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## ESTIMATION OF TRAFFIC FLOW PARAMETERS IN URBAN TRAFFIC NETWORKS\*

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### Abstract

The basic problem addressed in this paper is the development of better software for the computer control of urban traffic. Previous research indicates that the computer software, and especially the filtering and prediction algorithm, is the limiting factor in computerized traffic control. Recently developed techniques for the filtering, prediction and control of point processes, the type of statistics that occur in traffic flow, are used to develop filtering and prediction algorithms to facilitate the development of traffic responsive urban traffic control systems.

### I. Introduction:

At present, it is estimated that "... more than 200 cities around the world are either operating computer traffic control systems or are planning to install them" [1]. Of these, more than 100 are in the United States [2]. One of these, the Urban Traffic Control System in Washington, D. C., was developed under the auspices of the Office of Research of the Federal Highway Administration "for the purpose of advancing the state of the art of computerized urban traffic control" [2]. One of the issues that was specifically addressed by the UTCS was the amount of improvement that could be achieved by using a traffic responsive control system instead of a so-called Time of Day (open-loop) system. It was a source of some surprise and disappointment that only marginal improvements were found [2]. The purpose of this paper is to describe some research on improvements to the UTCS software that, the authors believe, can significantly improve the performance of traffic responsive systems.

A basic component of computer controlled, traffic responsive, urban traffic control systems is an algorithm for estimating and predicting the traffic flow based on the data from traffic sensors. There is evidence [2] that the filtering and prediction algorithms are the major cause of the failure of the traffic responsive systems to perform up

to expectations. In the early versions of UTCS, the traffic signal timing pattern could only be changed every 10-15 minutes and so the data was aggregated for 10-15 minutes. For this type of aggregated data, diffusion approximations with Gaussian statistics are probably valid and were used in the design of the UTCS filters and predictors [3]. However, it must be remembered that the system was operating open-loop for 10-15 minutes at a time. In later versions of UTCS, where signal timing patterns were updated every 3-6 minutes and where critical intersections were controlled in real time, these approximations were no longer valid and the filter/predictors were ineffective [2].

The authors of this paper believe that the difficulty is that the data from traffic sensors (normally loop detectors) are either:

- i) a sequence of times  $t_1, t_2, \dots (t_i < t_{i+1})$  representing the activation times of the detector or,
- ii) the data in (i) together with some auxiliary observations, such as the characteristics of each pulse (e.g. duration).

In any case, the data is a point process that, in the urban traffic case, is not Poisson.

Thus, the purpose of the research reported herein is to use point process techniques to develop improved filter/predictors for use in traffic responsive (nearly real time) computer control of urban traffic. Two such filter/predictors have been developed. The first is aimed primarily at critical intersection control and is based on a time-varying Markov chain model that represents a linearization and discretization of the nonlinear traffic dynamics. The second is based on a platoon motion model that utilizes an exponential distribution for headways between platoon leaders and a log-normal distribution for headways within a platoon. These are described in detail in the next two sections of the paper. The paper concludes with a description of current research on this problem.

### II. Markov Chain Model:

#### II. 1. An example of model description

The Markov Chain model described below can, with minor modifications, be applied to a wide variety of practical traffic situations. In the interest of clarity of exposition, the model is derived for only one of them. The extension of this deriva-

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tion to the other cases is discussed later.

The simplest practical traffic flow estimation problem occurs in the case of the single, isolated, intersection of two one-way, single lane streets. In order to adjust the traffic light to, in some sense, optimize (or even improve) the flow of traffic it is necessary to obtain fairly good estimates of the traffic queues upstream from the intersection. In practical systems, the estimate needs to be based on a minimal amount of historical data and on the signals from one, or more, detectors positioned as shown in Fig. 1.

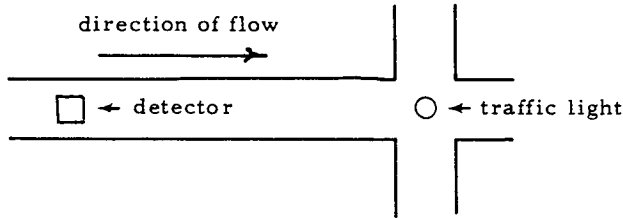


Figure 1. Detector Location

Assume, for simplicity, that the light operates on a simple, known, red-green cycle (no amber), that there is only one detector and that the detector is located  $n$  car lengths from the stop line.

