Arch Structures

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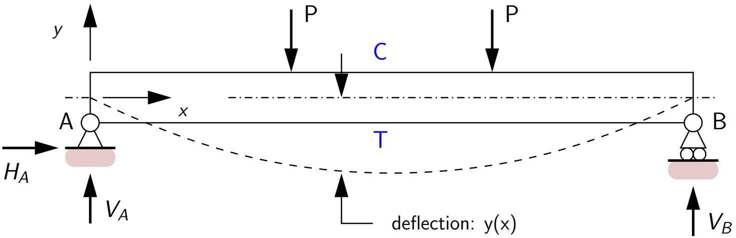
Overview

Motivation for Arch Structures

Motivation for Arch Structures

Reminder: Flexural behavior of simply supported beam ...

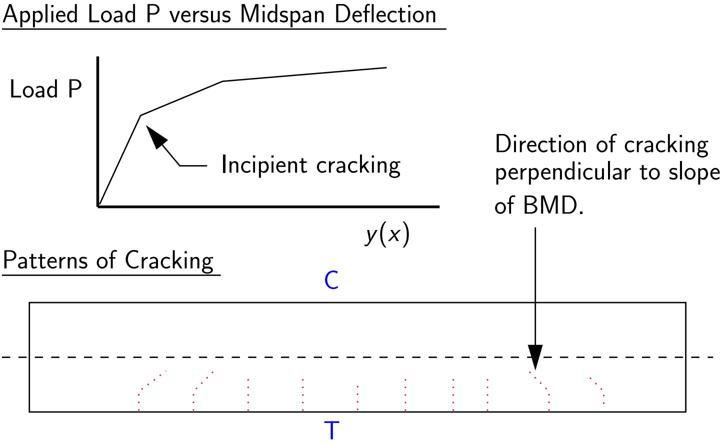
Experimental Setup



Bending Moment Diagram (BMD)

Motivation for Arch Structures

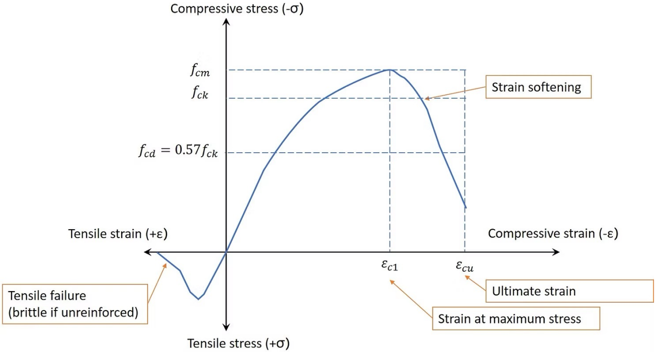
Reminder: Flexural behavior of simply supported beam ...



Motivation for Arch Structures

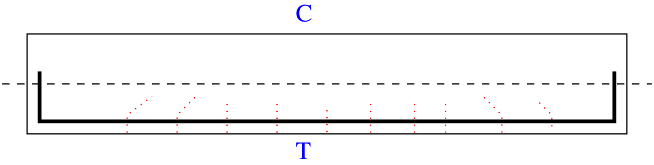
Stress-Strain Behavior of Concrete:

- Concrete materials have an ability to carry loads in compression but are very poor at carrying loads in tension.



Motivation for Arch Structures

Strategy 1: Reinforced Concrete Beam (post- 1850)

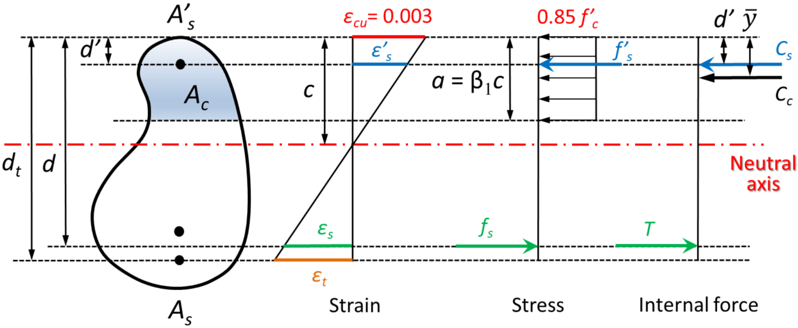


Steel bars strengthen concrete and resist tensile stress.

