Homework 4

(Due: April 24, 2026)

Question 1: 10 points

Figure 1 is a front elevation view of a simply supported beam that carries a triangular load.

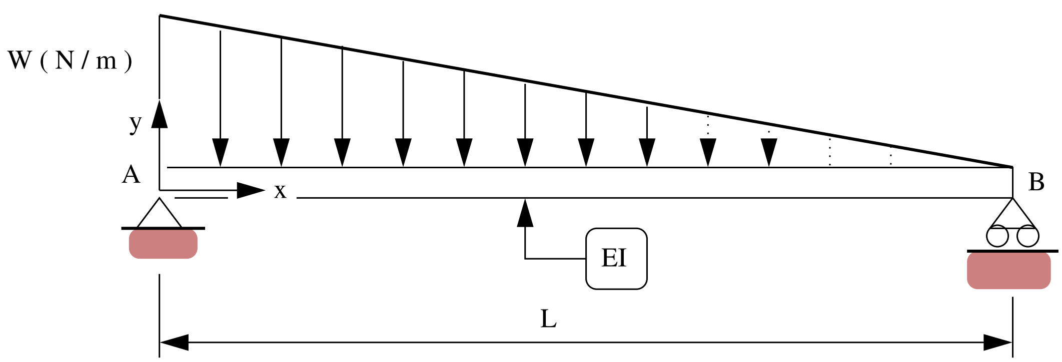

Figure 1: Simply supported beam carrying a triangular load.

The load decreases from W (N/m) at point A to zero at point B. Thus, the total beam loading is WL/2.

[1a] (4 pts). Starting from first principles of engineering, show that the bending moment at point x is:

$$M(x) = \left[\frac{W}{6L} \right] x \left(L - x \right) \left(2L - x \right). \tag{1}$$

[1b] (4 pts). Show that the elastic curve for beam deflection is given by (notice that in Figure 1, the y axis is pointing upwards):

$$y(x) = \left[\frac{WL^4}{360EI} \right] \left(3 \left[\frac{x}{L} \right]^5 - 15 \left[\frac{x}{L} \right]^4 + 20 \left[\frac{x}{L} \right]^3 - 8 \left[\frac{x}{L} \right] \right) \tag{2}$$

[1c] (2 pts). Show that the maximum beam curvature occurs at x = $\left[1 - \frac{1}{\sqrt{3}} \right] L$.

Question 2: 10 points

The cantilever beam structure shown in Figure 2 carries a uniform load w (N/m) along its entire length.

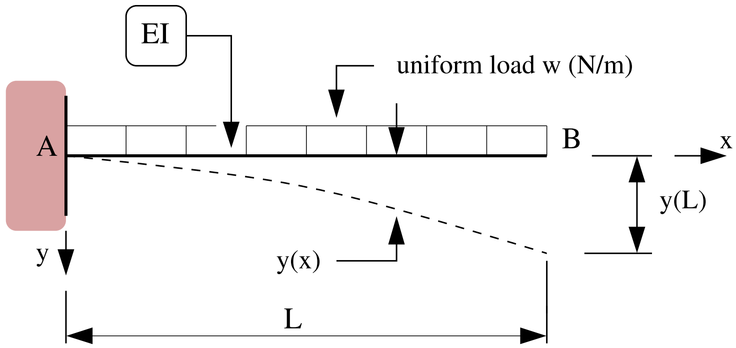

Figure 2: Cantilever beam carrying a uniform load.

The beam is fully fixed at point A and the flexural stiffness EI is constant along the beam. The coordinate system is positioned at point A.

[2a] (5 pts) Starting from the differential equation,

$$\frac{d^2y}{dx^2} = \left[\frac{M(x)}{EI}\right], \tag{3}$$

and appropriate boundary conditions, show that:

$$y(x) = \left(\frac{w}{24EI}\right)\left(6L^2x^2 - 4Lx^3 + x^4\right). \tag{4}$$

[2b] (5 pts) Using the results of question [1a] as a starting point, compute the support reactions at A and B for the propped cantilever shown in Figure 3

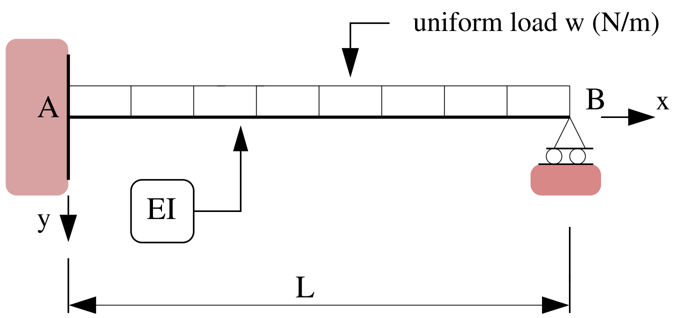

Figure 3: Propped cantilever beam carrying a uniform load.

Question 3: 20 points

Consider the cantilever shown in Figure 4.

