

Optimization-Based, Direct Frequency Domain Design of  
Controllers for Linear Distributed Systems

John S. Baras  
Electrical Engineering Department  
University of Maryland  
College Park, Maryland 20742

Abstract:

A CAD methodology is developed for the design of controllers for linear distributed systems incorporating the following key components: (i) Transfer function models of distributed systems and their approximations; (ii) Recent frequency domain design methodology for multi-variable control systems; (iii) Interactive optimization packages (DELIGHT) in the search for feasible designs satisfying practical engineering specifications. Primarily specifications on the shape of the frequency response and of the unit step response will be considered. The application of the method for large space structures is explained via simple examples including beams and plates.

## 1. Identification of the Problem and Its Significance

Satisfactory performance of large space structures, especially antennae, depends on the competence of their control systems. Several kinds of control systems are required for each device including: shape, attitude, deployment, and orbital transfer and stationkeeping. In a global sense the control systems cannot be totally divorced since the operation of each affects the dynamics of the others. In particular, since the attitude motion is described by coordinates depending on time alone and the elastic deformation of the device by coordinates depending on the spatial position and time, large flexible spacecraft are hybrid dynamical systems [1]. If the device is a composite of several articulated structures, then the complexity is compounded. For example, it is impossible to choose principal coordinate axes in a continuous way as the configuration of the rigid segments is changed [2]. This means that the principal coordinate systems in which observations and control actions are made will not always be the most natural. This is one major difference between the behavior of these systems and the usual rigid body dynamics.

Another consideration in the design of large, high-precision antennae is achieving the minimum feasible mass per unit area. Reflector surface mass densities as low as several grams per square meter can be approached with the use of a membrane reflector such as a wire mesh or a metalized plastic film [3][4]. To achieve low antenna mass densities, the mechanical rigidity of the reflector is sacrificed. This, however, does not preclude the maintenance of a very precise reflector configuration provided the deflection of many points on the reflector can be rapidly and precisely controlled. Guidelines for estimating the number  $N$  of control points necessary to achieve a particular tolerance may be readily determined [3][5]. In typical cases  $N$  will be on the order of 1000 [3].

There are several different kinds of disturbances which tend to distort the shape of the flexible body following deployment and attitude acquisition. These include solar wind effects, thermal gradients, residual aerodynamical momenta, controller errors, gravitational gradients, and material non-uniformities. To counter these disturbances, one can implement pointwise active figure control systems using  $N$  control elements, or one can consider the use of distributed control based on electrostatic forces as described in [3][4]. In either case the resulting control problem requires techniques which go beyond the state of current control theory.

In this research we plan to concentrate on the problem of controller design for large space structures. Actuators in space structures can be implemented in a localized fashion, such as thrusters or torquers at a finite number of locations, or in a distributed fashion such as the electrostatic control of antenna shape [3,4]. The controllers utilize available measurements, which may be noisy or incomplete, from on board sensors to estimate critical parameters of the large space structure. The typical formulation of controller design for large space structure follows the so called "Linear Quadratic Gaussian" (LQG) approach. Namely, the dynamics of the flexible parts are described by a continuous, distributed state which satisfies a self-adjoint, linear boundary value problem, together with a set of observations and actuators applied at a number of discrete points on the structure. The performance criteria is quadratic, leading thus to an infinite dimensional regulator problem [19]. Resolution of this problem requires effective interaction of structural dynamics modeling algorithms with control and estimation algorithms. Sophisticated analytical methods are needed to develop this interaction to the advanced level required - recent surveys of the state of the art in the control of large flexible structures clearly indicate that this level has not been achieved [1,8,9].

