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# FEEDBACK COMPENSATION WITH IRRATIONAL TRANSFER FUNCTIONS: AN ALGEBRAIC APPROACH

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## Abstract

We present here extensions of earlier work [1] on dynamic feedback compensation of infinite dimensional systems. The extensions are in two directions: (a) Unstable (in the input-output sense) plans are allowed (b) New classes of irrational transfer functions are being considered (other than meromorphic of bounded type).

## Summary

The dynamic feedback compensation problem considered here can be described simply as follows (refer to figure 1): Given the plant transfer function $T_0$, construct input and output compensators $H_i$, $H_0$, so that the overall closed loop transfer function has desired properties.

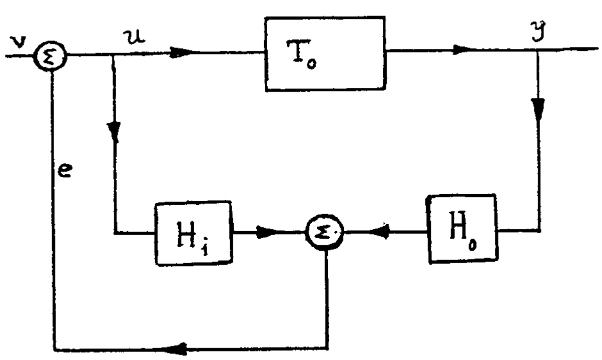


Figure 1. Dynamic Feedback Compensation

Formally if

$$T_0 = N_0 D_0^{-1} \tag{1}$$

is a matrix fraction description for $T_0$ and

$$H_i = K_i Q^{-1}$$
$$H_0 = K_0 Q^{-1} \tag{2}$$

the closed loop system is described by

$$(QD_0 - K_i D_0 - K_0 N_0)y = N_0 Q v \tag{3}$$

One wishes to choose $H_i$, $H_0$ so that (3) has desired specifications in the frequency domain. In (1)-(3) $N_0, D_0, K_i, K_0, Q$ are analytic (scalar or matrix functions in some left half-plane).

In [1] we solved the problem for the classes $H_m^\infty(D)$, $H_m^\infty(\Pi^+)$, $H^\infty_{m\ pxm}(D)$ and $H^\infty_{m\ pxm}(\Pi^+)$.

We also identified in [1] the necessary algebraic tools to solve the problem:

(a) A Banach algebra of $\mathcal{G}$ of analytic functions in some unbounded subset $\Omega$ of the complex plane, including always a right half-plane. A subalgebra of $\mathcal{G}$, $\mathcal{G}_s$ corresponding to stable systems. One considers also the set $\mathcal{G}_{pxm}$, pxm matrices with elements in $\mathcal{G}$. Similarly $\mathcal{G}_{s,pxm}$.

(b) Matrix fraction descriptions for the elements of $\mathcal{G}_{pxm}$, $\mathcal{G}_{s,pxm}$ with the factors analytic in the complement of $\Omega$. We denote by $\tilde{\mathcal{G}}$, the algebra of scalar factors.

(c) Notion of coprimeness in $\tilde{\mathcal{G}}$ which will imply a Bezout identity:

"if $N, D \varepsilon \tilde{\mathcal{G}}$ are coprime, there exist $X, Y \varepsilon \tilde{\mathcal{G}}$ such that

$XN + YD = 1$".

(d) Global parametrization of zeros of elements of $\tilde{\mathcal{G}}$.

(e) An algorithm to extract strictly proper parts from matrix fractions.

Abstractly, having properties (a) - (e) allows a direct solution to the problem. However in all interesting cases which are intrinsically induced



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by infinite dimensional systems the development of (a) - (e) in concrete examples may require rather advanced mathematical results. For example in [1] we used the Carlesan corona theorem.

Here we extend the result of [1] to $\mathcal{Q} = \Pi^{\infty}$, in the terminology of [1] (i.e. we allow unstable plants). Furthermore we extend the results of [1] to certain algebras $\mathcal{G}$ of analytic functions on strips or wedges in the complex plane.


References

[1] J.S. Baras, "Frequency Domain Design of Linear Distributed Systems", Proc. IEEE Decision and Control Conference, 1980, pp. 728 - 732.