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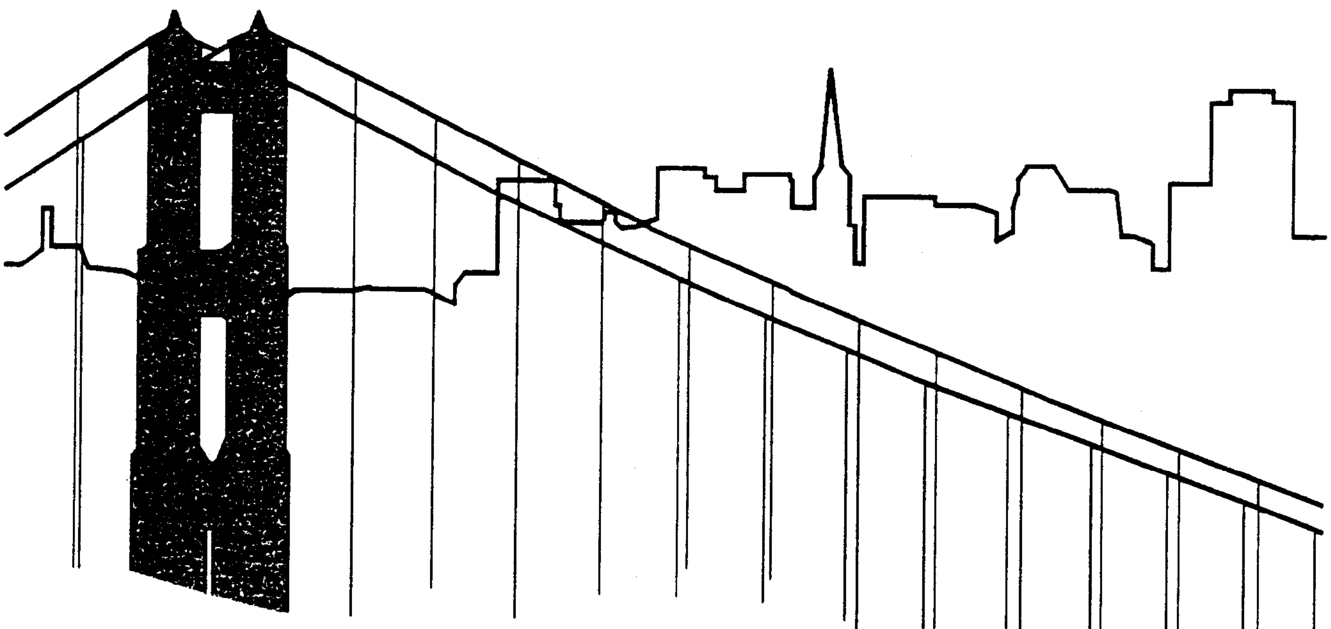

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REDUCED COMPLEXITY NONLINEAR H_∞ CONTROLLERS: RELATION TO CERTAINTY EQUIVALENCE

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Abstract. This paper considers the problem of constructing reduced complexity controllers for output feedback nonlinear H_∞ control. Conditions are obtained under which such controllers achieve the closed-loop performance requirements. These controllers are non-optimal in general. However, in case optimality holds, they are in fact the certainty equivalence controllers. Conditions under which certainty equivalence holds are simplified, and linked to the solvability of a functional equation.

Keywords. Nonlinear control systems, Robust control, H-infinity control, Discrete-time systems, Disturbance rejection, Game theory.

1. INTRODUCTION

Since, Whittle (Whittle, 1981) first postulated the minimum stress estimate for the solution of a risk-sensitive stochastic optimal control problem, it has evolved into the certainty equivalence principle. The latter states that under appropriate conditions, an optimal output feedback controller can be obtained by inserting an estimate of the state into the corresponding state feedback law. The certainty equivalence property is known to hold for linear systems with a quadratic cost (Basar and Bernhard, 1991). The recent interest in nonlinear H_∞ control has led researchers to examine whether, certainty equivalence could be carried over to nonlinear systems. If certainty equivalence were to hold, it would result in a tremendous reduction in the complexity of the problem. In a recent paper (James *et al.*, 1994), sufficient conditions were given for certainty equivalence to hold in terms of a saddle point condition. Also, in (James, 1994), a simple example is given to demonstrate the non-optimal nature of the certainty equivalence controller. An implementation of a certainty equivalence controller can be found in (Teolis *et al.*, 1993).