Compression and Tension Failure:

Motivation for Arch Structures

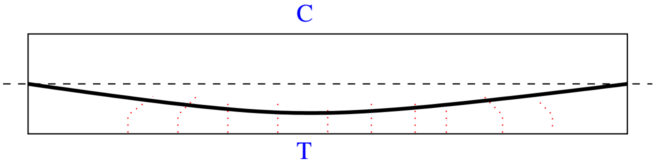
Preview to Analysis:



Maximum strain at the extreme concrete compressive fiber is assumed to be $\eta_c = 0.003$.

Motivation for Arch Structures

Strategy 2: Prestressed Beam (post-1960)

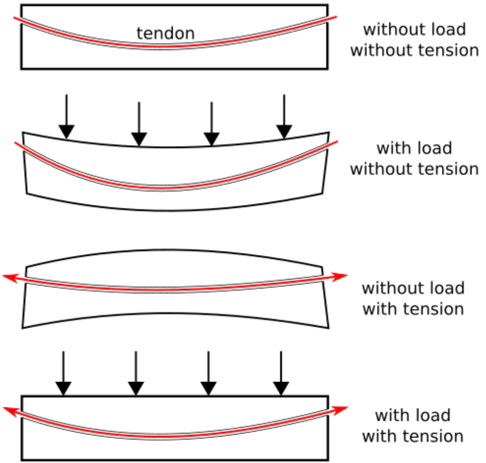


Design Procedure:

- High strength tendons are threaded through the concrete and then pulled tight.
- This puts the beam in compression (and reduces/eliminates the tensile stresses.
- Cable profiles can be designed to counter bending moments due to self weight.

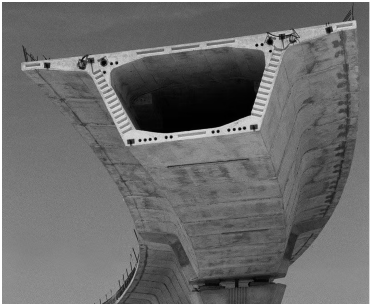
Motivation for Arch Structures

Post-Tensioned Concrete:



Motivation for Arch Structures

Application: Prestressed Concrete Box-Girder Bridge ...

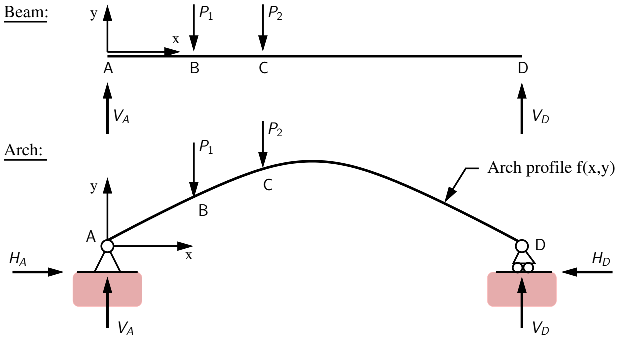


Benefits of Prestresssing:

- Increased load capacity; improved resistance to cracking.
- Longer beam spans are feasible.

Motivation for Arch Structures

Strategy 3: Remove constraint that beam element is straight !!!



Within the segment A-B:

- For the beam: $M(x) = V_A x$.
- For the arch: $M(x, y) = V_A x - H_A y$.

Motivation for Arch Structures

Main Takeaway:

- We can reduce bending moments by removing the constraint that beams are straight.
- But now we need to deal with the horizontal reactions H_A.