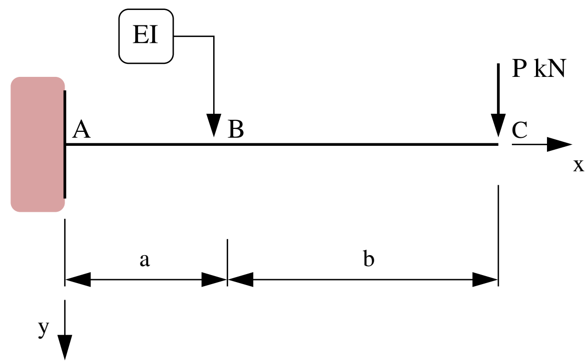

Figure 4: Front elevation view of a cantilever.

The cantilever has constant section properties, EI, along its entire length (a+b). A vertical load P (kN) is applied at point C.

[3a] (3 pts) Use the method of moment area to show that the vertical deflection at point C is:

$$y_C = \frac{P(a+b)^3}{3EI}. \tag{5}$$

[3b] (3 pts) Use the method of moment area to show that the vertical deflection at point B is:

$$y_B = \frac{Pa^2}{6EI}[3b + 2a]. \tag{6}$$

Now suppose that a roller support is inserted below point B as follows:

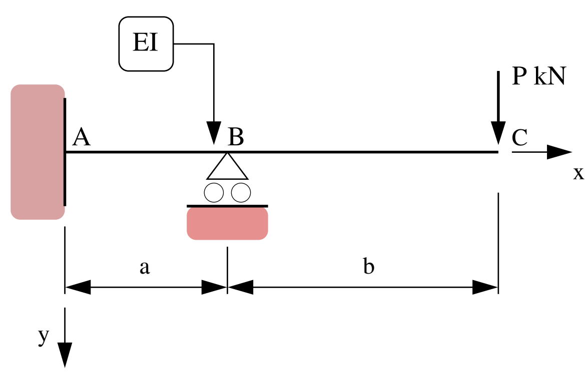

Figure 5: Front elevation view of a cantilever supported by a roller at point B.

[3c] (3 pts) Show that the vertical support reaction at B is:

$$V_b = \frac{P}{2}\left[\frac{3b + 2a}{a}\right]. \tag{7}$$

[3d] (3 pts) Hence, derive a simple expression for the bending moment at A.

Finally, let's replace the roller support below point B with a spring.

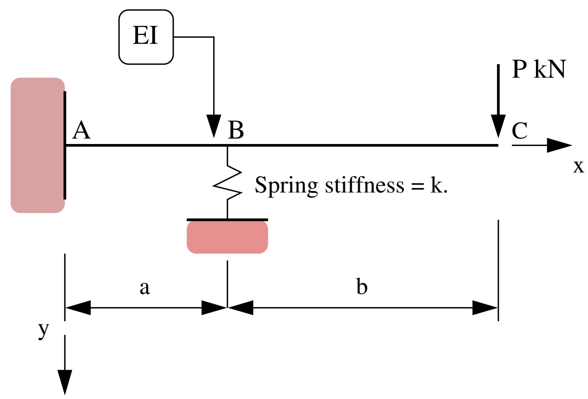

Figure 6: Cantilever supported by a spring at point B.

[3e] (4 pts) Show that the support reaction, V_b, is now given by the equation:

$$V_b\left[\frac{1}{k} + \frac{a^3}{3EI}\right] = \frac{Pa^2}{6EI}[3b + 2a]. \tag{8}$$

[3f] (4 pts) Explain why V_b for spring support (i.e., equation 8) is always lower than for roller support (i.e., equation 7).

Question 4: 10 points

The simple beam shown in Figure 7 has length L and uniform section properties EI. A point load P is applied at distance a from the left-hand support.

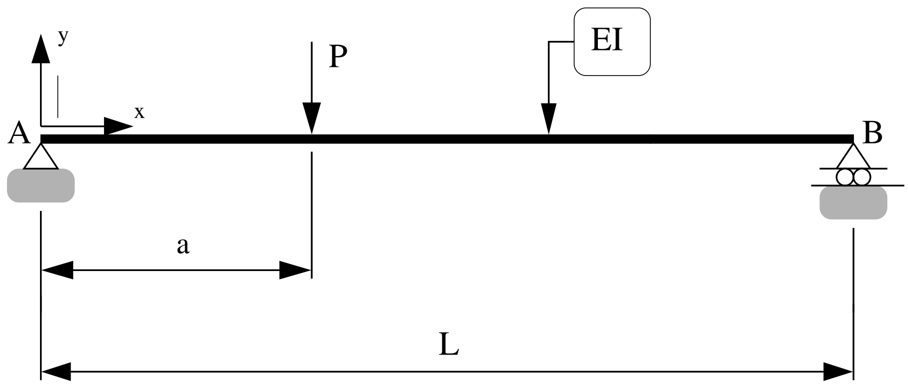

Figure 7: Front elevation view of a simple beam structure.

[4a] (2 pts) Draw and label the M(x)/EI diagram in terms of the problem parameters (i.e., P, EI, L and a).

[4b] (5 pts) Use the method of moment-area to show that the beam rotation at A is:

$$\theta_A = \left[\frac{P}{6EI}\right]\left[\frac{a\,(L-a)\,(2L-a)}{L}\right]. \tag{9}$$

[4c] (3 pts) Show that the maximum value of beam rotation at A occurs when:

$$a = L\left[1 - \frac{\sqrt{12}}{6}\right]. \tag{10}$$

Devise a simple strategy to test whether your answer makes sense (or not).