The observed signal from the detector will be denoted by  $y(t)$ ,

$$y(t) = \begin{cases} 0 & \text{if no vehicle is over the detector} \\ 1 & \text{if a vehicle is over the detector} \end{cases}$$

In practice, time is discretized with a small enough discretization interval ( $1/32$  second in UTCs) for each vehicle to be over the detector for several samples. For simplicity, it is assumed here that the data are sampled so that each vehicle produces exactly one pulse (one 1).

Let  $z(t)$  denote the number of vehicles in the queue at time  $t$ . It is well known that the velocity with which vehicles cross the detector, and hence the rate at which 1's appear in  $y(t)$ , is related to  $z(t)$  [4]. One way to model this dependence is to let

$$\lambda(k, t) = \text{rate at which vehicles arrive at the detector given that the queue length is } k$$

Equivalently,

$$\lambda(k, t) = \Pr\{y(t)=1 \mid z(t)=k, t\}$$

$$\lambda(n, t) = \Pr\{y(t)=1 \mid z(t)=n, t\} = 0$$

Also

$$\lambda(k, t) = \begin{cases} \lambda_{kr} & \text{when upstream traffic light is red} \\ \lambda_{kg} & \text{when upstream traffic light is green.} \end{cases}$$

Similarly, let

$$\mu(k, t) = \text{rate at which vehicles depart from the queue given that the queue length is } k.$$

So,

$$\mu(k, t) = \Pr\{1 \text{ departure} \mid z(t)=k, t\}$$

$$\mu(0, t) = 0 \quad \text{for all } t$$

and

$$\Pr\{\text{more than 1 departure}\} = 0$$

$$\mu(k, t) = \begin{cases} \mu_{kr} & \text{when downstream traffic light is red} \\ \mu_{kg} & \text{when downstream traffic light is green.} \end{cases}$$

Furthermore, assume that arrivals and departures, conditioned on  $z(t)$ , are independent.

Examination of real traffic data shows that the assumption of conditionally inhomogeneous Poisson arrivals and departures is not strictly correct. It is also obvious that the coarse time discretization is throwing away useful information about the velocity with which vehicles cross the detector. There are three very good reasons for making these assumptions despite the inaccuracies they introduce. First, it will be seen that the filter/predictor based on these assumptions tends to ignore the extra randomness inherent in the conditionally Poisson assumption. Second, examination of real traffic data shows that the time dependence of vehicle arrivals caused by upstream traffic signals is a dominant effect and this is accurately modelled. Finally, the filter/predictor based on these assumptions is extremely easy to implement in a micro-processor, and can easily be made adaptive.

It should also be noted that both the assumption of a single lane street and the assumption that the departure rate ( $\mu$ ) is independent of the downstream queue are inessential. They have been made to simplify the development and can be removed.

In any case, one now has an inhomogeneous queueing problem with queue dependent arrivals and departures. Define

$$\pi_k(t) = \Pr\{z(t) = k\}$$

and

$$\underline{\pi}(t) = [\pi_0(t) \ \pi_1(t) \ \dots \ \pi_n(t)]^T$$

Obviously,

$$\begin{aligned} \pi_k(t+1) = & \Pr\{1 \text{ arrival, } 0 \text{ departures} \mid z(t)=k-1\} \\ & \Pr\{z(t)=k-1\} \\ & + \Pr\{0 \text{ arrivals, } 1 \text{ departure} \mid z(t)=k+1\} \\ & \Pr\{z(t)=k+1\} \\ & + \{\Pr\{0 \text{ arrivals, } 0 \text{ departure} \mid z(t)=k\} \\ & + \Pr\{1 \text{ arrival, } 1 \text{ departure} \mid z(t)=k\}\} \\ & \Pr\{z(t)=k\} \end{aligned}$$

or,

$$\begin{aligned} \pi_k(t+1) = & \lambda(k-1)(1-\mu(k-1))\pi_{k-1}(t) + (1-\lambda(k+1))\mu(k+1)\pi_{k+1}(t) \\ & + [(1-\lambda(k))(1-\mu(k)) + \lambda(k)\mu(k)]\pi_k(t) \end{aligned}$$