Implicit Assumption:

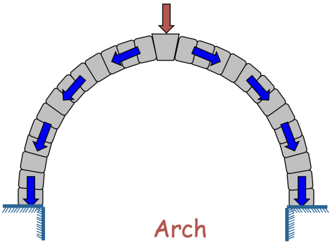
- Arch supports will not move ...

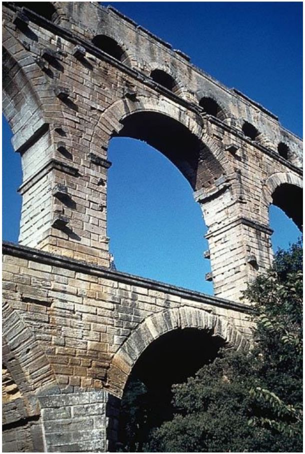
To see what happens when things go wrong:

- Google: bridge collapse taiwan.

Motivation for Arch Structures

Design and Analysis:



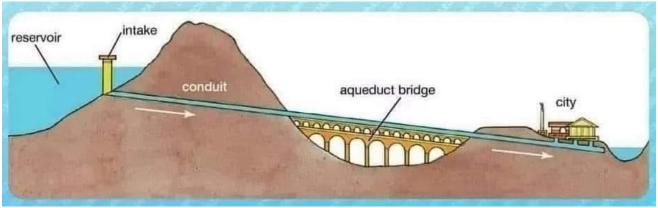


Design objective: Structure needs to work and be aesthetically pleasing!!
Analysis objective: What shape should the arch be so that forces can be transferred to the foundation through compression mechanisms alone?

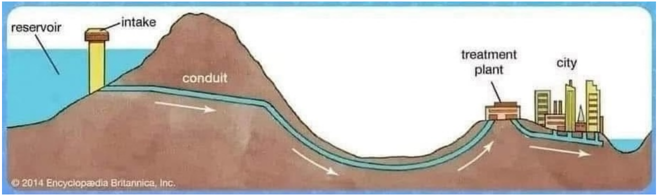
Brief History

History of Arch Structures

Basic Problem: Here's why the Romans built Aqueducts ...



Today we would use a pump to get the job done ...



History of Arch Structures

Aqueduct: Pont du Gard (circa first century) ...

Transported water approximately 50 km.

History of Arch Structures

Roman Aqueduct: of Segovia, Spain (circa 100-120 AD)

Transported water 10 miles during the first Century.

History of Arch Structures

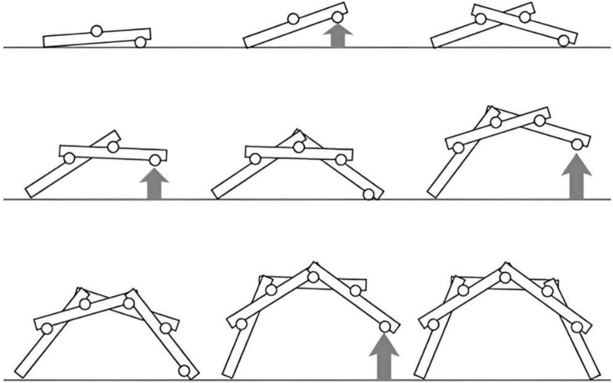
Example: Leonardo Da Vinci Bridge (1500s)

- Bridge is held together by its own weight without requiring any ties or connections.
- Bridge members interlock and tighten together through element shear and bending.

History of Arch Structures

Example: Leonardo Da Vinci Bridge (1500s)

Designed for Quick Assembly in Military Applications:



Case Studies

Key Bridge: Georgetown-Reston (1923)

Case Studies

Bixby Bridge: California Coastline (1932)

Case Studies

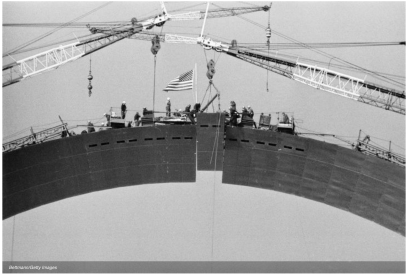
St Louis Gateway Arch:

Rough timeline of development:

- Initial planning occurred in the mid-1930s.
- Design competition held in the late 1940s and early 1950s.
- Construction began in 1963; completed 1965.

