Department of Civil and Environmental Engineering,

Fall Semester, 2024

ENCE 353 Midterm 2, Open Notes and Open Book

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Exam Format and Grading. This take home midterm exam is open notes and open book. You need to comply with the university regulations for academic integrity.

There are three questions. Partial credit will be given for partially correct answers, so please show all your working.

Please see the **class web page for instructions on how to submit your exam paper**.

Question	Points	Score
1	15	
2	10	
3	15	
Total	40	

Question 1: 15 points

Moment-Area Method. Figure 1 is a front elevation view of a cantilevered beam carrying a single point load P. EI is constant along the beam structure A-B-C-D.

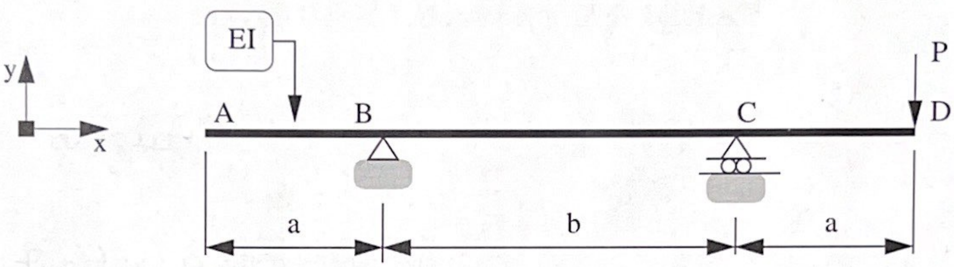
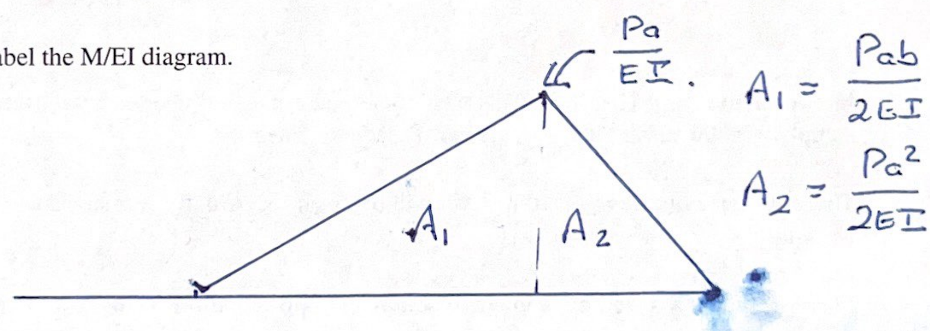
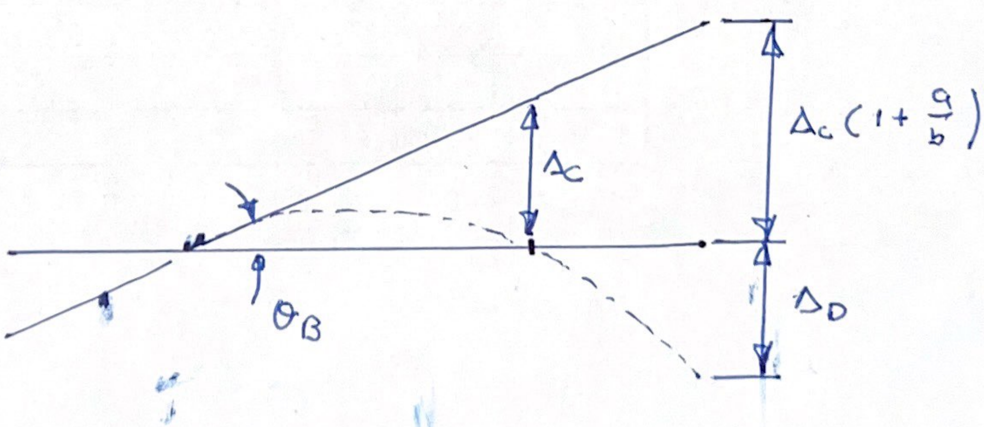


Figure 1: Cantilevered beam carrying a single applied load P.

[1a] (2 pts) Draw and label the M/EI diagram.



[1b] (2 pts) Draw and label the **moment area analysis diagram** (i.e., with rotations, tangents, displacements, etc) for this problem.



