}$ be the approximation coefficients and the detail coefficients of the Haar wavelet. We have the following relations:*

$$a_{j,2k} = \frac{a_{j+1,k} + d_{j+1,k}}{\sqrt{2}}, \quad (10)$$

$$a_{j,2k+1} = \frac{a_{j+1,k} - d_{j+1,k}}{\sqrt{2}}. \quad (11)$$

With the uncorrelated assumption of $a_{j,k}$ and $d_{j,k}$ for every j, we have

$$Var[a_j] = \frac{Var[a_{j+1}] + Var[d_{j+1}]}{2}. \quad (12)$$

Note that the plot $\log_2 Var[d_j]$ $v.s$ j is the *Logscale* diagram. On the other hand, let A_j be the total arrival bytes in the interval $[0, 2^j\Delta)$. From the definition of Haar wavelet:

$$A_j = a_j 2^{j/2}. \quad (13)$$

Thus, the variance of workload $Var[A_j]$ can be computed recursively for all j by using the *Logscale* diagram.

Lemma *Given the Logscale diagram $log_2 Var[d_j]$ and the variance of A_0, the variance of A_j, $j = 1, 2, ...$ is*

$$Var[A_j] = 2^j Var[a_j]$$
$$Var[a_j] = 2Var[a_{j-1}] - Var[d_j].$$

Assuming that A_j has the *Lognormal* distribution for all j with mean $M_j := E[A_j] = m2^j\Delta$ and variance $V_j := Var[A_j]$, the probability density function of the *Lognormal* distribution is:

$$f_{A_j}(x) := \frac{1}{x\sigma_j\sqrt{2\pi}} \exp[-\frac{(\ln x - \mu_j)^2}{2\sigma_j^2}], \; x > 0. \quad (14)$$

Since the r^{th} moment of the *Lognormal* distribution has a closed-form:

$$EA_j^r = \exp(r\mu_j + \frac{r^2\sigma_j^2}{2}), \quad (15)$$

the parameters μ_j and σ_j can be easily calculated by the following equations:

$$\sigma_j^2 = \ln(\frac{M_j^2 + V_j}{M_j^2}), \quad (16)$$

$$\mu_j = \ln(M_j) - \frac{\sigma_j^2}{2}. \quad (17)$$

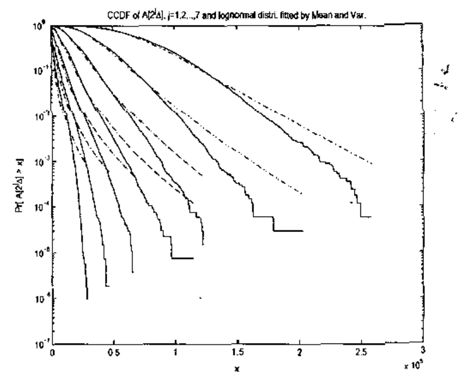

Fig. 2. The CCDF of workload A_j $j = 1, 2, ..., 7$ and the fitted *Lognormal* distribution

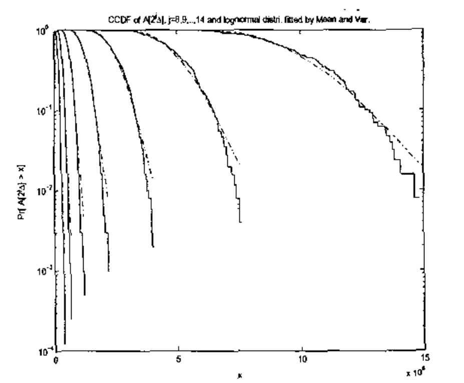

Fig. 3. The CCDF of workload A_j $j = 8, 2, ..., 14$ and the fitted *Lognormal* distribution

Figure 2 and Figure 3 are the complementary CDF $Pr[A_j > x]$ of the real traffic trace and the corresponding *Lognormal* distribution with the parameters estimated from the Logscale diagram.