Case Studies

St Louis Gateway Arch: Construction ...



Case Studies

Principal Architect: Eero Saarinen (1910-1961)

Note: Saarinen also designed the terminal building at Dulles Airport.

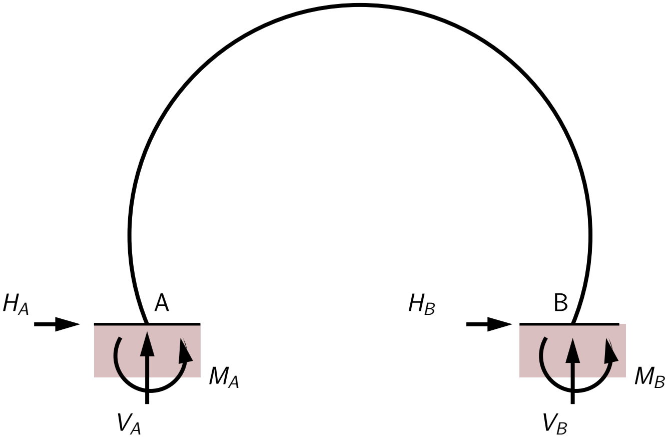
Case Studies

Frederick Douglass Memorial Bridge: Washington DC (2017-2022).

Types of Arch Structure

Fully-Fixed Arch

Analysis: Statically indeterminate: $\widehat{i} = 3$.

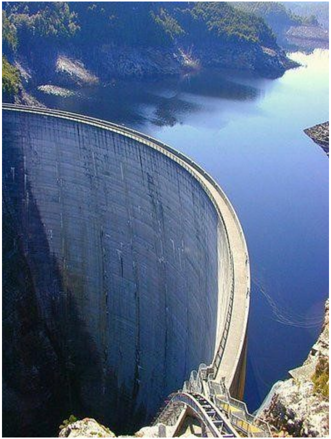


Fully-Fixed Arch

Arch Dam: Uses arching principle to resist water pressure.

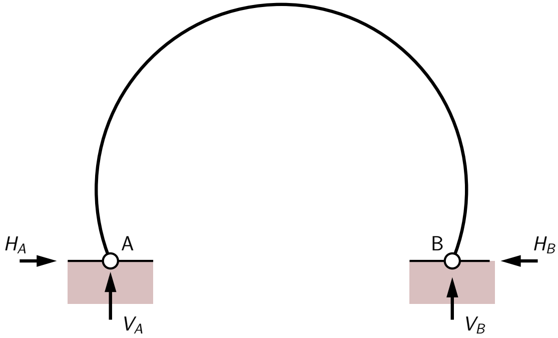
The main loads are:

- Dead load.
- Hydrostatic load generated by upstream reservoir.
- Temperature and earthquake loads.



Two-Hinged Arch

Analysis: Statically indeterminate: $\widehat{i} = 1$.



Support Hinge:

- Prevents transfer of moment into foundation.
- Free to rotate – this is important for thermal expansion.

Two-Pinned Arch

Sydney Harbour Bridge: Arch-based steel frame, construction 1923-1932.

Bridge System: Rail, vehicular, bicycle and pedestrian traffic.