While the control theory for large space structures is relatively young, dating perhaps from reports like [17], there has been an extensive investigation of the use of state variable control theory for the design of controllers. Most of this work has, at some point in the analysis, invoked a finite dimensional approximation and then used finite dimensional linear control theory, e.g., the "LQG" theory, to arrive at specific control algorithms. These techniques, which almost always involve some form of a modal expansion in terms of the eigenfunctions of the linear operators involved in the model, have led to two kinds of problems: the "spillover effect" articulated by Balas [8], and a high dimensionality of the resulting model. While ad hoc techniques have been found to deal with these kinds of problems, including the use of bandpass prefilters in the controllers, and while the theory of modal control has been advanced to a considerable degree [18], basic problems still remain, including the solution of large systems of linear equations and Riccati equations, and the lack of an effective framework in which to assess the convergence of the resulting finite dimensional control structures to the optimal infinite dimensional ones. (In [19][20] Gibson has shown that convergence is in fact impossible if the most commonly used models are not augmented to include the system's natural damping which is a parasitic effect in most designs.) We believe that the lack of a consistent approximation theory incorporating approximation bounds as required by the design criteria and objectives is the major stumbling block for the development of an efficient, computationally feasible design methodology for controllers in large space structures. In fact it is quite apparent that the currently employed finite dimensional approximations [8][9], based either on modal truncation or finite element analysis cannot be linked tightly to such practical engineering specifications as shape of the transfer function (in steady state, as function of frequency), or shape of the step response (e.g. as it results from specifications on settling time, rise time, overshoot, etc). The so called "spillover effect" is a precise

manifestation of this phenomenon, and is due to the inability of current design methodology to predict the effect of "closing the loop" with a controller designed on an ad hoc approximate model of the true system.

Our approach to the design problem is based on the combination of three methodologies from control theory: (i) the systematic analytical and numerical investigation of the input-output description of the infinite dimensional part of the system using transfer functions [25]; (ii) the employment of recent advances in the design theory of multivariable control systems and in particular the resulting parametrization of stabilizing compensators [21]; (iii) the utilization of advanced optimization methods (e.g. infinite programming formulations) via highly interactive optimization packages [34], to incorporate in the design realistic engineering specifications for the controllers [32].

In the proposed research program we are interested in developing this design methodology for the whole spectrum of large space structures control problems: antenna shape control, control during slewing maneuvers, attitude control. A particularly desirable feature of our approach is that requirements for decentralization (an increasingly attractive specification for the control of large space structures) can be implemented as a simple constraint on the form of controller transfer function (i.e. it must be block diagonal) and accounted for directly in the design process.

As we shall argue in a later section, the reformulation of the controller design problem in the frequency domain using optimization based design software, permits the use of fast and efficient algorithms, including fundamental methods like the fast Fourier transform, to determine the transfer functions of the controllers, in a way which can be easily programmed in a production grade software.

It is clear that the further unrestricted use of finite dimensional models for the analysis and design of control laws is not productive. The usual formulation of the linear quadratic Gaussian control problem even in the distributed parameter case involves the solution of a nonlinear operator Riccati differential equation; in view of the above remarks there is little incentive to pursue this formulation of the problem.

It is well known that the linearized dynamical equations describing the distributed parameter subsystems (e.g. panels, antennae, flexible appendices) are hyperbolic partial differential equations capable of producing very weakly damped oscillatory solutions. On the other hand the controllers designed will be finite dimensional control systems. The first objective of our work is to demonstrate that the recent developments in the frequency domain design of multivariable control systems [21][25-31]

can be extended to the class of transfer functions appearing in large space structures (which are not going to be rational). Recently Baras [35][36] has demonstrated that extensions of fundamental constructs in the transfer function theory of linear systems design (such as minimality, coprime factorizations, Bezout identities, dynamic compensator design) can be extended to some very interesting classes of irrational transfer functions. We are particularly interested in extending the work of Youla et al. [21], since it incorporates several design criteria very appropriate for large space structures design, and because the resulting controllers have additional desirable properties such as adjustable stability margins, robustness with respect to system model perturbations. The main reason for our interest in this methodology, is our conviction that it provides a fundamental framework for the consistent development of approximate models on which to base the design of finite dimensional controllers for the infinite dimensional system representing the large space structure. Consistent here means that approximation errors in modeling can be quantitatively linked to performance measures or engineering specifications degradation.