Consider a FIFO queue with an infinite buffer size and the service rate C (bytes/Δ). Assuming that the distribution of A_j is known for all j, Riedi [22] proposes an upper bound for the overflow probability $P[Q > b]$.

Lemma [22] *Assume that $E_j := \{A_j < b + C2^j\Delta\}$ are independent to each other and the r.v. Q is the queue length in steady state. An upper bound of the overflow probability of a FIFO queue is:*

$$\begin{aligned} P[Q > b] &= 1 - P[Q \le B] \approx 1 - P[\cap_{j=1}^K E_j] \\ &= 1 - P[E_0] \prod_{j=1}^K P[E_j | E_{j-1}, ..., E_0] \\ &\le 1 - \prod_{j=0}^K P[E_j] \\ &= 1 - \prod_{j=0}^K P[A_j < B + C2^j\Delta], \quad (18) \end{aligned}$$

where K is the maximum octave and $2^K\Delta$ is the maximum time scale.

Figure 4 shows simulation results of the queue length distribution and the upper bound. The simulation result shows that this upper bound provides a good approximation of the steady state queue length distribution. We will apply this result to predict the jitter variance of a CBR traffic at the bottleneck router.

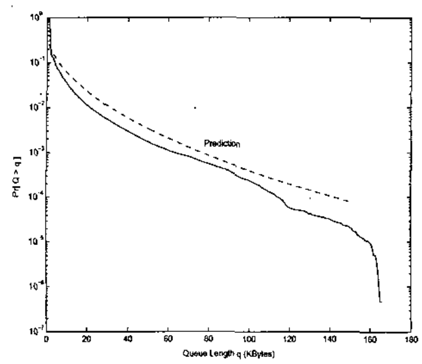

Fig. 4. The queue length distribution with utilization $\rho = 0.4$.

IV. Upper Bound for Jitter Variance

In this section, we propose an efficient method to predict the jitter variance of a CBR connection based on the Logscale diagram of the background traffic at the bottleneck router. Consider the following scenario. In Figure 5, there is one CBR process sharing the bandwidth and buffer with the background traffic at a FIFO queue.

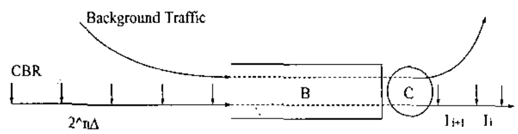

Fig. 5. The target process and the background traffic

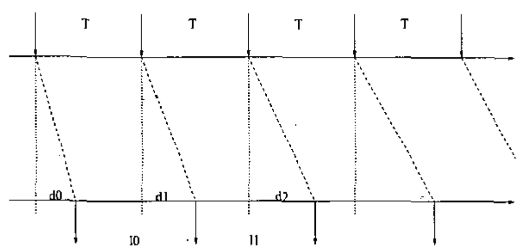

Fig. 6. The arrival and departure time of CBR traffic

The CBR process periodically sends out a small packet every $2^n\Delta sec$. Note that the Δ is the finest time resolution in the wavelet analysis and we set $\Delta = 0.001sec$ in this paper. The packet size of the CBR process is set to be small enough so that its mean rate is negligible to the bandwidth at the bottleneck link. The statistical property of the background traffic is described by its mean rate m $(bytes/\Delta)$ and the Logscale diagram $L_j := \log_2 Var[d_j]$ of the wavelet analysis. The delay jitter of CBR is defined as follows:

Definition: *Let the random sequence I_i be the interdeparture times of the target process. The jitter is defined as the difference of two consecutive interdeparture times:*

$$J_i := I_{i+1} - I_i. \tag{19}$$

We also define $A_{n,i}$ as the total arrival bytes of the background traffic in the i^{th} time slots. The duration of each time slot is $T := 2^n\Delta$ sec.

