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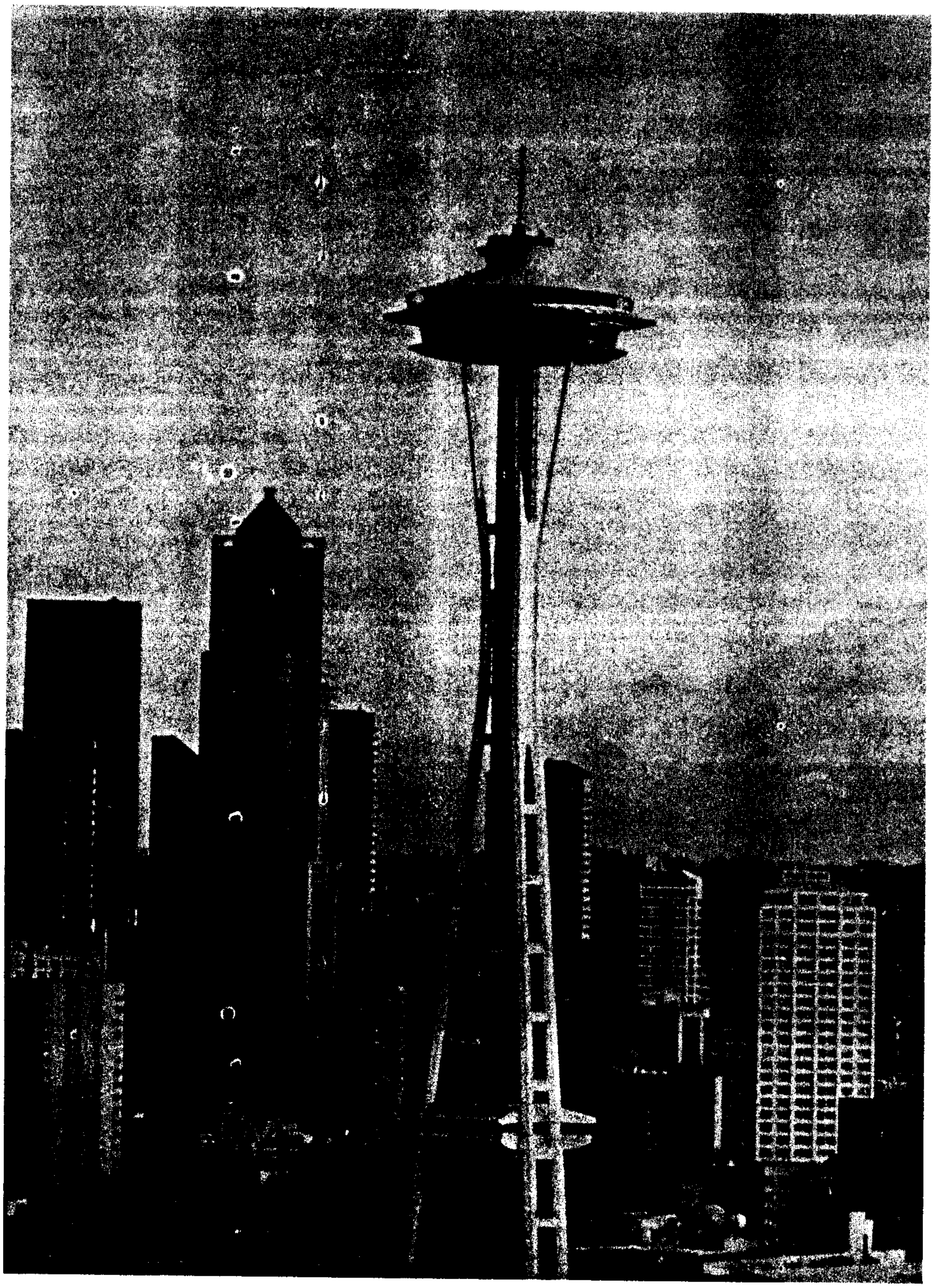

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Robust Control of Discrete Time Generalized Dynamical Systems: Finite and Infinite Time Results[1]

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Abstract

This paper deals with the output feedback robust control of discrete time systems modelled as dynamic inclusions. This formulation allows for the more typical in practice situation of non-additive distrubances. The control problem is solved via dynamic programming. Separation between estimation and control is achieved by using the concept of an information state. The nature of the information state is investigated.