This paper, considers the infinite time case, and deals with establishing sufficiency conditions for a reduced complexity controller to exist. These conditions apply for both optimal and non-optimal policies. In general, obtaining an optimal solution to the output feedback problem, involves solving an infinite dimensional dynamic programming equation (James and Baras, 1995). Hence, one may be satisfied with a reduced complexity non-optimal policy, which guarantees asymptotic stability of the nominal (no exogenous inputs) closed-loop system, as well as achieves a pre-specified disturbance attenuation level γ. In the special case, it is shown that the policies so obtained are certainty equivalence policies. Furthermore, in doing so, one obtains an equivalent sufficiency condition for certainty equivalence which may be more tractable than the one given in (James *et al.*, 1994). The approach is based on establishing dissipativity results, since these guarantee under detectability assumptions, asymptotic stability of the closed-loop system when exogenous inputs are zero. Lastly, it is shown that the condition for certainty equivalence to hold is equivalent to the existence of a (unique) solution to a functional equation.

2. PROBLEM STATEMENT

Consider the following system:

$$\Sigma \begin{cases} x_{k+1} = f(x_k, u_k, w_k), & x_0 \in \mathbf{R}^n \\ y_{k+1} = g(x_k, u_k, w_k) \\ z_{k+1} = l(x_k, u_k, w_k), & k = 0, 1, 2, \ldots \end{cases}$$

where, $x_k \in \mathbf{R}^n$ are the states, $y_k \in \mathbf{R}^t$ are the measurements, $u_k \in \mathbf{U} \subset \mathbf{R}^m$ are the controls, $z_k \in \mathbf{R}^q$ are the regulated outputs, and $w_k \in \mathbf{R}^r$ are the exogenous inputs. Furthermore, assume that 0 is an equilibrium point of Σ, and $\mathbf{U}$ is compact. Denote the set of feasible policies as O, i.e. if $u \in O$, then $u_k = u(y_{1,k}, u_{0,k-1})$, where $s_{i,j}$ denotes a sequence $\{s_i, s_{i+1}, \ldots, s_j\}$. Also assume that f, g, and l are continuous. The output feedback problem is, given $\gamma > 0$, find a control policy $u^* \in O$, so as to ensure that there exists a finite $\beta^{u^*}(x) \geq 0$, $\beta^{u^*}(0) = 0$, such that

$$\sup_{w \in l^2([0,\infty), \mathbf{R}^r)} \sup_{x_0 \in \mathbf{R}^n} \{p_0(x_0) + \sum_{i=0}^{\infty} | z_{i+1} |^2 - \gamma^2 | w_i |^2\} \leq \sup_{x \in \mathbf{R}^n} \{p_0(x) + \beta^{u^*}(x)\}. \quad (1)$$

where, $p_0 \in \mathcal{E}$, with $\mathcal{E}$ defined as

$$\mathcal{E} \triangleq \{p \in C(\mathbf{R}^n) \mid p(x) \leq R \text{ for some finite } R \geq 0\}$$

Here, $|\cdot|$ denotes the Euclidean norm. Also assume that for such $u^* \in O$, Σ^{u^*} is z-detectable. Equation (1) is based on the dynamic game interpretation of the nonlinear H_∞ control problem. It ensures that if $x_0 = 0$ then

$$\sup_{w \in l^2([0,\infty), \mathbf{R}^r), w \neq 0} \frac{\|z\|_{l^2}}{\|w\|_{l^2}} \leq \gamma \quad (2)$$