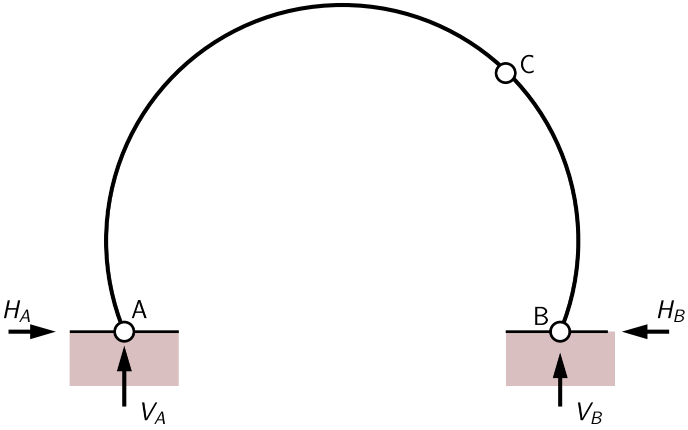
Two-Pinned Arch

Pin Supports in Sydney Harbour Bridge:

Key Advantages: Hinge provides built-in support for thermal expansion.

Three-Pinned Arch

Analysis: Statically determinate: $\widehat{i} = 0$.



Three-Pinned Arch

Basic Questions:

- What are the support reactions?
- What forces are transferred across the internal hinges?
- What do M(x), V(x) and N(x) look like?
- Does the relationship $V(x) = \frac{dM}{dx}$ still work?

Harder Questions:

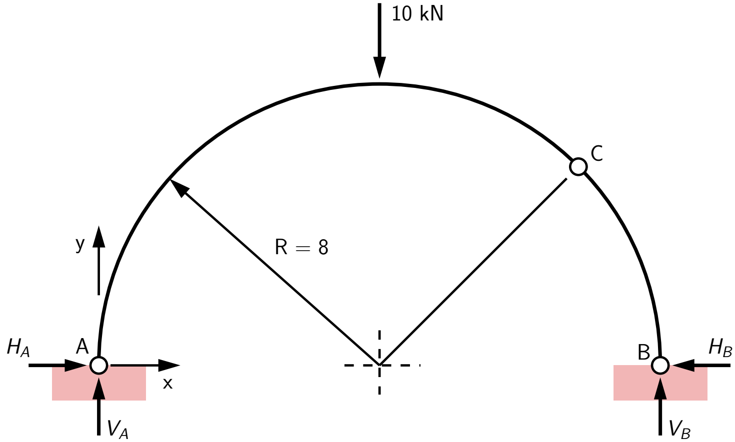
- Does the position of the internal hinge matter?
- What is the optimal shape for the arch?

Analysis

Part 1: Circular Arch

Analysis of Circular Arch

Problem Setup:



Analysis of Circular Arch

Equations of Equilibrium:

$$\sum F_y = 0, \qquad V_A + V_B - 10 = 0, \tag{1}$$

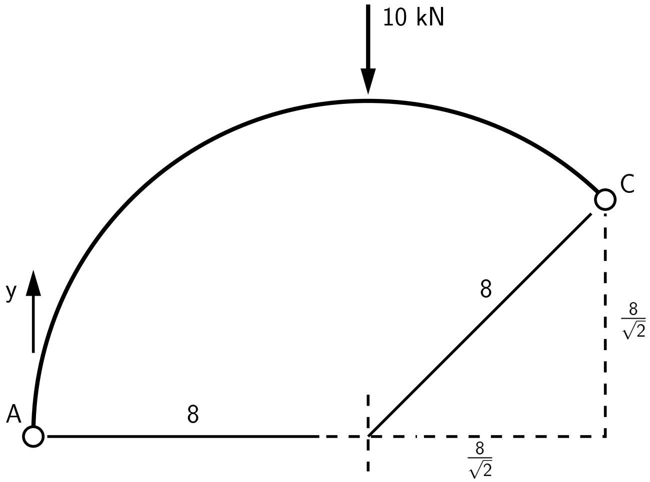
$$\sum M_A = 0, \qquad 10R - V_B(2R) = 0, \tag{2}$$

$$\sum F_x = 0, \qquad H_A = H_B \quad \text{(does not help)}. \tag{3}$$

From equations 1 and 2, $V_A = V_B = 5.0$ kN. We also know $M_C = 0$. For the left-hand substructure:

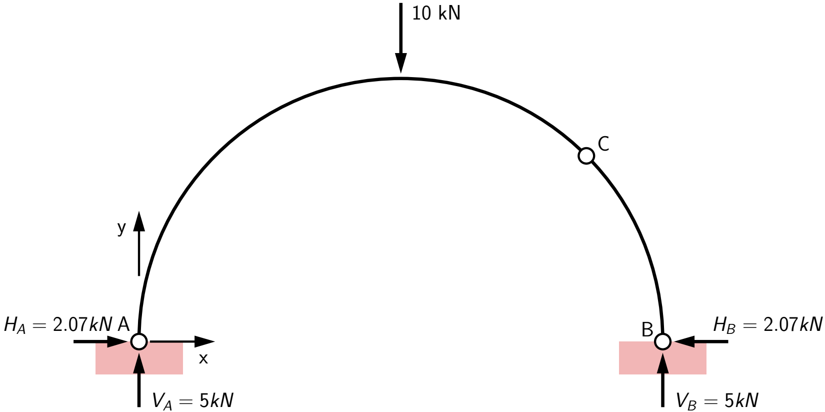
$$(10 + H_A)\frac{8}{\sqrt{2}} = V_A\left(8 + \frac{8}{\sqrt{2}}\right) \longrightarrow H_A = H_B = 2.07 \quad \text{kN}. \tag{4}$$

Analysis of Circular Arch



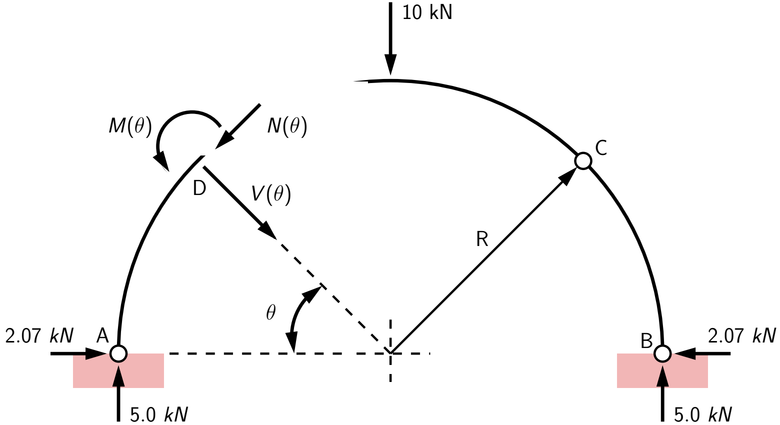
Analysis of Circular Arch

Summary of Support Reactions:



Analysis of Circular Arch

Bending Moments and Shear Forces: Polar coordinates (R,θ):



Analysis of Circular Arch

Bending Moments and Shear Forces: $\sum M_D = 0$,

$$M(\theta) = 5R\left[1 - \cos(\theta)\right] - 2.07R\sin(\theta). \tag{5}$$

$$V(\theta) = 5\sin(\theta) - 2.07\cos(\theta). \tag{6}$$

Axial Force:

$$N(\theta) = -\left[5\cos(\theta) + 2.07\sin(\theta)\right]. \tag{7}$$

Note: N($\theta = 0$) = -5 kN, so the system is in equilibrium.

Analysis of Circular Arch

Relationship between Shear Forces and Bending Moments:

Question: Does $V = \frac{dM}{dx}$ still work? Well, sort of ...