Thus,

$$\left. \begin{aligned} \underline{\pi}(t+1) &= \underline{Q}^T(t) \underline{\pi}(t) \\ \text{and } \Pr[y(t) = 1] &= \underline{\lambda}^T(t) \underline{\pi}(t) \end{aligned} \right\} \quad (M)$$

where

$$\begin{aligned} Q_{i,i}^T(t) &= (1-\lambda(i))(1-\mu(i)) + \lambda(i)\mu(i), \\ Q_{i,i-1}^T(t) &= \lambda(i-1)(1-\mu(i-1)), \\ Q_{i,i+1}^T(t) &= \mu(i+1)(1-\lambda(i+1)), \quad i=0,1,\dots,n \\ Q_{i,j} &= 0 \quad \text{elsewhere.} \end{aligned}$$

Where the argument,  $t$ , has been suppressed in both  $\lambda$  and  $\mu$ .

$$\underline{\lambda}^T(t) = [\lambda(0,t) \lambda(1,t) \dots \lambda(n-1,t) 0]$$

The problem is now formulated in such a way that the filter/predictor derived by Segall [5] or, equivalently, by Smallwood and Sondik [6] can be applied directly. First, it will be shown, following Segall, that this is a problem with linear dynamics so that the Kalman filter as well as the minimum variance filter, can be given.

Define

$$x_k(t) = \begin{cases} 0 & \text{if } z(t) \neq k \\ 1 & \text{if } z(t) = k \end{cases}$$

$$\underline{x}^T(t) = [x_0(t) \ x_1(t) \ \dots \ x_n(t)]$$

Segall then shows that

$$\begin{aligned} \underline{x}(t+1) &= \underline{Q}^T(t) \underline{x}(t) + \underline{u}(t) \\ y(t) &= \underline{\lambda}^T(t) \underline{x}(t) + w(t) \end{aligned}$$

where  $u(t)$  and  $w(t)$  are "noise" processes which only take on values 0 or 1. It is an important technicality that both  $u(t)$  and  $w(t)$  are martingale difference sequences with respect to the  $\sigma$ -algebra generated by the sequences  $\{y(0), y(1), \dots, y(t-1)\}$  and  $\{\underline{x}(0), \underline{x}(1), \dots, \underline{x}(t)\}$ . One consequence of this technicality is that  $u(t)$  and  $w(t)$  are "white" noise.

## II. 2. Optimal filter and predictor

Define:  $\hat{\underline{x}}(t+1|t)$  = least squares estimate of  $\underline{x}(t+1)$  given the observation sequence  $\{y(0), y(1), \dots, y(t)\}$

Then, the one-step predictor, from Segall, is

$$\hat{\underline{x}}(t+1|t) = \underline{Q}^T(t) \hat{\underline{x}}(t|t-1) + \frac{[\underline{S}^T(t) \hat{\underline{x}}(t|t-1) - \underline{Q}^T(t) \underline{\Sigma}(t) \underline{\lambda}(t)]}{[\underline{\lambda}^T(t) \hat{\underline{x}}(t|t-1) - (\underline{\lambda}^T(t) \underline{\Sigma}(t) \underline{\lambda}(t))^2]} (y(t) - \underline{\lambda}^T(t) \hat{\underline{x}}(t|t-1)) \quad (2.1)$$

$$\hat{\underline{x}}(1|0) = \underline{\pi}(0) \quad (2.2)$$

$$\underline{\Sigma}(t) = \hat{\underline{x}}(t|t-1) \hat{\underline{x}}^T(t|t-1) \quad (2.3)$$

and  $\underline{S}(t)$  is defined by

$$S_{ij}(t) = \Pr[x_j(t+1)=1, y(t)=1 | x_i(t)=1]$$

It is easily shown that

$$\left. \begin{aligned} S_{ii}(t) &= \lambda(i,t)\mu(i,t) \\ S_{i,i+1}(t) &= \lambda(i,t)(1-\mu(i,t)) \\ S_{ij}(t) &= 0 \quad j \neq i, j \neq i+1 \end{aligned} \right\} \quad (2.4)$$

And, the filter [7] is

$$\hat{\underline{x}}(t|t) = \hat{\underline{x}}(t|t-1) + \frac{[\text{diag}(\hat{\underline{x}}(t|t-1) - \underline{\Sigma}(t)) \underline{\lambda}(t)]}{[\underline{\lambda}^T(t) \hat{\underline{x}}(t|t-1) - (\underline{\lambda}^T(t) \underline{\Sigma}(t) \underline{\lambda}(t))^2]} (y(t) - \underline{\lambda}^T(t) \hat{\underline{x}}(t|t-1)) \quad (2.5)$$

where  $\text{diag}(\hat{\underline{x}}(t|t-1)) = \text{diag}(\hat{x}_0(t|t-1), \dots, \hat{x}_n(t|t-1))$ .