1. Introduction

In this paper, we deal with the robust control of systems modelled as dynamic inclusions. Systems of this type occur, for example in the following cases. (i) When we have real parametric uncertainty. In the linear context, the stability analysis of these systems can be carried out by techniques evolving from Kharitonov's theorem, or the zero exclusion principle [5]. (ii) In the case of hybrid systems, where an upper logical level switches between different plant models, depending on observed events [8]. (iii) When the system is discontinuous in the states, e.g. systems subject to friction. (iv) When the information available about the system is insufficient to enable one to generate a reliable nominal model. In this case, one must then deal with the entire set of viable models.

Our work is motivated by recent results obtained in the nonlinear H_∞ context in [10]. We will use the dynamic game framework developed in [6],[10]. Furthermore, to establish the ultimate boundedness of trajectories, we employ the theory of dissipative systems [13] to write down a version of the bounded real lemma. We employ the concept of an information state to obtain a separation between estimation and control. The exact form of the information state recursion was derived from an analogous set-valued stochastic control problem in [3]. This paper is a summary version of [4], and we present only the major results here.

Formally, the system under consideration (Σ) is expressed as

$$\Sigma \begin{cases} x_{k+1} & \in & \mathcal{F}(x_k, u_k)\ , \quad x_0 \in X_0 \\ y_{k+1} & \in & \mathcal{G}(x_k, u_k) \\ z_{k+1} & = & l(x_{k+1}, u_k)\ , \quad k = 0, 1, \ldots \end{cases} \tag{1}$$

Here, $x_k \in \boldsymbol{R}^n$ are the states, $u_k \in U \subset \boldsymbol{R}^m$ are the control inputs, $y_k \in \boldsymbol{R}^t$ are the measured variables, and $z_k \in \boldsymbol{R}^q$ are the regulated outputs. The following assumptions are made on the system Σ:

1. $0 \in X_0$.
2. $\mathcal{F}(x,u)$, $\mathcal{G}(x,u)$ are compact for all $x \in \boldsymbol{R}^n$ and $u \in U$.
3. The origin is an equilibrium point for $\mathcal{F}$, $\mathcal{G}$ and l. i.e .
$$\mathcal{F}(0,0) \ni 0\ ;\ \ \mathcal{G}(0,0) \ni 0\ ;\ \ l(0,0) = 0$$
4. There exists an $\bar{\epsilon} > 0$, such that for all $x \in \boldsymbol{R}^n$, $u \in U$, $B_\epsilon(r) \bigcap \mathcal{F}(x,u) \neq \phi$ for all $r \in \mathcal{F}(x,u)$, $\bar{\epsilon} > \epsilon > 0$. Here $B_\epsilon(r)$ is the open ball of radius ϵ centered at r.
5. $l(\cdot, u) \in C^1(\boldsymbol{R}^n)$ for all $u \in U$ and is such that, $\exists\ \gamma_{min} > 0$, such that the set
$$\left\{ s \in \boldsymbol{R}^n | \exists\ u \in U \text{ s.t. } \left| \frac{\partial}{\partial x} l(s,u) \right| \leq \gamma \right\}$$
is bounded and contains the origin $\forall\ \gamma \geq \gamma_{min}$.

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6. $U \subset \boldsymbol{R}^m$ is compact.