The second objective of our work is the utilization of powerful optimization packages for numerical study of design procedures in multivariable control systems. The system used is the DELIGHT [32] [34] system developed at the University of California Berkeley. The approach is aimed at combining the consistent parametrization of controllers developed by the first objective with DELIGHT's capability to search through parametrizations (i.e. controllers) satisfying a plethora of practical engineering specifications such as frequency shape of the transfer function, shape of the unit step response, robustness tolerances, etc. It is our conviction that with the combination of these two objectives many intuitive design ideas (as may occur to a designer at various intermediate steps) can be quickly and inexpensively tried out prior to the final design recommendations.

## 2. Main Technical Contributions

The main technical contributions of the paper are:

- (i) Determine how transfer functions for the distributed parameter parts, of the large space structure can be computed efficiently from the dynamical equations model of the system. Determine also necessary sampling rates (in frequency, i.e.

for how many frequencies we need to know the value) for transfer function data.

This part of the work is directed at the following questions?

1. What type of irrational transfer functions arise in such systems?
  2. What is an efficient approximation of such transfer functions by sampling or rational interpolation?
  3. What are appropriate error estimates and the numerical performance of these approximations?
- (ii) Extend the frequency domain design of Youla et al. [21] and of Baras [35,36] for this class of transfer functions.

This research is directed at the following questions:

1. Can the necessary numerical computations be supported by theory when rational or sampling approximations are used?
  2. What are the appropriate parametrizations of the controller for use in the optimization phase?
  3. Can consistent error estimates be developed reflecting the design philosophy and engineering specifications? For example can coprime factorizations of a rational approximant be considered as approximants of the true irrational factors? Also can spectral factors of an approximant be considered as approximants of the true spectral factors?
- (iii) Develop a combined methodology using the structure of the controller developed in (ii) and optimization based design in order to satisfy explicit engineering constraints on frequency shape of transfer functions, or time domain specifications on unit step responses.

This research is directed at the following questions:

1. Can we show that the two methodologies will work synergistically? Is the optimization phase and the related feasible directions search consistent with the structural constraints on the controller?
2. Can we develop a quantitative (even if computable by numerical methods) relationship between the degradation of performance and engineering specifications on the one hand and the approximation errors for the transfer function on the other?

3. Is the overall method computationally efficient and accurate? Does it lend itself to efficient implementation of the resulting controllers?

### 3. Outline of the approach

As already mentioned the proposed methodology combines three ingredients of control systems modeling and design theory.

The first component on which the methodology is based is the transfer function modeling of distributed parameter systems which constitute subsystems of large space structures.

Following Balas [8], the usual abstract model for a flexible large space structure (LSS) is the distributed forced oscillator equation

$$(3.1) \quad m(x)u_{tt}(x,t) + D_0 u_t(x,t) + A_0 u(x,t) = F(x,t)$$

where  $u(x,t)$  is a vector of generalized displacements of the structure from its equilibrium configuration - displacements caused by transient disturbances and the applied force distribution  $F(x,t)$ . The mass distribution  $m(x)$  is positive and bounded on the spatial domain  $\Omega$  occupied by the LSS;  $\Omega$  is simply connected. Since the mass density of most of the structure may be very small [3][4], the system (3.1) may be "stiff" in a suitably defined sense [23]. The term  $A_0 u$  with  $A_0$  a differential operator represents the internal restoring forces of the structure. It is generally assumed that  $A_0$  has a discrete spectrum, i.e.,

$$(3.2) \quad A_0 \phi_k = \omega_k^2 \phi_k, \quad k = 1, 2, \dots$$

with  $\omega_k$  the "mode frequencies" and  $\phi_k(x)$  the mode shapes. The domain of  $A_0$  is generally a dense subset  $D(A_0)$  of some Hilbert space  $H_0$  equipped with an "energy" inner product  $\langle \cdot, \cdot \rangle$ . The damping term in (3.1) is generated by an appropriate ( $A_0$ -bounded) differential operator representing gyroscopic damping mechanisms of the LSS. (As Gibson [19][20] has shown, it may be critical to represent this term to produce a stable control system.)