Furthermore, define the following sup-pairing

$$(p, q) \triangleq \sup_{x \in \mathbf{R}^n} \{p(x) + q(x)\}$$

and the function $\delta_x \in \mathcal{E}$, $\delta_x : \mathbf{R}^n \to \mathbf{R}^*$

$$\delta_x(\xi) \triangleq \begin{cases} 0 & \text{if } \xi = x \\ -\infty & \text{else} \end{cases}$$

An information state based solution was recently obtained in (James and Baras, 1995). The information state is defined by the following recursion

$$\begin{aligned} p_{k+1} &= H(p_k, u_k, y_{k+1}), \; k = 0, 1, \ldots \\ p_0 &\in \mathcal{E} \end{aligned}$$

where

$$H(p_k, u_k, y_{k+1})(x) \triangleq \sup_{\xi \in \mathbf{R}^n} \{p_k(\xi) + \sup_{w \in \mathbf{R}^r} (| l(\xi, u_k, w) |^2 - \gamma^2 | w |^2 | \; x = f(\xi, u_k, w), y_{k+1} = g(\xi, u_k, w))\}.$$

The problem is solved via dynamic programming, where the upper value function M satisfies

$$M(p) = \inf_{u \in \mathbf{U}} \sup_{y \in \mathbf{R}^t} \{M(H(p, u, y))\} \quad (3)$$

for all $p \in \mathcal{E}$, with $M(p) \geq (p, 0)$, and $M(-\beta^u) = 0$, for some $\beta^u(x) \geq 0$, $\beta^u(0) = 0$. In particular, $M(p)$ is the least possible worst case cost to go, given $p_0 = p$. Now, supposing such a solution M exists to equation (3), one has the following result.

Theorem 1. ((James and Baras, 1995)). Let $u^* \in O$, be such that $u_k^* = \bar{u}(p_k)$, where $\bar{u}(p_k)$ achieves the minimum in (3) for $p = p_k$. Then $u^* \in O$ solves the output feedback problem. Here, p_k is the information state trajectory initialized by $p_0 = -\beta^u$, and is such that $M(p_k)$ is finite for all k.

Such a policy, obtained via the dynamic programming equation (3) is called an optimal policy. Now, assume $u(p)$ is a non-optimal policy, then there exists a function $W : \mathcal{E} \to \mathbf{R}$, $W(p) \geq (p, 0)$, and $W(-\beta^u) = 0$ ($\beta^u(x) \geq 0$, $\beta^u(0) = 0$), and W satisfies for all $p \in \mathcal{E}$

$$W(p) \geq \sup_{y \in \mathbf{R}^t} W(H(p, u(p), y))$$

Such a W is called a storage function for the output feedback policy u. Conversely, if such a function exists, then the corresponding control policy $\bar{u}(p)$ solves the output feedback problem.

In the well known, state feedback case, denote by V the upper value function of the state feedback problem. Furthermore, $V \geq 0$, $V(0) = 0$, and V satisfies

$$V(x) = \inf_{u \in \mathbf{U}} \sup_{w \in \mathbf{R}^n} \{| l(x, u, w) |^2 - \gamma^2 | w |^2 + V(f(x, u, w))\}$$

for all $x \in \mathbf{R}^n$. The policy u_F, such that $u_F(x) = u^*$, where $u^* \in U$ achieves the infimum in the above equation is called an optimal state feedback policy. For non-optimal state feedback policies u, there exists a $U : \mathbf{R}^n \to \mathbf{R}$, with $U \geq 0$, $U(0) = 0$, and satisfies

$$U(x) \geq \sup_{w \in \mathbf{R}^r} \{| l(x, u(x), w) |^2 - \gamma^2 | w |^2 + U(f(x, u(x), w))\}$$

for all $x \in \mathbf{R}^n$. Such a U is called a storage function for the state feedback policy u.

From now on, define $\mathcal{I} \subset \mathcal{O}$, to be the set of output feedback policies which have the separated structure, i.e. depend only on the information state p_k. Such policies are called information state feedback policies. The control policy generated by the dynamic programming equation (3) is of this type.