Some of the notation employed in the paper will be as follows: $|\cdot|$ denotes the Euclidean norm, $\|\cdot\|$ denotes the l^2 norm, $\Gamma^u(x_0)$ denotes the forward cone of the point $x_0 \in \boldsymbol{R}^n$ [1]. In particular

$$\Gamma^u(x_0) \triangleq \{x | x_{j+1} \in \mathcal{F}(x_j, u_j), j = 0, \ldots\}.$$

and $\Gamma^u_{j,k}(x)$ with $j \leq k$ refers to the section of the forward cone of the point x, between time instances j and k. Similarly, $\Delta^u(x_0)$, denotes the forward cone of the measured variable (y), provided that the initial state is x_0. Finally, we denote the set of control policies as O. Hence, if $u \in O$, then $u_k = h(y_{1,k}, u_{0,k-1})$.

The robust control problem can now be stated as: Given $\gamma \geq \gamma_{min}$, find a controller $u \in O$, such that the closed loop system Σ^u satisfies the following three conditions:

1. Σ^u is weakly asymptotically stable, in the sense that for each k, there exists an $\alpha_k \in \mathcal{F}(x_k, u_k)$ such that, the sequence $\alpha_k \longrightarrow 0$ as $k \longrightarrow \infty$.
2. Σ^u is ultimately bounded.
3. There exists a finite $\beta^u(x)$, with $\beta^u(0) = 0$ such that

$$\sup_{r,s\in\Gamma^u(x_0), r\neq s} \sum_{i=0}^{\infty}\{| l(r_{i+1}, u_i) - l(s_{i+1}, u_i) |^2$$
$$-\gamma^2 | r_{i+1} - s_{i+1} |^2\} \leq \beta^u(x_0),$$

$\forall\, x_0 \in X_0$. In particular, if $x_0 = 0$, and r, s are such that $r - s \in l^2$, then the above guarantees that

$$\frac{\|l(r,u) - l(s,u)\|}{\|r - s\|} \leq \gamma$$

2. Finite Time Case

For the finite time case, we are only interested in the satisfaction of 3. Hence, the problem can be restated as: Given $\gamma \geq \gamma_{min}$, and a finite time interval $[0, K]$, find a control policy $u \in O_{0,K-1}$, such that there exists a finite quantity $\beta^u_K(x)$, with $\beta^u_K(0) = 0$ and

$$\sum_{i=0}^{K-1}\{| l(r_{i+1}, u_i) - l(s_{i+1}, u_i) |^2 -$$
$$\gamma^2 | r_{i+1} - s_{i+1} |^2\} \leq \beta^u_K(x_0), \qquad (2)$$

$\forall\, r, s \in \Gamma^u_{0,K}(x_0)$, $\forall\, x_0 \in X_0$.

Before proceeding further, we introduce a function space $\mathcal{E} = \{p : \boldsymbol{R}^n \longrightarrow \boldsymbol{R}^*\}$, and define for each $x \in \boldsymbol{R}^n$ a function $\delta_x : \boldsymbol{R}^n \longrightarrow \boldsymbol{R}^*$ by

$$\delta_x(\xi) \triangleq \begin{cases} 0 & \text{if } \xi = x \\ -\infty & \text{if } \xi \neq x \end{cases}$$

Furthermore, we introduce the following pairing: $(p, q) = \sup_{x\in\boldsymbol{R}^n}\{p(x) + q(x)\}$.

2.1. Information state

Motivated by results obtained in the set-valued stochastic control problem [3], and by the formalism of [10], for a fixed $y_{1,k} \in \Delta^u_{1,k}(X_0)$, and $u_{1,k-1}$, we define the *information state* $p_k \in \mathcal{E}$ by

$$p_k(x) \triangleq \sup_{x_0\in X_0}\ \sup_{r,s\in\Gamma^{u,y}_{0,k}(x_0)} \{p_0(x_0) + \sum_{i=1}^{k} | l(s_i, u_{i-1})$$
$$- l(r_i, u_{i-1}) |^2 - \gamma^2 | r_i - s_i |^2 \mid r_k = x\} \qquad (3)$$

where, $\Gamma^{u,y}_{0,k}(x_0)$ is the set of state trajectories compatible with the observed $y_{1,k}$ and $u_{0,k-1}$, given that the initial state was x_0. Here, the convention is that the supremum over an empty set is $-\infty$. Since, $x_0 \in X_0$, we may assume that $p_0(x) = -\infty$, for all $x \notin X_0$.