Lemma: *Let the current time be $t = iT$ and assume that the current queue length is $q(t) \geq b := 2^{n+1}\Delta(C - m)$. The conditional variance of jitter is*

$$Var[J|q(t) \geq b] = \frac{1}{C^2} Var[(A_{n,i+1} - A_{n,i})]. \tag{20}$$

Proof: Without loss of generality, let the CBR packets arrive at times 0, T, and $2T$, which have queuing delay d_0, d_1 and d_2, respectively. As shown in Figure 6, the total arrival bytes of the background traffic in the i^{th} time slot $(t \in [iT, (i+1)T))$ is $A_{n,i}$. Since the current length $q(0) \geq b$ is quite large, it is reasonable to say that the output link is always busy during the 0^{th} and 1^{th} time slots. Moreover, the buffer size is infinite so that there is no packet loss event. The Lindley equation:

$$q(t) = max_{0 \leq s \leq t}[A(t) - A(s) - C(t-s)], \forall t \geq 0 \tag{21}$$

can be simplified as

$$q((i+1)T) = q(iT) + A_{n,i} - CT. \tag{22}$$

Hence, the packet delay $d_i = q(iT)/C$ and the interdeparture time is

$$\begin{aligned} I_i &= T + d_{i+1} - d_i \\ &= T + \frac{q((i+1)T) - q(iT)}{C} \\ &= \frac{A_{n,i}}{C}. \end{aligned} \tag{23}$$

The jitter variance under this condition is

$$\begin{aligned} Var[J|q(t) \geq b] &= Var(I_{i+1} - I_i) \\ &= \frac{Var[(A_{n,i+1} - A_{n,i})]}{C^2}. \end{aligned} \tag{24}$$

According to the definition of wavelet analysis, one may easily obtain the value of $Var[(A_{n,1} - A_{n,0})]$ from the Logscale diagram L_j

$$E[d_{j+1,k}^2] = \frac{E[(A_{j,2k+1} - A_{j,2k})^2]}{2^{j+1}}, \tag{25}$$

for every j and k.

Hence, the conditional jitter variance of the CBR process is

$$Var[J|q(t) \geq b] = \frac{2^{n+1+L_{n+1}}}{C^2}. \tag{26}$$

On the other hand, if the current queue length is small ($q(t) < b$), we assume that there is at least one idle server event happening in the next two time slots. The simple relations of eq.(22) and eq.(23) do not hold. Since there is at least one idle event in this period, the sequence of interdeparture times I_i and the sequence of packet delays d_i can be treated as uncorrelated random sequences respectively. We have the following approximation:

$$\begin{aligned} Var[J|q(t) < b] &= Var[I_{i+1} - I_i] \approx 2Var[I_i] \\ &= 2Var[d_{i+1} - d_i] \approx 4Var[d_i] \\ &\leq \frac{4}{C^2} \max_{0 \leq j \leq n} Var[(A_j - C2^j\Delta)^+]. \end{aligned} \tag{27}$$

Lemma: *Let A be the Lognormal random variable with parameter (μ, σ) and $d > 0$ be any real number, we have*

$$\begin{aligned} E[(A-d)^+] &= \frac{e^{\mu+\sigma^2/2}}{2} erfc(\frac{\ln d - \mu - \sigma^2}{\sqrt{2}\sigma}) \\ &- d\bar{F}(d) \end{aligned} \tag{28}$$

$$\begin{aligned} E[((A-d)^+)^2] &= \frac{e^{2\mu+2\sigma^2}}{2} erfc(\frac{\ln d - \mu - 2\sigma^2}{\sqrt{2}\sigma}) \\ &- de^{\mu+\sigma^2/2} erfc(\frac{\ln d - \mu - \sigma^2}{\sqrt{2}\sigma}) \\ &+ d^2\bar{F}(d), \end{aligned} \tag{29}$$

where $\bar{F}(d) := Pr[A > d]$.

The probability of $Pr[q(t) \geq b]$ is based on the prediction of the steady state queue length distribution. We applied Riedi's approach [22] to calculate the overflow probability.

