ENCE 353 Solutions to Midterm 1

Question 1 (15 points): Support Reactions and Bending Moments in a Connected Beam Structure.

Consider the multi-span connected beam structure shown in Figure 1.

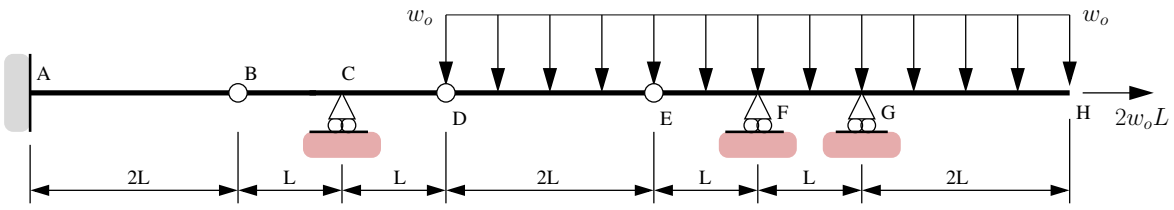


Figure 1. Front elevation view of multi-span connected beam structure.

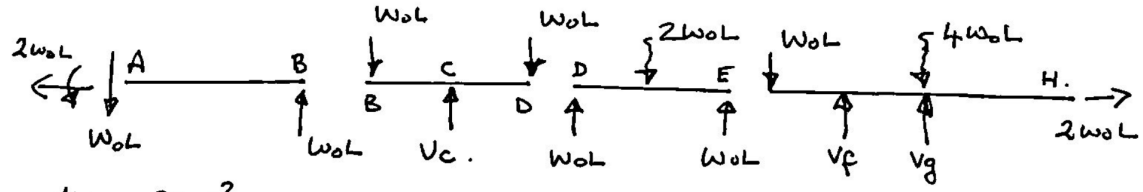
The cantilever is fully-fixed to the wall at Point A. Points B, D and E are hinges. The beam segment D-E-F-G-H carries a uniformly distributed load w_o (force/per unit length). Finally, a horizontal point load $2w_oL$ is applied at point H.

Part [1a]: (3 pts) Compute the degree of indeterminacy for the beam structure.

Soln: support reactions r = 12, n = 4 substructures, hence:

$$\hat{i} = r - 3n = 0 \quad \longrightarrow \quad \text{statically determinate.} \tag{1}$$

Part [1b]: (3 pts) Compute the vertical reaction forces at nodes C, F and G.



$M_A = -2w_oL^2$

$\sum M_A = 0$

$\Rightarrow M_A = -2w_oL^2$

$\sum M_B = 0$

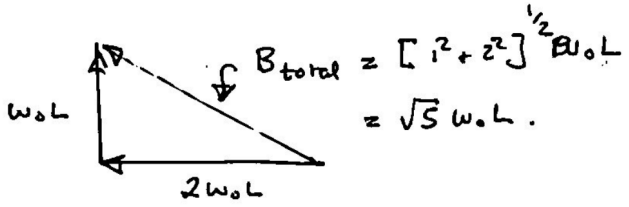
$\Rightarrow V_C = 2w_oL$.

$\sum M_f = 0 \Rightarrow V_g = 3w_oL$

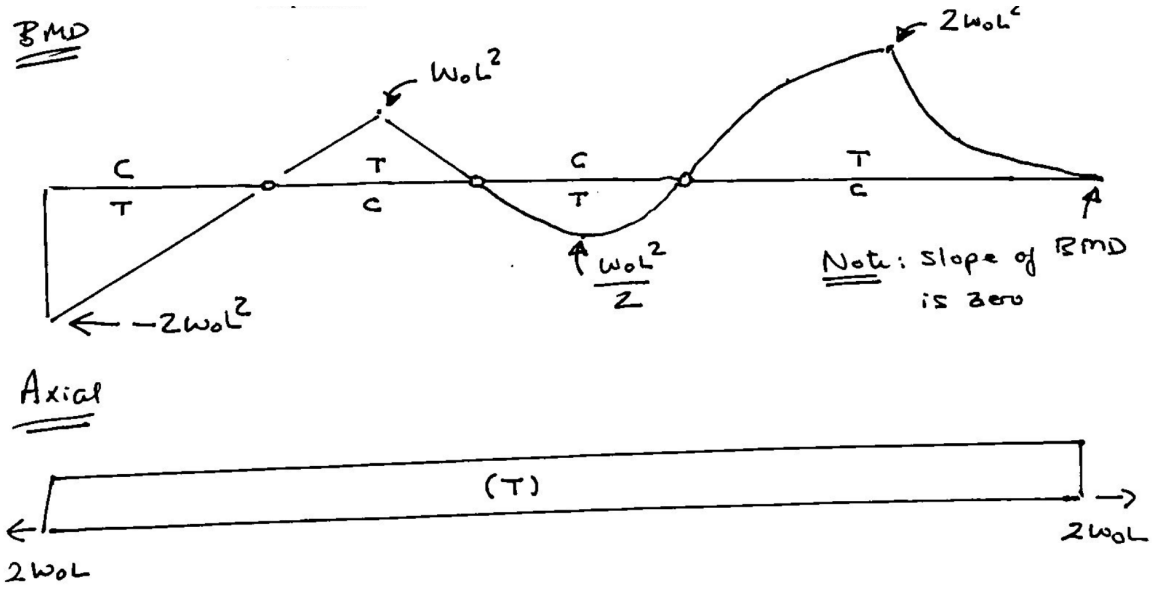
$\sum V = 0 \Rightarrow V_f + V_g = 5w_oL$