In most studies [1][8] the applied force distribution is given by

$$(3.3) \quad F(x,t) = F_D(x,t) + \sum_{i=1}^m b_i(x) f_i(t)$$

where  $F_D$  represents external forces and the sum includes the control forces due to discrete actuators, with  $b_i(x)$  the actuator influence functions. Observations are generally of the form

$$(3.4) \quad y_j(t) = \langle c_j, u \rangle + \langle c'_j, u \rangle, \quad j = 1, 2, \dots, n$$

with  $c_i, c_i'$  the  $x$ -dependent influence functions (e.g., delta functions for point sensors). Accelerometers may also be used. Distributed controllers like the electrostatic system described in [3][4] require different models.

By taking  $v(x,t) = [u(x,t), u_t(x,t)]$  in  $H \equiv D(A_0) \times H_0$  with the energy norm

$$(3.5) \quad ||v||^2 = \langle mu_t, u_t \rangle + \langle A_0^{1/2} u, A_0^{1/2} u \rangle$$

produces the state variable form

$$(3.6) \quad \begin{aligned} v_t &= Av + Bf \\ y &= Cv \end{aligned}, \quad A = \begin{bmatrix} 0 & I \\ -A_0 & -D_0 \end{bmatrix}$$

with  $B$  and  $C$  clear from (3.3)(3.4) (and  $F_D = 0$  assumed). The homogeneous system is very oscillatory in the sense that the semigroup generated by  $A$  has very little damping [8]. Since the LSS is distributed over a large volume with low mass density, its dominant modes are slow and oscillatory.

There are two other aspects of the model (3.1) which have received little attention in the past. These are the fact that the low mass density of the LSS makes the mathematical system singularly perturbed, or stiff in the sense of numerical analysis. This means that there are boundary layer and initial layer effects associated with any transient motions. Fattorini has described the mathematics of these phenomena for a related class of equations in [23]. As one might expect the analysis is considerably more sophisticated than the variations on finite dimensional methods which have previously been used to analyze the dynamics of large flexible structures [8], p. 528, [24]. The transfer function of the system is obtained by taking Laplace transforms of (3.6) and is the operator valued (in general) function

$$(3.7) \quad P(s) = C(sI - A)^{-1} B.$$

In actual applications since the number of controllers (or actuators) and the number of sensors is finite, the function (3.7) is matrix valued. For the sake of specificity, let us say it is  $n \times m$ , i.e.,  $m$  actuators and  $n$  sensors. The major difficulty with (3.7) is that  $P(s)$  will not be rational as a function of the complex variable  $s$ . It may have discrete singularities (typical in the case of distributed controls), or it may even display branch points (typical in the case of boundary (i.e. point) actuators and sensors. On the other hand, due to the slight damping which is always present in (3.1) the matrix valued function  $P(s)$  will be analytical in the right half plane,  $\text{Re } s > 0$  (denoted  $\Pi^+$ ), and uniformly bounded there (a manifestation of the



Bounded Input Bounded Output Stability of (3.1)). In other words,  $P \in H_{\eta \times m}^{\infty}(\mathbb{T}^n)$  in the terminology of Baras [35,36]. As it was demonstrated in [35], appropriate and efficient approximation schemes for such transfer functions exist, although the work in [35,36] concentrated on some important subclasses of  $H^{\infty}$  only. Further work is necessary to demonstrate that the results of Baras, and efficient approximation schemes can be developed for the transfer functions arising from flexible space structures.