Lemma 1 *For any output feedback controller $u \in O_{0,K-1}$, the closed loop system Σ^u is finite gain on $[0, K]$ if and only if the information state p_k satisfies*

$$\sup_{y_{1,k}\in\Delta^u(X_0)} \{(p_k, 0) \mid p_0 = \delta_{x_0}\} \leq \beta^u_K(x_0), \qquad (4)$$

for all $k \in [0, K]$ and for some finite $\beta^u_K(x_0)$, with $\beta^u_K(0) = 0$.

Now, define $H(p, u, y) \in \mathcal{E}$ by $H(p, u, y)(x) \triangleq \sup_{\xi\in\boldsymbol{R}^n}\{p(\xi) + B(\xi, x, u, y)\}$ with the function B defined by

$$B(\xi, x, v, y) \triangleq \sup_{s\in\mathcal{F}(\xi,v)} \{| l(x, v) - l(s, v) |^2 -$$
$$\gamma^2 | x - s |^2\}$$

if $x \in \mathcal{F}(\xi, v)$, $y \in \mathcal{G}(\xi, v)$, and is equal to $-\infty$ else. Then, we obtain

Lemma 2 *The information state is the solution of the following recursion*

$$\begin{cases} p_{k+1} &= H(p_k, u_k, y_{k+1}) \\ p_0 &\in \mathcal{E} \end{cases} \qquad (5)$$

for $k = 0, \ldots, K-1$.

The information state dynamics (5) may be regarded as a new (infinite dimensional) control system Ξ, with control u and uncertainty parameterized by y. The state p_k, and the disturbance y_k are available to the controller, so the original output feedback dynamic game is equivalent to a new game with *full information.* The cost is now given by

$$\sup_{y_{1,k}\in\Delta^u(X_0)} \{(p_k, 0) \mid p_0 = p\},\ k \in [0, K]$$

Assuming Σ^u is finite gain, define

$$\mathcal{J}_K^u \triangleq \{p \in \mathcal{E} \mid (p,0), (p, \beta_K^u) \text{ is finite}\}$$

and define for a function $M : \mathcal{E} \longrightarrow \boldsymbol{R}^*$,

$$dom\ M \triangleq \{p \in \mathcal{E} \mid M(p) \text{ finite}\}$$

We now need an appropriate class $I_{i,K-1}$ of controllers, which feedback this new state variable. A control u belongs to $I_{i,K-1}$, if for each $k \in [i, K-1]$, there exists a map $\bar{u}_k$ from a subset of $\mathcal{E}^{k-i+1}$ (sequences $p_{i,k}$) into U, such that $u_k = \bar{u}(p_{i,k})$. Note that, $I_{0,k-1} \subset O_{0,k-1}$, for $k = 1, \ldots, K$.

2.2. Solution to the finite time output feedback robust control problem

We use dynamic programming to solve the game. Define the value function by

$$M_k(p) = \inf_{u\in O_{0,k-1}} \sup_{y\in\Delta^u_{1,k}(X_0)} \{(p_k, 0) \mid p_0 = p\} \tag{6}$$

for $k \in [0, K]$, and the corresponding dynamic programming equation is

$$M_k(p) = \inf_{u\in U} \sup_{y\in \boldsymbol{R}^t} \{M_{k-1}(H(p,u,y))\},\ k \in [1, K] \tag{7}$$

with the initial condition $M_0(p) = (p, 0)$. Then, we obtain the following necessary and sufficient conditions.