$\Rightarrow V_f = 2w_oL$.

Part [1c]: (3 pts) Compute the **total force** at hinge B.



Part [1d]: (6 pts) Draw and label diagrams showing how the **bending moment** and **axial force** vary along the beam, nodes A through H. Clearly indicate on your bending moment diagram, regions that are in tension/compression.



Question 2 (15 points): Tension, Compression and Zero-Force Members in a Truss Structure.

Consider the truss structure shown in Figure 2.

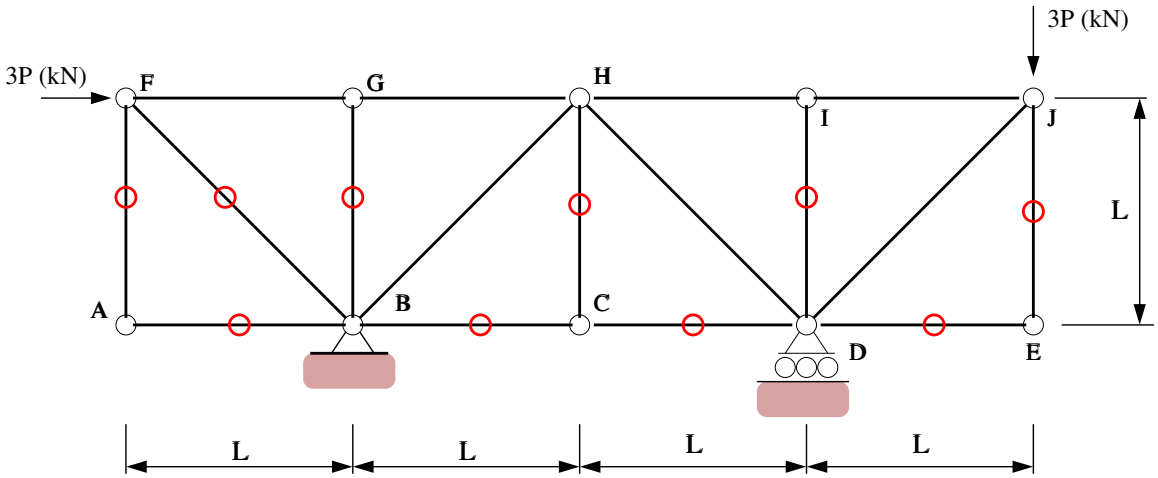


Figure 2. Elevation view of 17 bar truss structure.

Horizontal and vertical loads of **3P** kN are applied at nodes F and J, respectively.

Part [2a]: (3 pts) Compute the **magnitude** and **direction** of the **total support reactions** at points B and D.