With respect to approximations we have two techniques in mind:  
 (i) Rational interpolation based on minimizing the  $L^{\infty}$ -norm on the  $j\omega$  axis between the true transfer function and the approximant. That is find a rational  $P_a$  such that

$$\sup_{\omega} \| P_a(j\omega) - P(j\omega) \| \text{ is minimized,}$$

where  $\| \cdot \|$  is a matrix norm. There will be infinite approximants depending on the upper bound we are willing to place on  $P_a$ . (ii) Matching the singularities of the true transfer function up to a certain region of the complex plane. This second method is less developed and requires further work in developing measures of the approximation error. It is needed because on the surface it resembles modal approximation. For both methods the behavior of sampled functions has to be investigated. By that we understand the following problem. The computation of  $P(\cdot)$  in (3.7) will be performed numerically (for example by FFT - Fast Fourier Transform), resulting in a sequence of sampled values  $P(j\omega_i)$ ,  $i = 1, \dots, N_s$ , where  $N_s$  will be large. We need to investigate how approximations developed on the basis of these samples behave under changes in sampling rate and model perturbations. This is necessary for both approximation methods.

The second component of the proposed methodology is the extension of certain recent developments in the frequency domain design of multivariable control systems to transfer function classes appearing in large space structures. A major development in the theory of multivariable control systems design was established in [21]. The solution developed in [21] is based on a least squares Wiener-Hopf minimization of an appropriately chosen cost functional. The methodology provides an analytic frequency-domain design, which is valid for inherently open loop unstable systems, improper and non-minimum phase systems. Although the method is somewhat "theoretical", it produces a multivariable controller design that is able to cope with disturbance rejection, plant saturation, measurement noise, process lag, sensitivity reduction, and at the same time incorporate suitable engineering specifications on transient behavior and steady state performance - obviously qualities of great practical value. The derived optimal controller is proper and guarantees a dynamical asymptotically stable closed-loop design possessing a proper sensitivity matrix equal to the identity matrix at  $s = \infty$ . The methodology and problem formulation presented in the seminal paper

[21], is quite different from the "Linear Quadratic Gaussian" (LQG) methodology that dominates the literature on design of controllers for large space structures [7-20]. It is worth emphasizing that the methodology permits incorporation of feedback transducers such as tachometers, rate gyros, and accelerometers with nondynamical transfer functions.

The basic multivariable closed loop configuration considered in [21], is shown in figure 1 below.