Theorem 1 *(Necessity) Assume that $\bar{u} \in O_{0,K-1}$ solves the finite time output feedback robust control problem. Then there exists a solution M to the dynamic programming equation (7) such that $\mathcal{J}_K^{\bar{u}} \subset dom\ M_k$, $M_k(\delta_0) = 0$, $M_k(p) \geq (p, 0)$, $k \in [0, K]$.*

Theorem 2 *(Sufficiency) Assume there exists a solution M to the dynamic programming equation (7) such that $\delta_x \in dom\ M_k$ for all $x \in X_0$, $M_k(\delta_0) = 0$, $M_k(p) \geq (p, 0)$, $k \in [0, K]$. Let $u^* \in I_{0,K-1}$ be a policy such that $u_k^* = \bar{u}^*_{K-k}(p_k)$, $k = 0, \ldots, K-1$; where $\bar{u}_k^*(p)$ achieves the minimum in (7), $k = 1, \ldots, K$. Then u^* solves the finite time output feedback robust control problem.*

3. Infinite Time Case

We pass to the limit as $K \longrightarrow \infty$ in the dynamic programming equation (7)

$$\lim_{k\longrightarrow\infty} M_k(p) = M(p)$$

where $M_k(p)$ is defined by (6), to obtain a stationary version of equation (7)

$$M(p) = \inf_{u\in U} \sup_{y\in\boldsymbol{R}^t} \{M(H(p,u,y))\} \tag{8}$$

3.1. Dissipation inequality

The following lemma is a consequence of Lemma 1.

Lemma 3 *For any $u \in O$, the closed loop system Σ^u is finite gain if and only if the information state satisfies*

$$\sup_{k\geq 1} \sup_{y\in\Delta^u_{1,k}(x_0)} \{(p_k, 0) \mid p_0 = \delta_{x_0}\} \leq \beta^u(x_0) \tag{9}$$

for some finite $\beta^u(x_0)$, with $\beta^u(0) = 0$.

Hence, we say that the information state system Ξ^u ((5) with information state feedback $u \in I$) is finite gain if and only if the information state p_k satisfies (9) for some finite $\beta^u(x_0)$, with $\beta^u(0) = 0$. Furthermore, if Σ^u is finite gain, we write

$$\mathcal{J}^u \triangleq \{p \in \mathcal{E} \mid (p,0), (p, \beta^u) \text{ finite}\}$$

We say that the information state system $\Xi^{\bar{u}}$ is *finite gain dissipative* if there exists a function (storage function) $M(p)$, such that $dom\ M$ contains δ_x for all $x \in X_0$, $M(p) \geq (p, 0)$, $M(\delta_0) = 0$, and satisfies the following dissipation inequality

$$M(p) \geq \sup_{y\in\boldsymbol{R}^t} \{M(H(p, \bar{u}(p), y))\} \tag{10}$$

Note, that if $\Xi^{\bar{u}}$ is finite gain dissipative, then $p_0 \in dom\ M$, implies $p_k \in dom\ M$, $\forall k > 0$.

We will need the following assumption.

A: Assume that for a given $\gamma > 0$, the system $\Sigma^{\bar{u}}$ is such that

$$\lim_{k\longrightarrow\infty} \left| \frac{\partial}{\partial x} l(\bar{x}_{k+1}, \bar{u}_k) \right| \leq \gamma$$

implies $0 \in \liminf_{k\longrightarrow\infty} \mathcal{F}(\bar{x}_k, \bar{u}_k)$.

Remark: The assumption above, can be viewed to be analogous to the *detectability* assumption often encountered in H_∞ control literature.
We now state a version of the bounded real lemma for the information state system Ξ.

Theorem 3 *Let $u \in I$. Then the information state system Ξ^u is finite gain if and only if it is finite gain dissipative. Furthermore, if Ξ^u is finite gain dissipative, then*
(i) Ξ^u is stable for all feasible $x \in R^n$.
(ii) Σ^u is ultimately bounded.
(iii) If Σ^u satisfies assumption A, then Σ^u is weakly asymptotically stable.

3.2. Solution to the output feedback robust control problem

It can be inferred from theorem 3, that the controlled dissipation inequality (10) is both a necessary and sufficient condition for the solvability of the output feedback robust control problem. However, the following two theorems give necessary and sufficient conditions in terms of the dynamic programming equation (8).