## II. 3. Realization of optimal filter and predictor

A straightforward calculation shows that Eq. (2.5) reduces to:

$$\hat{x}_i(t|t) = \begin{cases} \frac{(1-\lambda(i,t)) \hat{x}_i(t|t-1)}{\sum_{i=0}^n (1-\lambda(i,t)) \hat{x}_i(t|t-1)} & \text{if } y(t) = 0 \\ \frac{\lambda(i,t) \hat{x}_i(t|t-1)}{\sum_{i=0}^n \lambda(i,t) \hat{x}_i(t|t-1)} & \text{if } y(t) = 1 \end{cases} \quad (2.6)$$

where use has been made of the easily demonstrated fact that

$$\sum_{i=0}^n \hat{x}_i(t|t-1) = 1$$

Pre-multiplying Eq. (2.5) by  $\underline{Q}^T(t)$  and comparing with Eq. (2.1) shows that

$$\hat{\underline{x}}(t+1|t) = \underline{Q}^T(t) \hat{\underline{x}}(t|t) + \frac{[\underline{S}^T(t) \hat{\underline{x}}(t|t-1) - \underline{Q}^T(t) [\text{diag}(\hat{\underline{x}}(t|t-1)) \underline{\lambda}(t)]]}{[\underline{\lambda}^T(t) \hat{\underline{x}}(t|t-1) - (\underline{\lambda}^T(t) \underline{\Sigma}(t) \underline{\lambda}(t))^2]} (y(t) - \underline{\lambda}^T(t) \hat{\underline{x}}(t|t-1)) \quad (2.7)$$

Next, by factoring  $\underline{S}^T(t)$ , it is possible to rewrite  $\underline{S}^T(t) \hat{\underline{x}}(t|t-1)$  as

$$\underline{S}^T(t)\hat{\underline{x}}(t|t-1) = \begin{bmatrix} 0 & 0 & & & & & & & & 0 \\ 1 & \mu(1) & 0 & 0 & & & & & & 0 \\ 0 & (1-\mu(1)) & \mu(2) & 0 & & & & & & 0 \\ 0 & 0 & (1-\mu(2)) & \mu(3) & & & & & & \vdots \\ & & & 0 & \ddots & \ddots & & & & \vdots \\ & & & & \ddots & \ddots & \ddots & & & \vdots \\ 0 & 0 & \dots & \dots & \dots & \dots & \mu(n-1) & 0 & & 0 \\ 0 & 0 & \dots & \dots & \dots & \dots & \dots & 1-\mu(n-1) & 0 & 0 \end{bmatrix}$$

$$(\text{diag } \hat{\underline{x}}(t|t-1))\underline{\lambda}(t) \quad (2.8)$$

$$= \underline{M}^T(t)(\text{diag } \hat{\underline{x}}(t|t-1))\underline{\lambda}(t) \quad (2.9)$$

Substituting Eq. (2.9) into Eq. (2.7) and doing a calculation similar to that which leads to Eq. (2.6) gives

$$\underline{x}(t+1|t) = \begin{cases} \underline{M}^T(t)\hat{\underline{x}}(t|t) & \text{if } y(t) = 1 \\ \underline{M}^T(t)\hat{\underline{x}}(t|t) + \frac{[\underline{Q}^T(t) - \underline{M}^T(t)]\hat{\underline{x}}(t|t-1)}{\sum_{i=0}^n (1-\lambda(i,t))\hat{x}_i(t|t-1)} & \text{if } y(t) = 0 \end{cases} \quad (2.10)$$

It should be apparent that Eqs. (2.6) and (2.10), coupled with Eq. (2.2) for initialization, represent an algorithm for the minimum variance filter/predictor which can be easily realized in a micro-processor, see figure 2. This is especially so since  $\lambda(t)$ ,  $\underline{Q}^T(t)$  and  $\underline{M}^T(t)$  are all piecewise constant and periodic. Even more importantly, since the filter/predictor depends in a simple way on  $\underline{\lambda}$  and  $\underline{\mu}$ , it is easy to make the filter/predictor adaptive. It will be shown in Section IV that an adaptive filter/predictor is most desirable in this application.