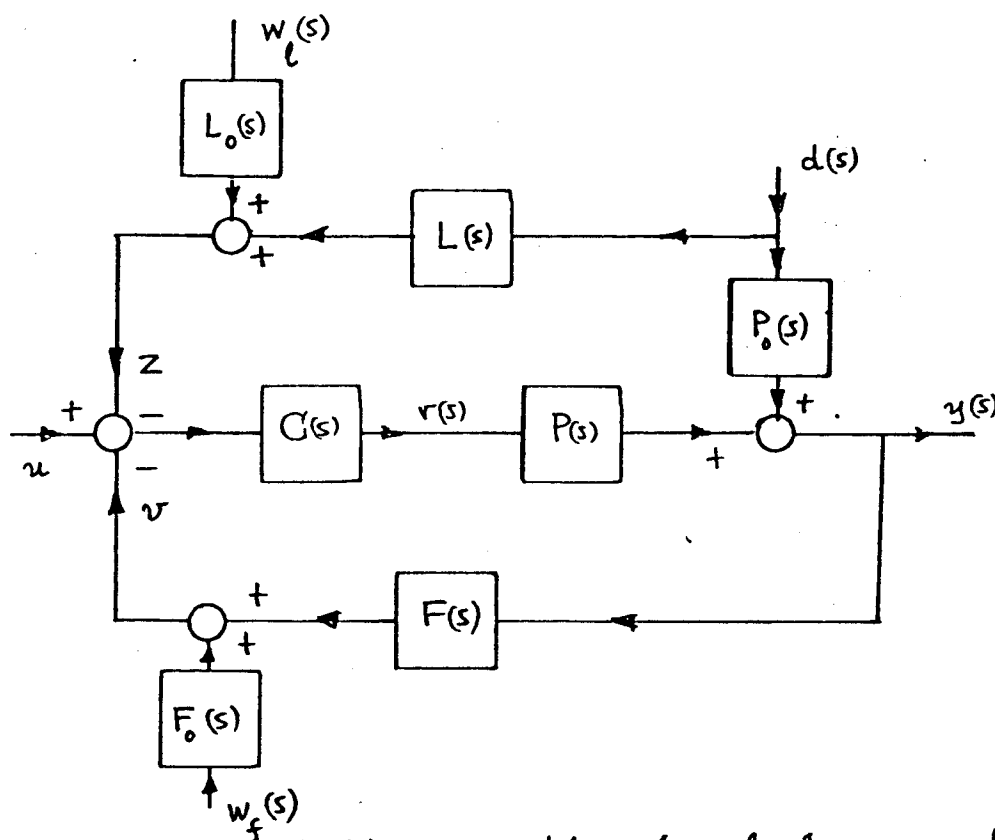


Fig 1. Multivariable closed-loop configuration

$P(s)$  is the system closed loop transfer function matrix which is assumed  $n \times m$ .  $F(s)$  is the transfer function of the feedback compensator and is assumed to be  $n \times n$ . In practice

$$(3.8) \quad F(s) = F_e(s)F_f(s)$$

where  $F_f$  represents feedback sensor dynamics, while  $F_e$  is a known pre-equalizer. The methodology of [21] assumes that  $F_e$  is known, although the designer has some flexibility in choosing  $F_e$ . Similarly,  $L(s)$  is the transfer function of the feedforward compensator which is assumed  $n \times n$  and also has the form

$$(3.9) \quad L(s) = L_e(s)L_f(s).$$

In applications to large space structures the available choices of physical sensing devices  $L_f(s)$ ,  $F_f(s)$  are severely restricted and more or less dictated by the problem at hand. In the sequel  $L$  will be assumed known also.  $C(s)$  is the  $m \times n$  transfer function matrix of the controller to be designed. Plant disturbance  $d(s)$  and instrument noise  $w_f(s)$ ,  $w_l(s)$  are incorporated into the model in a general way:

$$(3.10) \quad \begin{aligned} y(s) &= P(s)r(s) + P_o(s)d(s) \\ v(s) &= F(s)y(s) + F_o(s)w_f(s) \\ z(s) &= L(s)d(s) + L_o(s)w_l(s) \end{aligned}$$

The models  $P_o$ ,  $L_o$ ,  $F_o$  are assumed to be known in advance with certain accuracy. Furthermore the disturbance and noise models are known within certain class of models. Let

$$(3.11) \quad \begin{aligned} R(s) &= C(s)S(s) \\ S(s) &= (I + F(s)P(s)C(s))^{-1} \\ P_d(s) &= F(s)P_o(s) + L(s) \end{aligned}$$

In the absence of measurement noise and plant disturbances we have  $y = PRu$ , and therefore

$$T(s) = P(s)R(s)$$

is the closed loop transfer function matrix and  $S(s)$  is the sensitivity matrix, which has attracted quite an interest lately [37] in control theory design. Following [21] we shall call the pair of compensator  $F$  and plant  $P$  admissible if each individually is free of hidden modes and if there is no cancellation of unstable poles and zeros between  $P$  and  $F$ .