[1c] (3 pts) Use the method of **moment area** to show that the anticlockwise rotation of the beam at B is:

$$\theta_B = \frac{Pab}{6EI} \tag{1}$$

and the clockwise rotation of the beam at C is:

$$\theta_C = \frac{Pab}{3EI}. \tag{2}$$

Show all of your working.

Moment Area. $\Delta_c = \left(\frac{b}{3}\right) . A_1 = \frac{Pab^2}{6EI}.$

$$\theta_B = \left(\frac{\Delta_c}{b}\right) = \frac{Pab}{6EI}.$$

From the first theorem of Moment Area:

$$\theta_B + \theta_c = A_1 = \frac{Pab}{2EI}.$$

clockwise

anticlockwise,

$$\Rightarrow \theta_c = \frac{Pab}{2EI} - \frac{Pab}{6EI} = \frac{Pab}{3EI}.$$

[1d] (3 pts) Use the method of **moment area** to show that the vertical deflection of the beam at points A and D (measured downwards) is:

$$y_A = \frac{Pa^2 b}{6EI} \tag{3}$$

and

$$y_D = \frac{Pa^2}{3EI}(a+b). \tag{4}$$

Show all of your working.

Downwards deflection at $A = \theta_B \cdot a = \frac{Pa^2 b}{6EI}$.

Use moment area to find deflection at D.

$$\Delta_c = A_1 \cdot b/3$$

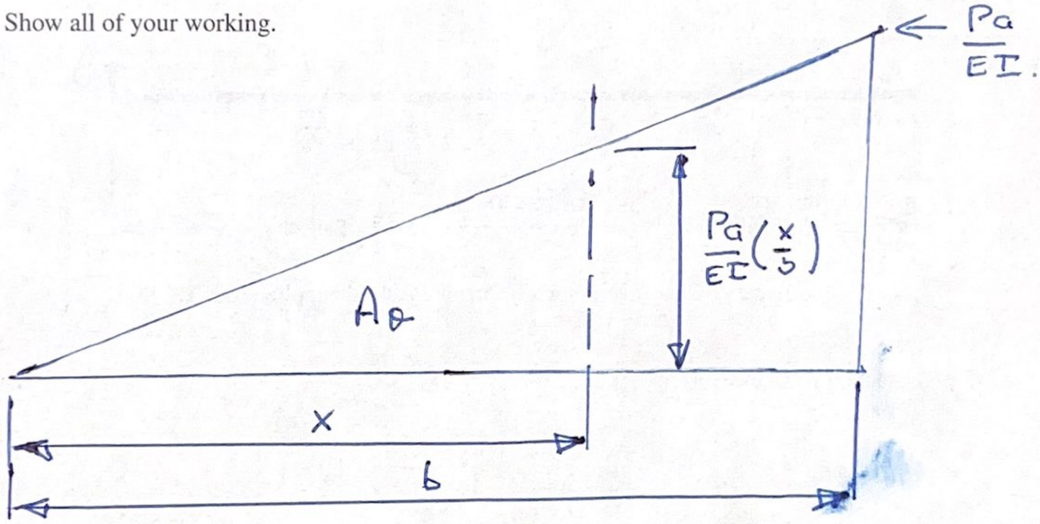
$$\Delta_c \left(1 + \frac{a}{b}\right) + \Delta_D = A_2 \cdot \left(\tfrac{2}{3} a\right) + A_1 \left(a + b/3\right)$$

$$\Rightarrow \Delta_D = \frac{Pa^2}{3EI}(a+b).$$

[1e] (5 pts) Show – do not simply assume – that the maximum upwards deflection of the beam occurs at a distance $b/\sqrt{3}$ from B, and that its value is:

$$y_{\text{maximum upward}} = \frac{Pab^2}{9\sqrt{3}EI}. \tag{5}$$

Show all of your working.



From geometry, $A_\theta = \frac{1}{2}\frac{Pa}{EI}\left(\frac{x}{b}\right).x = \frac{Pax^2}{2EIb}$.

At max upwards deflection, beam slope = 0, and $A_\theta = \theta_B$ (first theorem of Moment Area).

$$\Rightarrow \frac{Pax^2}{2EIb} = \frac{Pab}{6EI} \Rightarrow x = \left(b/\sqrt{3}\right)$$

Max upwards deflection $= \theta_B . x - A_\theta \left(x/3\right)$

$$= \frac{Pab^2}{9\sqrt{3}\,EI}.$$

Question 2: 10 points

Derive Elastic Curve for Beam Deflection. Figure 2 is a front elevation view of a cantilevered beam carrying a single point load P. EI is constant along the beam structure A-B-C-D.