#### II. 4. Optimal linear predictor

The optimal linear one step predictor (Kalman) is given by

$$\hat{\underline{x}}_{\ell}(t+1|t) = \underline{Q}^T(t)\hat{\underline{x}}_{\ell}(t|t-1) + k(t)(y(t) - \underline{\lambda}^T(t)\hat{\underline{x}}_{\ell}(t|t-1)) \quad (2.11)$$

At first glance, this appears to be substantially simpler than the optimal nonlinear filter. However, the gain,  $k(t)$ , presents a serious problem. If  $k(t)$  is pre-computed off-line one has the problem of storing a genuinely time-varying (not piecewise constant) quantity. Worse, if one attempts to make the filter/predictor adaptive pre-computing  $k(t)$  is impossible. The on-line calculation of  $k(t)$  is a very tedious business at best.

#### II. 5. Simulation results

The simulations that have been performed thus far have had two purposes. First, to provide a qualitative comparison between the minimum variance filter and the linear minimum variance (Kalman) filter. Second, to provide a qualitative verification for the time-varying model for the traffic queues.

Comparison of the two filters is complicated by the statistical nature of the result. Thus, the best method is to compute the two conditional

error covariance matrices directly and compare them. A less attractive alternative is to estimate the conditional error covariance matrices from a Monte-Carlo simulation. The better method was used here. However, to facilitate the calculation, two simplifications were made:

- 1)  $\underline{Q}^T(t) = \underline{Q}^T$  a constant matrix
- 2)  $S_{ij}(t) = q_{ij}(t)\lambda(i, t) = q_{ij}\lambda(i)$

The second simplification is equivalent to assuming  $y(t)$  and  $\underline{x}(t+1)$  are independent given  $\underline{x}(t)$ . This is not true in the traffic case. However, the purpose here is only to show that there exist situations where the nonlinear filter unequivocally outperforms the linear one.

Once the above simplifications were made, equations for the conditional error covariance of both filters were found. These equations were solved on the computer for several choices of  $\underline{Q}$  and  $\underline{\lambda}$  with data,  $y(t)$ , generated by Eqs. (M). The results of these calculations are shown in Figs. 3 and 4 in the abbreviated form of the trace of the error covariance. The values of  $\underline{Q}$ ,  $\underline{\lambda}$  and  $\underline{\pi}(0)$  used to obtain these Figs. are:

$$\underline{Q} = \begin{bmatrix} .998 & .001 & .001 \\ .002 & .996 & .002 \\ .003 & .001 & .996 \end{bmatrix}; \quad \underline{\lambda} = \begin{bmatrix} .2 \\ .5 \\ .8 \end{bmatrix}; \quad \underline{\pi}(0) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

It should be noted that there is a substantial performance difference in favor of the nonlinear filter. It should be noted that, for other choices of the parameters, the difference in performance of the two filters is not as large. Of course, the nonlinear filter always outperforms the linear one.

In order to obtain some verification of the time-varying model for the queue formation it is necessary to compare the  $y(t)$  generated by the model with the  $y(t)$  measured on an urban street. This has been done in the following rather crude manner. Examination of detector data obtained from FHWA Fairbank Research Center shows that the detector signal is roughly periodic with period equal to the cycle time of the traffic light. This variation has been approximately duplicated, using the time-varying model proposed above, in A. Keogh's thesis [7]. Obviously, this does not verify the model. It does suggest that the model is reasonable.

Ultimately, the significant question is: Does the filter/predictor based on this model perform adequately? Plans for research to answer this question are described in Section IV.

### III. Queue - Platoon Model

#### III. 1. Headway distributions

It has long been recognized that one of the most important parameters in the description of traffic flow is the distribution of headways [8] [9]. The subject has been studied extensively since the early days of traffic control and its importance is primarily due to two reasons: first, it is rel-

atively easy to collect headway data and several data bases exists, and second a successful description of the headway distribution is essential for the modeling of traffic patterns and thus for traffic signal setting. The statistical description of headways (interarrival times in the point process jargon) is the essential part in modelling the underlying point process and the point of departure of the modern theory [10] [11].