One of the most significant results in [21] is the parametrization of all stabilizing compensators for the above configuration. Namely, let  $P, F$  be admissible, and let

$$(3.12) \quad F(s)P(s) = A^{-1}(s)B(s) = B_1(s)A_1^{-1}(s)$$

where  $A, B$ , and  $B_1, A_1$  are left-right coprime factorizations of  $FP$ , respectively. Now choose polynomial matrices  $X$  and  $Y$  solving the Bezout equation

$$(3.13) \quad A(s)X(s) + B(s)Y(s) = I$$

Then the closed loop system of figure 1 is asymptotically stable if and only if

$$(3.14) \quad R(s) = H(s)A(s)$$

where

$$(3.15) \quad H(s) = Y(s) + A_1(s)K(s)$$

and  $K(s)$  is any  $m \times n$  real rational matrix, analytic in  $\text{Re } s > 0$ , and such that

$$(3.16) \quad \det (X(s) - B_1(s)K(s)) \neq 0.$$

Furthermore, the stabilizing controller associated with a particular choice of admissible  $K(s)$  has the transfer function

$$(3.17) \quad C = (Y + A_1 K) (X - B_1 K)^{-1}$$

This is the controller parametrization described earlier. The solution for "optimal choice" for  $C$ , provided in [21], is based on solving the optimization problem

$$(3.18) \quad \min_C E$$

where  $C$  is allowed to vary over all possible transfer functions defined by (3.17), and  $K$  constrained as above. Here  $E$  is a performance measure, or penalty function

$$(3.19) \quad E = E_t + kE_s$$

where  $E_t$  measures deviation of steady state response  $e(s) = u(s) - y(s)$  from zero, while  $E_s$  measures excitation of "sensitive" plant modes which must be especially guarded against excessive dynamic excursions. The complete solution to this optimization problem is provided in [21], as

$$(3.20) \quad C = H_0 (A^{-1}\Omega - FPH_0)^{-1}$$

$$H_0 = A_1 \Lambda^{-1} (\{\Lambda_*^{-1} I \Omega_*^{-1}\}_+ + \{\Lambda A_1^{-1} Y \Omega\}_-)$$

where  $\Lambda_*$ ,  $\Omega_*$  are matrix spectral factors of some positive definite spectral matrices (which can be explicitly computed from the system data),  $\{\}_+$  and  $\{\}_-$  denote the part of a rational matrix associated with its poles in  $\text{Re } s < 0$ , and  $\text{Re } s \geq 0$ , respectively. This is indeed an explicit solution and appropriate numerical algorithms have been developed as reported in [21] and elsewhere.

The objective of the work presented here is to establish a similar result and parametrization for irrational transfer functions of the type described by (3.7). The feasibility of these extensions has been demonstrated in recent work by Baras [35,36], who has provided extensions of the basic constraints for certain classes of  $H^\infty$  transfer functions. In addition we intend to investigate how the whole scheme behaves under approximations. Namely, suppose  $P_\alpha(s)$  is an approximation (rational) to  $P(s)$ . How close is the controller transfer function  $C_\alpha(s)$  based on  $P_\alpha(s)$ , to the true controller transfer function  $C(s)$  based on  $P(s)$ ? These estimates constitute the cornerstone for a consistent design methodology.

The third component of the proposed methodology involves development of programs for the controller design using advanced interactive optimization package DELIGHT [32,34]

The DELIGHT package is characterized by two main features that make it ideally suited as a supporting tool for the study we are proposing.

First, the package is designed as an extremely flexible tool, and a complete environment for interactive computer aided analysis and design. In particular, it has the following characteristics:

1. It is built around the RATTLE language that permits easy (on line) formulation of a program; the flow of computation can be affected at run time, in addition to the more usual preprogrammed sequence; configuration of the program is flexible and can be customized to many different applications. Just as an example, from the library of programs for matrix operations, only those needed for the particular application may be invoked and processed by the program; the others can still be used when and if needed, but until such time they reside somewhere on a disk, instead of the fast memory. This powerful property of the language was realized by combining most relevant features of several computer languages, including, C, MODULA, RATFOR, FORTRAN, PASCAL, and others.
2. The program is open ended, ie. by utilizing defines, and macros, often used body of code can be invoked by a single name.
3. Interfaces with fortran programs.
4. Allocation of memory to variables and programs is dynamic and in large part transparent to the user; this allows for great productivity in writing a program.