3. REDUCED COMPLEXITY CONTROLLERS

The dynamic programming equation (3), is infinite dimensional in general. Hence, this motivates the search for reduced complexity control policies, which preserve the stability properties, as well as the attenuation level γ of the closed-loop system (equation (2)).

For a given $x,\xi \in \mathbf{R}^n$, and $u \in \mathbf{U}$, define

$$\Omega(x, u, \xi) \triangleq \{w \in \mathbf{R}^r \mid x = f(\xi, u, w)\}.$$

Then, one has the following result.

Lemma 1. For any $\xi \in \mathbf{R}^n$, $u \in \mathbf{U}$, and a given function $h : \mathbf{R}^n \times \mathbf{R}^r \times \mathbf{R}^n \to \mathbf{R}$,

$$\sup_{x \in \mathbf{R}^n} \sup_{w \in \Omega(x,w,\xi)} h(x, w, \xi) \leq \sup_{w \in \mathbf{R}^r} h(f(\xi, u, w), w, \xi)$$

Proof: For any $\epsilon > 0$, there exists $x^\epsilon \in \mathbf{R}^n$, and $w^\epsilon \in \Omega(\xi, u, x^\epsilon)$ (i.e. with $x^\epsilon = f(\xi, u, w^\epsilon)$) such that

$$\begin{aligned}\sup_{x \in \mathbf{R}^n} \sup_{\Omega(\xi,u,x)} h(x, w, \xi) &< h(x^\epsilon, w^\epsilon, \xi) + \epsilon \\ &= h(f(\xi, u, w^\epsilon), w^\epsilon, \xi) + \epsilon \\ &\leq \sup_{w \in \mathbf{R}^r} h(f(\xi, u, w), w, \xi) + \epsilon\end{aligned}$$

Since, $\epsilon > 0$ is arbitrary, the result follows.

Define, $J_U^p : \mathbf{R}^n \times \mathbf{U} \to \mathbf{R}$ as

$$J_U^p(x, u) \triangleq \{p(x) + \sup_{w \in \mathbf{R}^r} \{| l(x, u, w) |^2 - \gamma^2 | w |^2 + U(f(x, u, w))\}$$

The following result is needed to establish conditions for the existence of reduced complexity policies which achieve the desired closed-loop performance.

Lemma 2. For any $u \in \mathbf{U}$, $U : \mathbf{R}^n \to \mathbf{R}$, and $p_k \in \mathcal{E}$,

$$\sup_{x \in \mathbf{R}^n} J_U^{p_k}(x, u) \geq \sup_{y \in \mathbf{R}^t} (H(p_k, u, y), U).$$

Proof:

$$\begin{aligned}&\sup_{y \in \mathbf{R}^t} (p_{k+1}, U) \\ &= \sup_{y \in \mathbf{R}^t} \sup_{x \in \mathbf{R}^n} \sup_{\xi \in \mathbf{R}^n} \{p_k(\xi) + \sup_{w \in \mathbf{R}^r} (| l(\xi, u, w) |^2 - \\ &\quad \gamma^2 | w |^2 | x = f(\xi, u, w), y = g(\xi, u, w)) + U(x)\} \\ &\leq \sup_{x \in \mathbf{R}^n} \sup_{\xi \in \mathbf{R}^n} \{p_k(\xi) + \sup_{w \in \mathbf{R}^r} (| l(\xi, u, w) |^2 - \\ &\quad \gamma^2 | w |^2 | x = f(\xi, u, w)) + U(x)\} \\ &= \sup_{\xi \in \mathbf{R}^n} \sup_{x \in \mathbf{R}^n} \sup_{w \in \Omega(\xi,u,x)} \{p_k(\xi) + | l(\xi, u, w) |^2 - \\ &\quad \gamma^2 | w |^2 + U(x)\} \\ &\leq \sup_{\xi \in \mathbf{R}^n} \sup_{w \in \mathbf{R}^r} \{p_k(\xi) + | l(\xi, u, w) |^2 - \\ &\quad \gamma^2 | w |^2 + U(f(\xi, u, w))\}, \text{ via lemma 1} \\ &= \sup_{\xi \in \mathbf{R}^n} J_U^p(\xi, u)\end{aligned}$$

The following theorem, gives a sufficient condition for the existence of dissipative reduced complexity policies.