In polar coordinates, the distance x measured around the circumference is:

$$x = R\theta \tag{8}$$

Applying the chain rule:

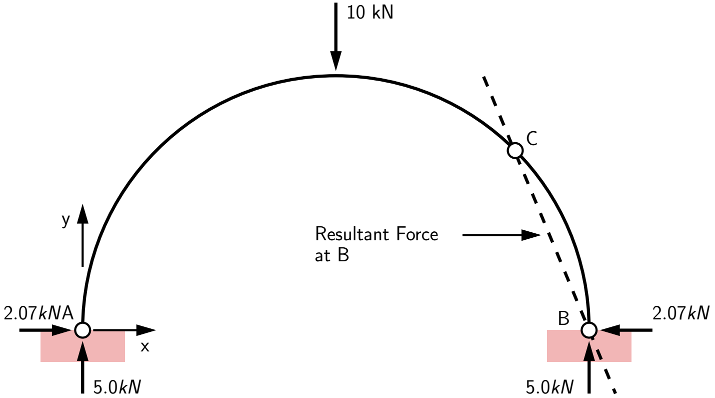
$$V(\theta) = \frac{dM}{dx} = \frac{dM}{dx}\frac{d\theta}{d\theta} = \frac{dM}{d\theta}\frac{d\theta}{dx} = \frac{dM}{d\theta}\frac{1}{R}. \tag{9}$$

Hence, we need to check:

$$V(\theta) = \frac{1}{R}\frac{dM}{d\theta}. \longrightarrow \text{It works!!} \tag{10}$$

Analysis of Circular Arch

Observation: There are no external loads acting on segment B-C of the arch. Hence, the resultant force at B must pass through the hinge at C.

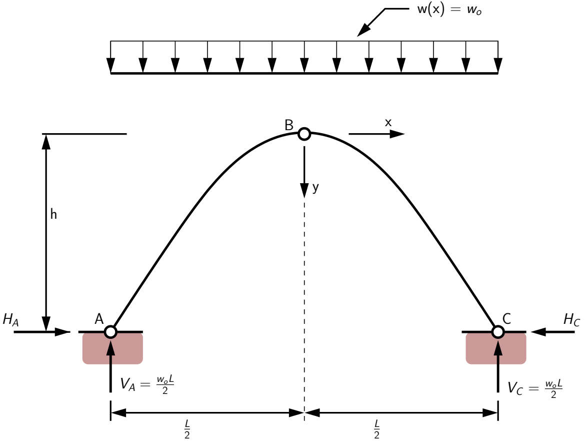


Analysis

Part 2: Parabolic Arch

Analysis of Parabolic Arch

Problem Setup: Let y(x) = k x^2.



Analysis of Parabolic Arch

Boundary Condition:

$$y = h \quad \text{at x} = \pm\frac{L}{2} \quad \longrightarrow \quad k = \frac{4h}{L^2}. \tag{11}$$

Also, note:

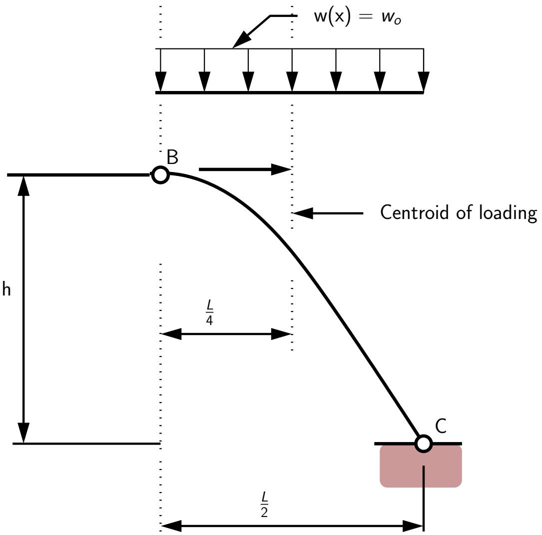
$$y(x) = \left[\frac{4h}{L^2}\right] x^2 \quad \longrightarrow \quad \frac{dy}{dx} = \left[\frac{8h}{L^2}\right] x. \tag{12}$$