Basically two types of distribution models have been proposed and tested for the description of headway distribution: simple models and mixed models. Of the simple models proposed, the most successful fit to actual data, as various studies indicate [9] [12] [13] was the lognormal distribution, shifted to provide a fixed minimum headway. The lognormal density function is given by

$$p_{\ell}(h) = \frac{1}{\sigma h \sqrt{2\pi}} \exp\left(-\frac{(\ln h - \mu)^2}{2\sigma^2}\right), h > 0 \quad (3.1)$$

while the shifted lognormal density is

$$p_{\ell s}(h) = \frac{1}{\sigma(h-a)\sqrt{2\pi}} \exp\left(-\frac{(\ln(h-a) - \mu)^2}{2\sigma^2}\right), h > a \geq 0. \quad (3.2)$$

There are various justifications of this fact. The primary reason given is that multiplicative, independent, identically distributed errors by various drivers attempting to follow each other combine to give a lognormal distribution. It should be noted that the simple lognormal distribution provides a very good fit for data from following (or queueing) traffic. The mixed distribution models are basically of the form

$$F_h(\xi) = \psi F_{hf}(\xi) + (1-\xi)F_{hnf}, \quad (3.3)$$

where  $F_{hf}$  is the headway distribution of following traffic and  $F_{hnf}$  is the headway distribution for free flowing traffic. The assumption of two subpopulations is clearly more realistic and several studies have indicated a superior fit to real data by distributions of mixed type [8]. The most successful model has been recently described by Branston [9] and utilizes a lognormal density for following headways. Then, utilizing the assumption of "random bunches" originally due to Miller [14] (i. e., that the gaps between platoon leaders follow an exponential distribution) Branston derives the density for nonfollowing headways. The resulting model has the form

$$p(h) = \psi g(h) + (1-\psi)\lambda \exp[-\lambda h] \int_0^h g(x) \exp(\lambda x) dx \quad (3.4)$$

Here  $\psi$  is the percentage of following cars in the overall traffic flow and  $1/\lambda$  is the mean interbunch (or interplatoon) gap, two parameters that can be rather easily estimated as will be discussed later. This model provided excellent fit to data from various traffic flow situations and is the one adopted here.

There are several reasons for choosing this model: a) the parameters introduced by the model

are natural and are important parameters for filtering/prediction and (or) control, b) the model can accommodate all traffic conditions (light, moderate, heavy) and is valid for practically all ranges of traffic flow and speed (a property that has been verified from real data and which is not true for simple models), c) the distributions involved imply underlying stochastic processes that can be completely described by a finite number of moments (at most two), an important fact for the development of simple but effective filter/predictors.

In addition to the parameters  $\psi, \lambda$ , the model requires two parameters  $\mu, \sigma$  for the lognormal density of the following headways, where  $\mu$  is the mean and  $\sigma$  the standard deviation of the natural logarithm of headway (a Gaussian random variable). To completely specify the model for a particular link or section of a link in a traffic network, it is important to understand the variation of the parameters with respect to traffic flow and speed. It was observed from real data [9] that  $\mu$  and  $\sigma$  both decrease as traffic flow increases, although often the variation is small enough to allow a very good fit with constant  $\mu$  and  $\sigma$ . The dependence on speed is more drastic. Both  $\mu$  and  $\sigma$  tend to increase with speed, but real data indicate that  $\mu$  can vary widely for the same speed for different traffic locations and or times, while  $\sigma$  did not show a similar wide variation. These established facts are encouraging, and actually imply that a periodic estimation of  $\mu$  and  $\sigma$  is likely to be an effective way of obtaining values of  $\mu$  and  $\sigma$  from on-line data. Moreover this adaptation of the parameters can be done at a much slower pace than the actual filter/predictor. This means that the resulting filter/predictor can be made adaptive.