Theorem 2. Given $U : \mathbf{R}^n \to \mathbf{R}$, $U \geq 0$, and $U(0) = 0$. If for all $p_k \in \mathcal{E}$

$$(p_k, U) \geq \inf_{u \in \mathbf{U}} \sup_{x \in \mathbf{R}^n} J_U^{p_k}(x, u)$$

then $\hat{u}(p_k) \in \arg\min_{u \in \mathbf{U}} \sup_{x \in \mathbf{R}^n} J_U^{p_k}(x, u)$, solves the output feedback problem, and the associated storage function is $W(p_k) = (p_k, U)$.

Proof:

$$\begin{aligned}(p_k, U) &\geq \inf_{u \in \mathbf{U}} \sup_{x \in \mathbf{R}^n} J_U^{p_k}(x, u) \\ &= \sup_{x \in \mathbf{R}^n} J_U^{p_k}(x, \hat{u}(p_k)) \\ &\geq \sup_{y \in \mathbf{R}^t} (H(p_k, \hat{u}(p_k), y), U), \text{ via lemma 2}\end{aligned}$$

Furthermore, $(p_k, U) \geq (p_k, 0)$, and $(-U, U) = 0$. Hence, (p_k, U) is a storage function, and $\hat{u}$ is a (non-optimal) solution to the output feedback problem, with the information state trajectory initialized via $p_0 = -U$.

Remark: One could have considered any $\hat{u}_k$ such that

$$\sup_{x \in \mathbf{R}^n} J_U^{p_k}(x, u(x)) \geq \sup_{x \in \mathbf{R}^n} J_U^{p_k}(x, \hat{u}_k).$$

Corollary 1. (Certainty Equivalence). Given $U \equiv V$, the upper value function of the state feedback problem, and the optimal state feedback policy u_F. If for all $p_k \in \mathcal{E}$

$$(p_k, V) = \inf_{u \in \boldsymbol{U}} \sup_{x \in \boldsymbol{R}^n} J_V^{p_k}(x,u) \tag{4}$$

then $u(p_k) = u_F(\hat{x})$, where $\hat{x} \in \arg\max_{x \in \boldsymbol{R}^n}\{p_k(x) + V(x)\}$, is an optimal control policy for the output feedback problem.

Proof: Clearly (4) implies that

$$\begin{aligned}\sup_{x \in \boldsymbol{R}^n} J_V^{p_k}(x, u_F(x)) &= \sup_{x \in \boldsymbol{R}^n} \inf_{u \in \boldsymbol{U}} J_V^{p_k}(x,u) \\ &= \inf_{u \in \boldsymbol{U}} \sup_{x \in \boldsymbol{R}^n} J_V^{p_k}(x,u)\end{aligned}$$

Hence, a saddle point exists, and so for any

$$\hat{x} \in \arg\max_{x \in \boldsymbol{R}^n} (p_k(x) + V(x)), \text{ and } \hat{u} = u_F(\hat{x}),$$

$$\begin{aligned}(p_k, V) = J_V^{p_k}(\hat{x}, \hat{u}) &= \sup_{x \in \boldsymbol{R}^n} J_V^{p_k}(x, \hat{u}) \\ &\geq \sup_{y \in \boldsymbol{R}^t} (H(p_k, \hat{u}, y), V)\end{aligned}$$

Hence, $W(p_k) = (p_k, V)$ is a storage function, and $W(\delta_x) = V(x)$, the optimal cost of the state feedback problem. Hence, the policy is optimal for the output feedback problem.