Theorem 4 *(Necessity) Assume that there exists a controller $\bar{u} \in O$ which solves the output feedback robust control problem. Then there exists a function $M(p)$, such that $\mathcal{J}^{\bar{u}} \subset dom\ M(p)$, $M(p) \geq (p, 0)$, $M(\delta_0) = 0$ and M solves the stationary dynamic programming equation (8) for all $p \in \mathcal{J}^{\bar{u}}$.*

Theorem 5 *(Sufficiency) Assume that there exists a solution M to the stationary dynamic programming equation (8) such that $\delta_x \in dom\ M$, $\forall x \in X_0$, $M(\delta_0) = 0$, and $M(p) \geq (p, 0)$. Let $\bar{u} \in I$ be a policy such that $\bar{u}(p)$ achieves the minimum in (8). Then, $\bar{u} \in I$ solves the information state feedback robust control problem, (and hence the output feedback robust control problem) if the closed loop system $\Sigma^{\bar{u}}$ satisfies assumption A.*

4. Example

As an example, we design a disturbance rejection controller for the following system, using both H_∞ and set-valued design techniques. The details of the design process will be given elsewhere [2]. The system is given by

$$\theta[k+2] + (c_1 - 2)\theta[k+1] + (1 - c_1)\theta[k] +$$

$$c_2 f \operatorname{sgn}(\theta[k+1] - \theta[k]) = c_3 u[k] \qquad (11)$$

with $\theta[1] = \theta[0] = 0$. Here θ is the position in radians, and the nominal values of the parameters are: $c_1^0 = 0.162$, $c_2^0 = 0.1$, $c_3^0 = 6.43 \times 10^{-4}$, and $f^0 = 7.2 \times 10^{-3}$. Furthermore, $-10 \leq u[k] \leq 10$, and we assume a $\pm 10\%$ variation in the parameter values. The above system corresponds to a sampling time of 0.01s. The controller is supposed to reject output additive disturbances (i.e. $y[k] = \theta[k] - r[k]$) of upto ± 0.25 radians, with a cutoff at 0.5 Hz. We first carry out the H_∞ design.

4.1. H_∞ design

We, first smoothen and approximately linearize the system (11) via dithering and nonlinear feedback, to obtain $G^0(z) = \frac{c_3^0}{(z-1)(z-1+c_1^0)}$ as the nominal plant. Due to the pole at $z = 1$, it is not possible to carry out an H_∞ design on this nominal plant. So, we apply a unity feedback, and shift the pole away from 1. Thus, we work with $P^0(z) = \frac{G^0(z)}{1+G^0(z)}$. Representing the parameter variations as a multiplicative perturbation ($\Delta_m(z)$), we obtain $\|\Delta_m\|_\infty < 0.25$. Transforming the discrete time plant into continuous time, using the inverse Tustin transform (which preserves the H_∞ norm), the problem can be stated as: Given $P^0(s)$, $W_1(s)$, $W_2(s)$, and $W_3(s)$, maximize ρ, while ensuring that $\left\| \begin{matrix} \rho W_1 T_{w,e} \\ W_2 T_{w,u} \\ W_3 T_{w,y} \end{matrix} \right\|_\infty < 1$, where $T_{w,e}$ is the sensitivity function, $T_{w,y}$ is the complementary sensitivity function, and $T_{w,u}$ is the transfer function from the disturbance to the controller output. Here, $W_3(s) = 0.25$, $W_2(s) = 10^{-2}\frac{s+10}{s+350}$, and $W_1(s) = \frac{\pi}{s+\pi}$. The solution is obtained via [7]. We get $\rho = 8.625$, and the corresponding controller $C_P(z)$ is

$$\frac{81.83z^4 - 137.86z^3 - 25.37z^2 + 137.86z - 56.45}{z^4 - 2.41z^3 + 1.45z^2 + 0.36z - 0.39}$$

which can now be transformed to correspond to the original plant $G(z)$ by $C_G(z) = C_P(z) + 1$.