Horizontal and Vertical Reactions:

$$\sum V = 0 \quad \longrightarrow \quad V_A = V_C = \frac{w_o L}{2}. \tag{13}$$

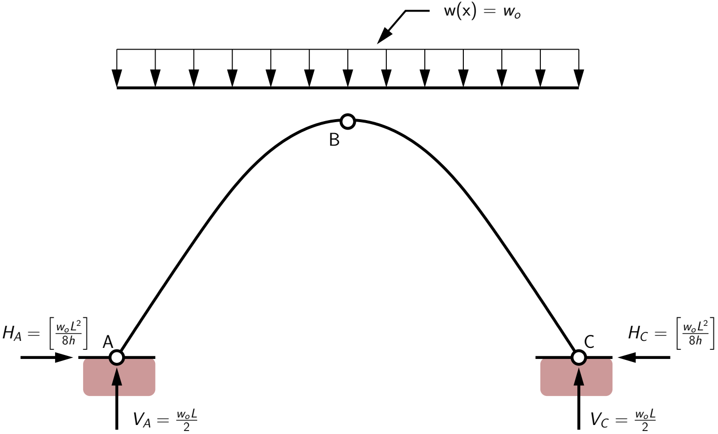
$$\sum M_B = 0 \quad \longrightarrow \quad H_C = \left[\frac{w_o L^2}{8h}\right]. \tag{14}$$

Analysis of Parabolic Arch



Analysis of Parabolic Arch

Summary of Reactions:



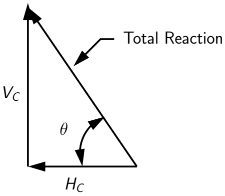
Analysis of Parabolic Arch

Reaction Magnitude:

$$R = \frac{w_o L}{2}\left[1 + \frac{L^2}{16h^2}\right]^{1/2}. \qquad (15)$$

Reaction Direction:

$$\tan(\theta) = \frac{V_C}{H_C} = \frac{4h}{L}. \qquad (16)$$



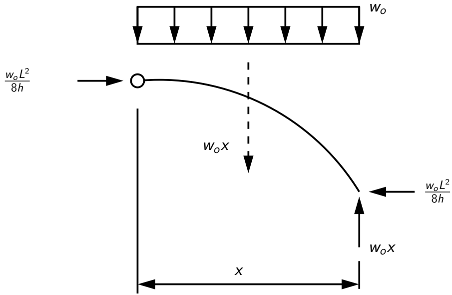
At $x = \frac{L}{2}$:

$$\frac{dy}{dx} = \left(\frac{8h}{L^2}\right)\frac{L}{2} = \left(\frac{4h}{L}\right). \qquad (17)$$

Equations 16 and 17 are identical $\longrightarrow$ pure compression (no bending, no shear) at the foundation support.

Analysis of Parabolic Arch

Analysis of an Arbitrary Section:



Bending Moment:

$$M(x) = w_o x \frac{x}{2} - \frac{w_o L^2}{8h} y = \frac{w_o x^2}{2} - \frac{w_o x^2}{2} = 0. \qquad (18)$$

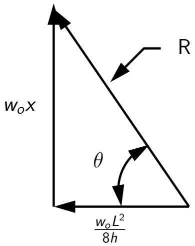
Analysis of Parabolic Arch

Analysis of an Arbitrary Section:

Resultant of the ground reaction forces:

$$R = w_o \left[x^2 + \left(\frac{L^2}{8h} \right)^2 \right]^{1/2} \qquad (19)$$

$$\tan(\theta) = \left(\frac{8xh}{L^2} \right) \qquad (20)$$



Slope of curve section:

$$\frac{dy}{dx} = 2kx = \left[\frac{8hx}{L^2} \right]. \qquad (21)$$

Equation 20 equals 21 $\longrightarrow$ pure compression throughout!

Analysis of Parabolic Arch

Practical Considerations: To eliminate bending moments, loading patterns need to be uniformly distributed.

To make this happen, put holes in the bridge ...

Humpback Bridge

Holes create uniform loading

Clever, huh!