Remark: It is sufficient that the conditions in theorem 2 and corollary 1 hold only for all p_k, $k = 0, 1, \ldots$. If this is the case, then U need not be a storage function for the state feedback problem. It is only when one needs the conditions to hold for $p_k \in \{\delta_x \mid x \in \boldsymbol{R}^n\}$ that U is forced to be a storage function.

In general, conditions for the optimal policy maybe difficult to establish. However, there may exist non-optimal state feedback policies such that their storage functions satisfy the conditions of theorem 2. In that case, using such reduced complexity policies based on the non-optimal state feedback policies will guarantee that the system is asymptotically stable whenever the exogenous inputs are zero. Moreover, such policies will also ensure that the closed-loop system satisfies the attenuation level γ (equation (2)).

Now consider the condition that characterizes certainty equivalence in terms of the upper value function of the output feedback problem. In (James *et al.*, 1994),(James, 1994) it is shown that certainty equivalence holds if, for all $k \geq 0$

$$M(p_k) = (p_k, V) \tag{5}$$

In general, since the trajectory p_k is not known *a priori*, one needs to check for all $p_k \in \mathcal{E}$. In fact one can show that if this were the case, then V is in fact the only function which will satisfy (5). To do so, one requires the following inequality.

Lemma 3. Let $\bar{u} \in \mathcal{I}$, with W its storage function. Then

$$W(p_k) \geq \inf_{u \in \boldsymbol{U}} \sup_{x \in \boldsymbol{R}^n} J_U^{p_k}(x,u), \ k = 0, 1, \ldots$$

where, $U(x) \triangleq W(\delta_x)$.

Proof:

$$\begin{aligned}W(p_k) \geq &\sup_{x \in \boldsymbol{R}^n} \{p_k(x) + \sup_{w \in l^2([0,\infty),\boldsymbol{R}^r)} \sum_{i=k}^{\infty} | z_{i+1} |^2 - \\ &\gamma^2 \mid w_i |^2 | \, x_k = x\} \\ = &\sup_{x \in \boldsymbol{R}^n} \{p_k(x) + \sup_{w_k \in \boldsymbol{R}^r} (| \, l(x, \bar{u}(p_k), w_k) |^2 - \\ &\gamma^2 \mid w_k |^2 + \sum_{i=k+1}^{\infty} | z_{i+1} |^2 - \gamma^2 \mid w_i |^2 | \\ &x_{k+1} = f(x, \bar{u}(p_k), w_k))\} \\ \geq &\sup_{x \in \boldsymbol{R}^n} \{p_k(x) + \sup_{w \in \boldsymbol{R}^r} (| \, l(x, \bar{u}(p_k), w) |^2 - \\ &\gamma^2 \mid w |^2 + U(f(x, \bar{u}(p_k), w)))\} \\ \geq &\inf_{u \in \boldsymbol{U}} \sup_{x \in \boldsymbol{R}^n} \{p_k(x) - \sup_{w \in \boldsymbol{R}^r} (| \, l(x,u,w) |^2 - \\ &\gamma^2 \mid w |^2 + U(f(x,u,w)))\} \\ = &\inf_{u \in \boldsymbol{U}} \sup_{x \in \boldsymbol{R}^n} J_U^{p_k}(x,u)\end{aligned}$$

Theorem 3. (Unicity). Let M be the upper value function of the output feedback problem. If there exists a function $U : \boldsymbol{R}^n \to \boldsymbol{R}$, such that $M(p_k) = (p_k, U)$, for all $p_k \in \mathcal{E}$, then $U \equiv V$, the upper value function of the state feedback problem.