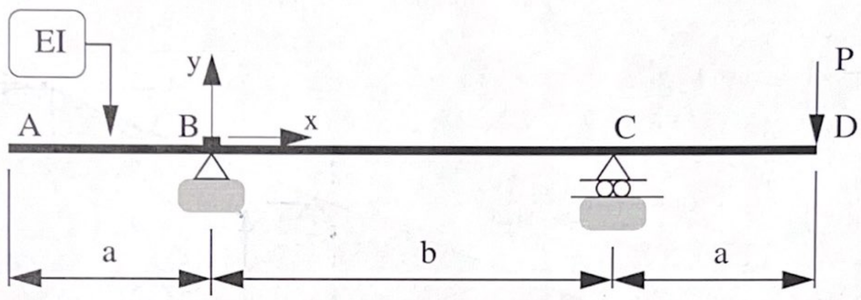


Figure 2: Cantilevered beam carrying a single applied load P.

[2a] (5 pts) Starting from the differential equation,

$$\frac{d^2y}{dx^2} = \left[\frac{-M(x)}{EI}\right], \tag{6}$$

(notice the minus sign on M(x)) and appropriate boundary conditions, show that that vertical beam deflection along the beam segment B-C is:

$$y(x) = \frac{Pa}{6EIb}(b^2 - x^2)\,x. \tag{7}$$

From part[1a], $M(x) = -P\left(\frac{a}{b}\right)x$ — A

$$\Rightarrow \frac{d^2y}{dx^2} = \left[\frac{M(x)}{EI}\right] \Rightarrow EI\frac{dy}{dx} = -\frac{Pa\,x^2}{2b} + A$$

$$\Rightarrow EI\,y(x) = -\frac{P}{6}\left(\frac{a}{b}\right)x^3 + Ax + B.$$

Bc. $y(0) = 0 \Rightarrow B = 0$, $y(b) = 0 \Rightarrow A = \frac{Pab}{6}$. $\Bigg\}$ $y(x) = \frac{Pa}{6EIb}(b^2 - x^2)x.$

[2b] (5 pts) Hence (i.e., using your results from part [2a] as a starting point), show that the maximum upwards deflection of the beam occurs at a distance $b/\sqrt{3}$ from B, and that its value is:

$$y_{\text{maximum upward}} = \frac{Pab^2}{9\sqrt{3}EI}. \tag{8}$$

Show all of your working.

Maximum upwards deflection occurs at:

$$\frac{dy}{dx} = 0 \Rightarrow \frac{Pa}{6b}(b^2 - 3x^2) = 0$$

$$\Rightarrow x = b/\sqrt{3}. \qquad \text{(A)}$$

Plug (A) into equation (8) gives:

$$y(b/\sqrt{3}) = \frac{Pa}{6EIb}\left(b^2 - \frac{b^2}{3}\right)\left(\frac{B}{\sqrt{3}}\right)$$

$$= \left\{\frac{Pab^2}{EI\,9\sqrt{3}}\right\}.$$

Question 3: 15 points

Principle of Superposition, Moment Area, Compatibility of Displacements. Figure 3 is a front elevation view of a multi-span beam structure that carries uniform load w_o (N/m) along beam segment B-C. EI is constant along the beam structure A-B-C-D.