Soln:

$$\sum M_B = 0, \longrightarrow \quad 3PL + 3P(3L) - V_d(2L) = 0 \longrightarrow \quad V_d = 6P. \tag{2}$$

$$\sum F_x = 0, \longrightarrow \quad H_B = -3P. \tag{3}$$

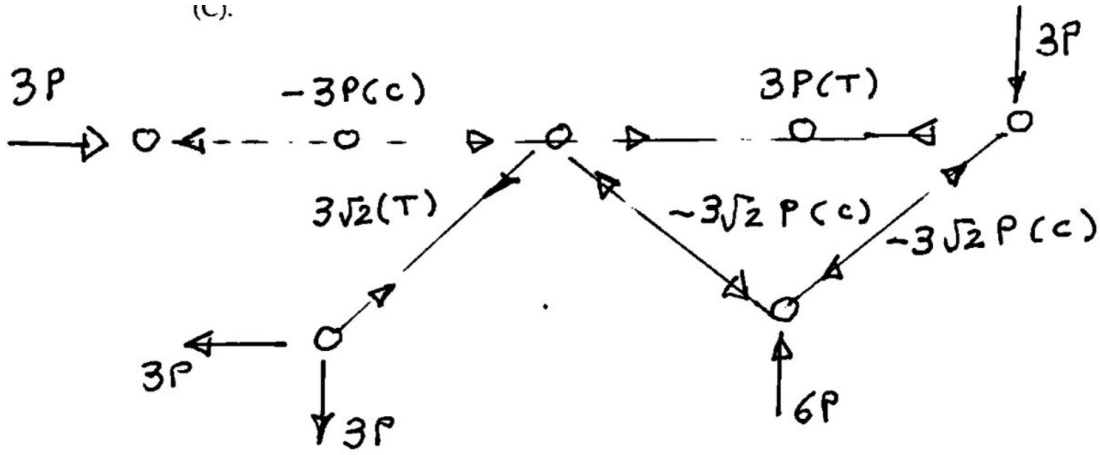
$$\sum F_y = 0, \longrightarrow \quad V_B = -3P. \tag{4}$$

Total reaction at B is $-3\sqrt{2}$ P acting at 45 degrees.

Part [2b]: (3 pts) Identify the zero-force members (If you wish, you can simply annotate Figure 2).

Soln: Notice that the total reaction force at B aligns perfectly with element $\overline{BH}$, hence $\overline{BC}$ = 0.0, a zero-force member.

Part [2c]: (7 pts) Using the method of joints (or otherwise) show that: (1) The maximum tensile force in the structure is $3\sqrt{2}P$ kN (T), and (2) The maximum compressive force in the structure is $-3\sqrt{2}\ P$ kN (C).



Part [2d]: (2 pts) Draw a simplified version of Figure 2 with **all of the zero force elements** removed.

Soln: See part 2c.

Question 3 (10 points): Method of Sections.

The frame tower carries horizontal and vertical loads P kN at joints C, E, G and H.

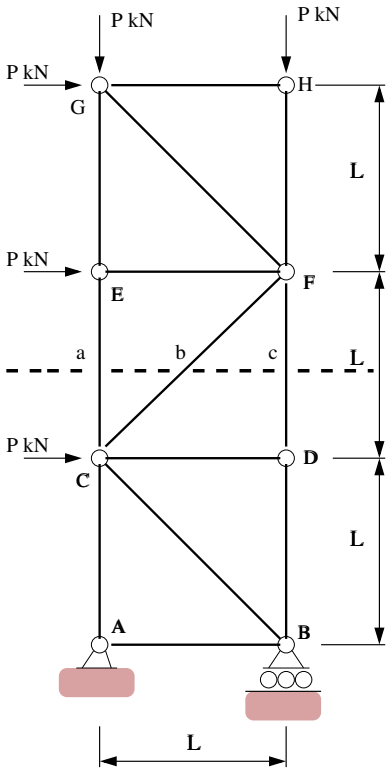


Figure 3. Elevation view of frame tower.

Use the **method of sections** to determine the forces in members a, b and c, as a function of the problem parameters L and P. Clearly indicate if the members are in tension or compression. Show all of your working.

Soln: Cut the structure at a-b-c (see Figure 3), then take moments about joint C.

$$\sum M_C = 0, \longrightarrow \quad \overline{DF} \cdot L + PL + 2PL + PL = 0 \longrightarrow \quad c = \overline{DF} = -4P(C). \tag{5}$$

Similarly, taking moments about joint F,

$$\sum M_F = 0, \longrightarrow \overline{CE} \cdot L + PL - PL = 0 \longrightarrow \quad a = \overline{CE} = 0P. \tag{6}$$

Summing forces in the vertical direction,

$$\sum V = 0, \longrightarrow \overline{CE} + \frac{\overline{CF}}{\sqrt{2}} + \overline{DF} = -2P, \longrightarrow \quad b = \overline{CF} = 2\sqrt{2}P(T). \tag{7}$$

Also note that the sum of forces in the horizontal direction is zero.