The second feature is a result of a particular application that motivated its development, which is optimization based computer aided design. Consequently, the package already contains an extensive library of routines (that exploit earlier mentioned extensibility property) to aid in solving even very complex (read realistic) optimization based problems. Typical of this capability is the following example [32]. Suppose we want to design a regulator for a given system, so that in addition to the system being stable the step response falls within certain region, like in Figure 2,

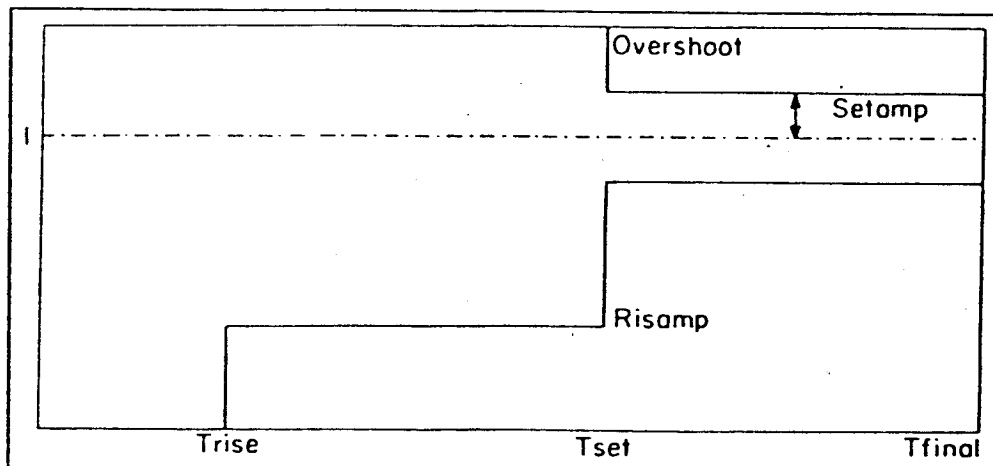


Fig 2. Engineering specifications on step response.

and the sensitivity to disturbances has certain properties as defined by a given band of the singular values of the return difference matrix across the frequency spectrum [32]. (Note the natural definition of the control problem, consistent with engineering intuition and

experience - not readily available for state space approaches, like LQG).

The solution of the problem by using DELIGHT requires that all the constraints be qualified as soft and hard. The soft constraints are modeled as a composite cost, while the hard constraints appear as simple inequalities

$$(3.21) \quad g_j(x) < 0, \text{ for } j = 1, 2, \dots$$

or as semi-infinite inequalities in the form

$$(3.22) \quad h^k(x, w_k) < 0, \quad k = 1, 2, \dots$$

where  $x$  is the design vector. The last thing required is a suitable initial design for the controller. For example, LQG design gives such a controller, that guarantees stability, but may (and most likely does) fail to satisfy the requirements posed through the step response quality, and the sensitivity. (Recently work has been done at the University of Maryland on coupling ORACLS, a NASA developed package for LQG design, and DELIGHT, and reportedly a successful solution of several optimization problems has been achieved through the sequence outlined above [38]).

The methodology described in this paper couples DELIGHT with the design methodology described above, based on frequency domain ideas. Namely the structure of the controller will be as defined earlier (i.e. equations (3.17) and (3.20)), and DELIGHT will be used to find admissible controls so as the closed loop transfer function satisfies various types of engineering constraints with respect to its shape, sensitivity etc (as described above). Questions related to approximations of transfer functions and their relation on engineering specs will also be addressed.

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