Finally for the determination of  $\psi$  and  $\lambda$  and their relation to traffic flow rate the following models were found to be in good agreement with real data [9]. Let  $\rho$  denote the traffic intensity

$$\rho = \frac{\text{mean following headway}}{\text{mean headway}} = \frac{\exp(\mu + \frac{1}{2}\sigma^2)}{h} \quad (3.5)$$

and  $\lambda^*$  the flow rate. Then

$$\lambda = \lambda^* - \frac{1}{2}\lambda^{*3/2} \quad (3.6)$$

$$\psi = \rho - \frac{1}{2}(\rho - 1)\lambda^{*3/2}. \quad (3.7)$$

Although, the above formulas are the results of curve fitting real data from specific traffic locations, they can be used as a first approximation to the relations between these parameters, because experimental evidence indicates low sensitivity to traffic location. In conclusion the model proposed above provides an acceptable model for headway distribution with many desirable properties.

### III. 2. A queue-platoon model for urban traffic flow

The model developed in this section for urban traffic flows is based on the headway distribution model adopted in the previous section. Each link will be divided in sections in accordance with the

detectorization of the link. For each section of the link the input and output traffic flows will have headway distributions as described in III.1. Notice that the headway distribution model can vary (and it should) from lane to lane. The required parameters of the model will be estimated at appropriate intervals from actual data, or from historical data as required. The effect of the link will be to alter the parameter values as traffic moves down stream.

The versatility of the proposed model is now briefly indicated along with the ability to incorporate all desired situations. If the next downstream section provides greater congestion than the current section of the roadway, this will appear as an increase in  $\psi$  for the next section, followed by a decrease in  $\mu$  and  $\sigma$ . Often, this change in  $\mu$  and  $\sigma$  will not be necessary. In case the current section is in front of a traffic light which just turned red, then the incoming flow parameters will be adjusted to that  $\psi$  will increase and  $\mu$  and  $\sigma$  will decrease gradually (according to time required to form the queue). In addition, for properly located detectors, the number of cars in this particular section can serve as a measure of the queue in front of the red traffic light. Similarly when the light turns green the  $\psi$  will decrease and  $\mu$  and  $\sigma$  will increase accordingly to reflect the transition from stopped queue to the level of traffic flow. By appropriate variation of  $\psi$ , one can thus create platoons or disperse platoons and thus realistically emulate traffic flow.

To complete the model, a distribution for the mark of the underlying point process, that is pulse length, is also needed. As a first approximation however, this is omitted here, although it will be incorporated in the final point process model.

To summarize, the model requires the determination of 4 parameters for each section, namely  $\psi, \lambda, \mu, \sigma$ , which will depend on traffic flow, speed, time and location in general. This provides a "local description" of the underlying point process which is realistic and allows the use of the modern theory [11] to obtain filter/predictors.

#### IV. Current Research

The most important item among the current research on this project is aimed at making the filter/predictors adaptive. There are two reasons for this. First, it is generally very difficult and expensive to collect, store and process data on vehicular traffic flows. Thus, a filter/predictor whose parameters are determined via processing of off-line data is a very expensive filter/predictor. Second, superimposed on the variation in traffic flow with signal settings and the high frequency stochastic fluctuations, there is a significant slow (half hour or slower) variation. It is thus feasible to let a filter/predictor estimate its own parameters, based on the data it receives, over a period of 15-30 minutes while, at the same time, it estimates the current traffic flow parameters.

Another question of some interest is the development of more effective control (signal timing) algorithms. This is a problem that had been

regarded as solved provided good estimates of traffic flow were provided [2]. This conclusion was based on the results for UTCS with a 10-15 minute up-date period for changing signal settings. Once the control system is made more nearly real time (3-6 minute up-date or less) the fact that one is controlling a point process becomes significant and the signal timing algorithms need to be optimized for this.