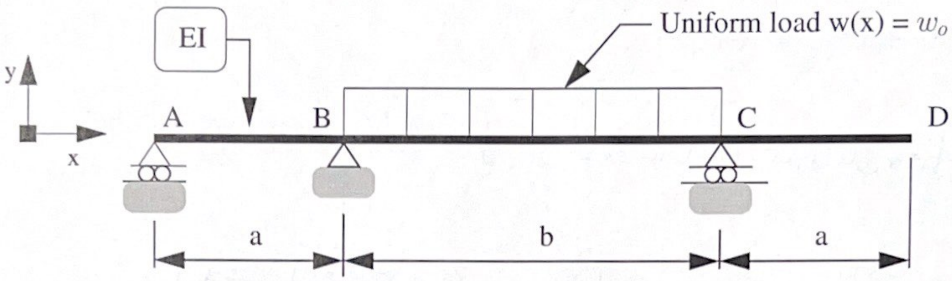


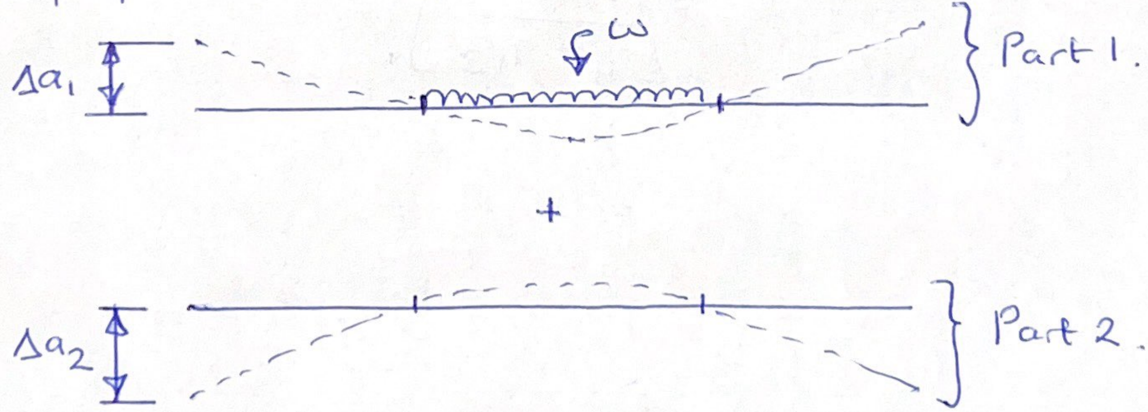
Figure 3: Multi-span beam carrying a uniform load on segment B-C.

[3a] (3 pts) Compute the degree of indeterminacy for this structure.

$\hat{i} = 1.$

[3b] (5 pts) Draw and label a diagram indicating how the **principle of superposition** can be used to simplify the analysis of the multi-span beam structure.

Here's one way – release the roller support at A. Apply superposition + displacement compatibility

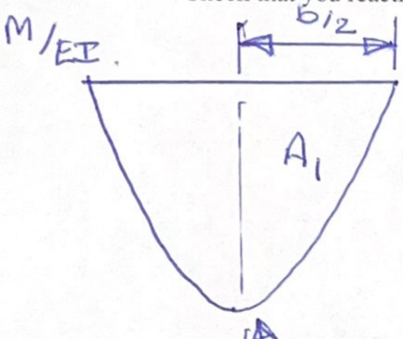


Displacement compatibility: $\Delta a_1 + \Delta a_2 = 0$

 $\Rightarrow$ find V_A.

[3c] (7 pts) Using the method of moment area or otherwise, compute the support reactions at supports A, B and C.

Check that you reaction forces are in equilibrium with the applied loads.



$$A_1 = \tfrac{2}{3}\left(\tfrac{b}{2}\right)\left(\frac{w_o b^2}{8EI}\right) = \frac{w_o b^3}{24EI}.$$

$$\Rightarrow \theta_B = \theta_C = \frac{w_o b^3}{24EI}$$

$$M_{max} = \frac{w_o b^2}{8EI}$$

Upwords deflection $\Delta a_1 = \theta_B \cdot a = \dfrac{w_o a b^3}{24EI} \uparrow$

From Question (1d):

$$\Delta a_2 = \frac{V_A \cdot a^2(a+b)}{3EI} \downarrow$$

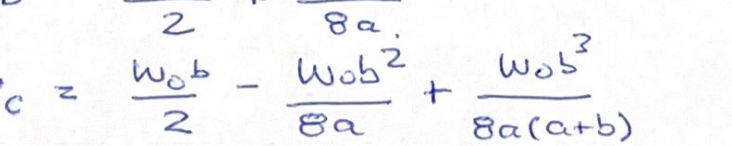
Compatibility of displacements $\Delta a_1 \uparrow + \Delta a_2 \downarrow = 0$

$$\Rightarrow V_A = \frac{w_o}{8} \cdot \frac{b^3}{a(a+b)} \downarrow$$

$$\Sigma' M_C = 0 \Rightarrow V_B = \frac{w_o b}{2} + \frac{w_o b^2}{8a}.$$

$$\Sigma' V = 0 \Rightarrow V_C = \frac{w_o b}{2} - \frac{w_o b^2}{8a} + \frac{w_o b^3}{8a(a+b)}$$

Suppose $a = b$.



Note: $V_A + V_B + V_C = w_o b$ ✓