Proof: It follows from lemma 3, that

$$(p_k, U) = M(p_k) \geq \inf_{u \in \boldsymbol{U}} \sup_{x \in \boldsymbol{R}^n} J_U^{p_k}(x,u).$$

Let $\hat{u}(p_k) \in \arg\min_{u \in \boldsymbol{U}} \sup_{x \in \boldsymbol{R}^n} J_U^{p_k}(x,u)$. Then

$$\begin{aligned}(p_k, U) &\geq \sup_{x \in \boldsymbol{R}^n} J_U^{p_k}(x, \hat{u}(p_k)) \\ &\geq \sup_{y \in \boldsymbol{R}^t} (H(p_k, \hat{u}(p_k), y), U) \\ &= \sup_{y \in \boldsymbol{R}^t} M(H(p_k, \hat{u}(p_k), y))\end{aligned}$$

Hence, $\hat{u}$ is an optimal policy since $(p_0, U) = M(p_0)$, $\forall\ p_o \in \mathcal{E}$. Thus,

$$M(p_k) = \sup_{y \in \boldsymbol{R}^t} M(H(p_k, \hat{u}(p_k), y))$$

which implies that

$$(p_k, U) = \inf_{u \in \boldsymbol{U}} \sup_{x \in \boldsymbol{R}^n} J_U^{p_k}(x, u).$$

Setting, $p_k = \delta_x$, one obtains

$$U(x) = \inf_{u \in \boldsymbol{U}} \sup_{w \in \boldsymbol{R}^r} \{|\ l(x,u,w)\ |^2 - \gamma^2\ |\ w\ |^2 + U(f(x,u,w))\}.$$

with $U(x) = M(\delta_x) \geq (0, \delta_x) = 0$, and $U(0) = M(\delta_0) = 0$. Hence, $U \equiv V$.

Corollary 2. If there exists a p_k such that $M(p_k) \neq (p_k, V)$, then there exists no function $Y : \boldsymbol{R}^n \rightarrow \boldsymbol{R}$, such that $M(p) = (p, Y)$ for all $p \in \mathcal{E}$.

Corollary 3. Let W be a storage function for a (non-optimal) information state feedback policy $\bar{u} \in \mathcal{I}$, and let $W(p_k) = (p_k, U)$, $k \geq 0$. Then $\hat{u}(p_k) \in \arg\min_{u \in \boldsymbol{U}} \sup_{x \in \boldsymbol{R}^n} J_U^{p_k}(x, u)$ solves the output feedback problem with the storage function $W(p)$. Furthermore, if one insists that $W(\delta_x) = (\delta_x, U)$, $\forall\ x \in \boldsymbol{R}^n$, then U is a storage function for a (non-optimal) state feedback policy. Also, if $W \equiv M$, the upper value function of the output feedback problem, then the controller is a certainty equivalence controller.

Remark: It is clear from the proof of theorem 3, that if (5) holds, then so does (4). However, (4) is a more tractable condition, since it does not involve the upper value function M, which is what one is trying to avoid having to compute in the first place.

The following alternate condition is a direct consequence of theorem 3.

Corollary 4. (Certainty Equivalence). The certainty equivalence controller is optimal, if there exists a solution $U : \boldsymbol{R}^n \rightarrow \boldsymbol{R}$ to the functional equation

$$M(p) = (p, U),\ \forall p \in \mathcal{E}$$

4. CONCLUSION

This paper has identified a strategy for generating reduced complexity output feedback policies. Sufficiency conditions have been stated, which guarantee asymptotic stability of the closed-loop system, in the absence of any exogenous inputs ($w \equiv 0$), as well as achieve the pre-specified attenuation level γ. In the optimal case, it is observed that the controller generated by such strategies reduces to the certainty equivalence controller. In doing so, one was able to obtain a more tractable version of the certainty equivalence condition stated in (James, 1994),(James *et al.*, 1994). Also, the certainty equivalence condition is shown to be equivalent to the existence of a solution to a functional equation.

Future research in this area pertains to showing whether (if at all) solvability of the output feedback problem implies existence of such reduced complexity controllers. Also, a more constructive approach to the problem needs to be developed. Finally, one can view the approach as trying to reduce complexity by considering storage functions that are evaluated by interpolation through the sup-pairing. Further investigation into alternate methods of interpolation may also prove fruitful.

5. ACKNOWLEDGMENTS

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