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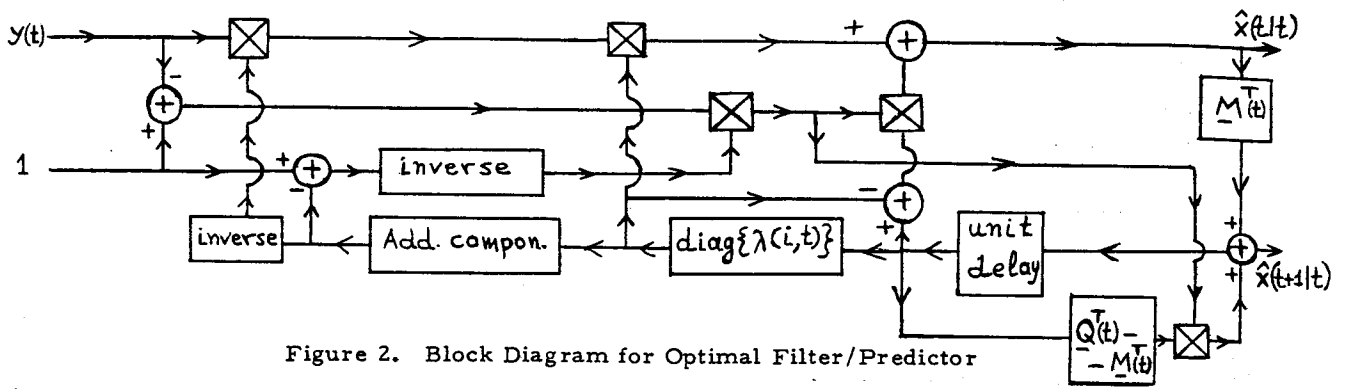


Figure 2. Block Diagram for Optimal Filter/Predictor

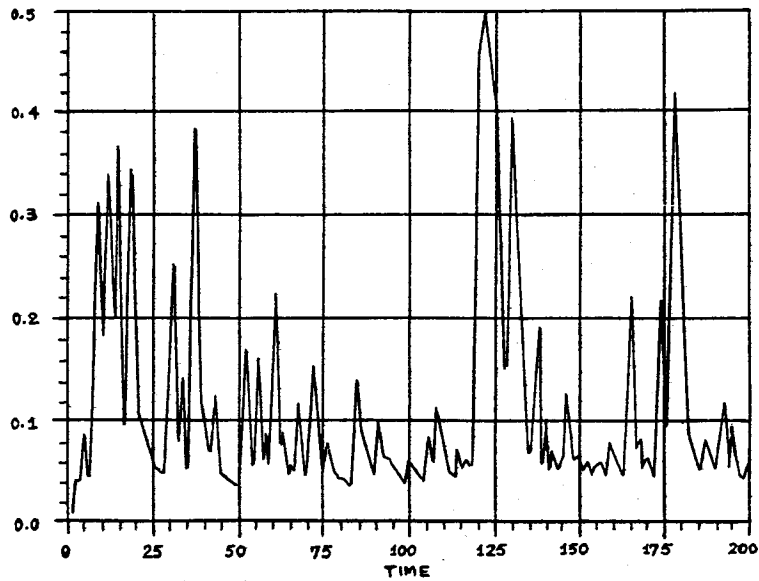


Figure 3. Trace of Conditional Error Covariance for Optimal Filter/Predictor

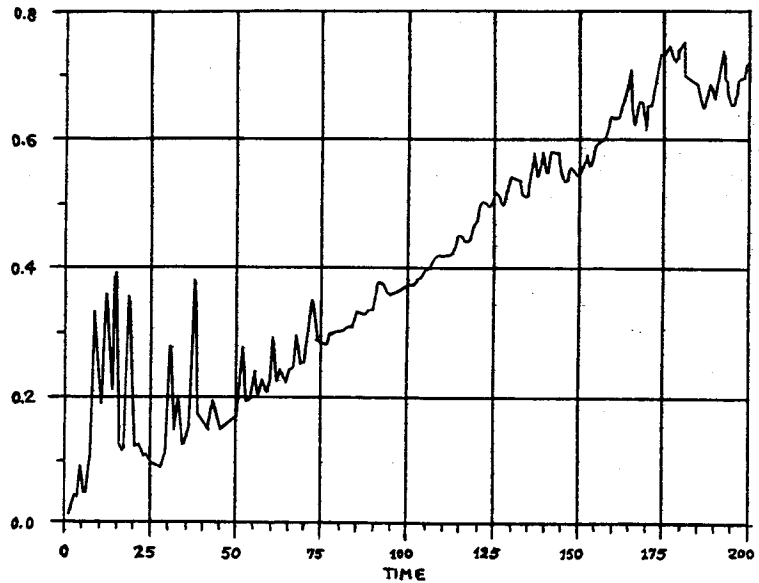


Figure 4. Trace of Conditional Error Covariance for Linear Optimal Filter/Predictor