4.2. Set-valued design

As in the H_∞ case, we pick $W_1(s) = \frac{\pi}{s+\pi}$. We discretize it using the Euler transform, which is known to preserve the H_∞ and l^1 norms [11]. Weighing the disturbance with W_1, we obtain $x_3[k+1] = 0.9686x_3[k] + 0.0314r$ with $r \in [-0.25, 0.25]$. Furthermore transforming (11) into its state space form, and allowing for parameter variations, we obtain

$$\begin{aligned} x_1[k+1] &\in \mathcal{A}x_1[k] + \mathcal{B}u[k] + \mathcal{C} \\ x_2[k+1] &= x_2[k] + 0.01x_1[k] \end{aligned}$$

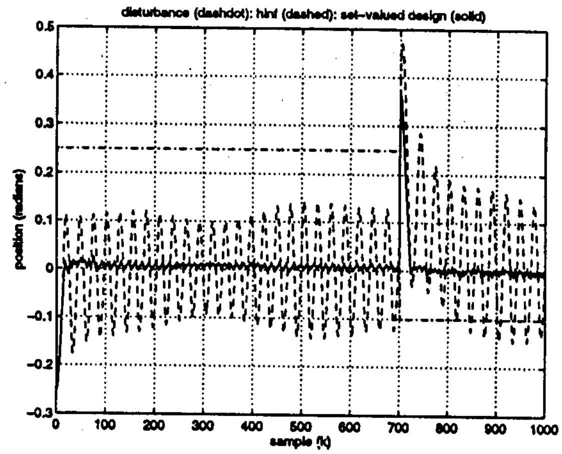

Figure 1: Error incurred during step disturbances

$$
\begin{aligned}
x_3[k+1] &\in 0.9686x_3[k] + 0.0314[-0.25, 0.25] \\
y[k+1] &= x_2[k] - x_3[k] \\
z[k+1] &= 10(x_2[k+1] - x_3[k+1])^2
\end{aligned}
$$

with $x_1[0] = x_2[0] = x_3[0] = 0$. Here, $\mathcal{A} = [0.8212, 0.8542]$, $\mathcal{B} = [0.05787, 0.07073]$, and $\mathcal{C} = [-0.08712, 0.08712]$. Note that $\theta[k] = x_2[k]$, and the error $e[k] = y[k]$. To avoid the infinite time dynamic programming, in practice we implement a certainty equivalence controller [6],[12]. Assuming, we have a solution to the state feedback control problem, with $\bar{u}$ its policy and V the value function, we estimate $\hat{x} \in \text{argmax}_x\{p_k(x) + V(x)\}$, where p_k is the information state at time k. Based on this, we then choose $u_k = \bar{u}(\hat{x})$ as the control value at time k. In general, the certainty equivalence controller is non-optimal.

We solve the state feedback problem with $x_1 \in [-2.5, 2.5]$, $\Delta x_1 = 0.1$, $x_2 \in [-0.3, 0.3]$, $\Delta x_2 = 0.01$, $x_3 \in [-0.35, 0.35]$, $\Delta x_3 = 0.05$, and we use $\Delta u = 0.5$. For this descretization, the optimal value of γ lies between 0.12 and 0.14. We pick $\gamma = 0.14$. The dynamic programming equation converges after 14 iterations. While carrying out the simulations, the plant parameters c_1, c_2, c_3, and f are sinusoidally varied between their exterme values at frequencies of 0.1, 0.2, 0.25, and 0.4 Hz respectively. Figure 1 shows the error incurred due to a step disturbance (dashdot) by the H_∞ controller (dashed), and the certainty equivalence controller (solid). The large error for the H_∞ controller is due to saturation of the input. However, using a more stringent $W_2(s)$ during the design process yields a controller that is ineffective in rejecting disturbances (due to extremely small values of ρ).

5. Conclusion

In this paper, we stated and solved the robust output feedback control problem for systems governed by dynamic inclusions. The problem remains computationally hard. One way of reducing the computation cost is via the certainty equivalence controller. However, conditions need to be established as to when certainty equivalence holds (see [9] for such results). Work is currently underway in this direction, and for developing efficient computational algorithms, as well as approximations to the original problem.

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