Proposition *Let the r.v. Q be the queue length in steady state. From eq. (26) (27) and (18), there is an upper bound of the jitter variance of the CBR traffic at the bottleneck router:*

$$\begin{aligned} Var(J) &\leq Var(J|Q \geq b)Pr[Q \geq b] \\ &\quad +Var(J|Q < b))Pr[Q < b]. \end{aligned} \tag{30}$$

V. Simulation results

The network topology in our experiment is the simple dumbbell with a single bottleneck link. One side of the bottleneck link consists of 800 web clients, each client sends a web request and has an *Exponential* think time with mean $50sec$ after closing the current session. The other side has 800 web servers. The server is running HTTP 1.1 protocol and has a *Pareto* file size distribution with parameters (K=$2.3Kbytes$, α=1.3). The propagation delay of each server link is uniformly distributed and the mean round-trip time is about $128ms$. Note that the mean arrival rate of the web (background) traffic is around $1.2Mbps$.

A. Jitter at a FIFO Queue

Figure 7 compares the predicted jitter standard deviation with the simulation results. The target CBR process has fixed interarrival times $2^n\Delta$ and $n = 3, 4, .., 8$. The link utilization is about $0.4, 0.6$ and 0.8 with the corresponding bandwidth $C = 3.0, 2.0$ and $1.5Mbps$ respectively.

The simulation results show that our prediction method indeed provides a tight upper bound even when the CBR interarrival time $2^n\Delta$ is getting larger.

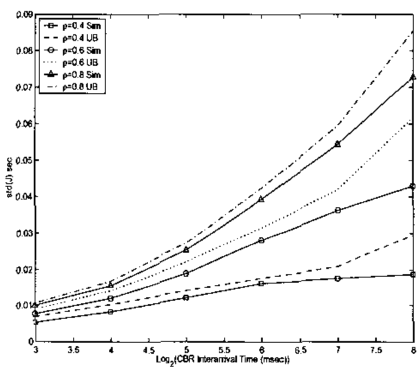

Fig. 7. The standard deviation of delay jitter $std(J)$ $v.s.$ n with a FIFO queue

B. Jitter at a RED Queue

We replace the FIFO queue by an adaptive RED queue at the bottleneck router. The adaptive RED queue [11] [10] will keep the average queue length located in a desired region by randomly dropping the TCP packets. Since the queue length is in the desired region, the link has a 100% utilization and no idle event happened. Hence, the jitter variance is bounded by equation (26).

Figure 8 shows that the prediction method also provides a tight bound for the jitter variance when a different queuing policy such as RED is employed.

VI. Conclusions

The performance of video and voice streams mainly depends on the delay jitter in the network. Recent studies show that Internet traffic is multifractal, which can not be

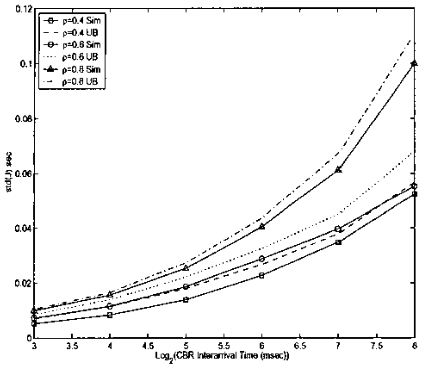

Fig. 8. The standard deviation of delay jitter $std(J)$ $v.s.$ n with a RED queue

modeled well by traditional Markovian models. Instead of using a Markovian approach, we applied wavelet analysis to analyze the background traffic at the bottleneck router. The second order statistical property of the background traffic is characterized by the Logscale diagram. Based on properties of wavelets and some reasonable assumptions, we provide an efficient and accurate method to predict the jitter variance of CBR traffic. Our simulation results also show that this method works well with a different queuing policy such as RED. In future work, we are considering the effects of finite buffer size in the FIFO queue.

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