Direct Stiffness Method

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Overview

Methods of Structural Analysis

Force and Displacement

Methods of Analysis

Methods of Structural Analysis

Force Method

- Displacements are proportional to external loads.
- Maintain equilibrium of forces.
- Use superposition and compatibility of displacements to solve problem.

Displacement Method

- Maintain compatibility of displacements.
- Use equilibrium of forces to solve problem.

Force (Flexibility) Method of Analysis

Solution Procedure:

- Convert an indeterminate structure to a determinate one by removing some unknown forces and/or support reactions and replacing them with (assumed) known unit forces.
- Problem solved for redundant forces.
- Coefficients of the unknowns in equations to be solved are flexibility coefficients.
- Additional steps needed to determine displacements and internal forces.

Computational Implementation:

- Can be programmed, but human input is needed to select primary structure and redundant forces.

Displacement (Stiffness) Method of Analysis

Solution Procedure:

- Express local (member) force-displacement relationships in terms of unknown member displacements.
- Coefficients of the unknowns in equations to be solved are stiffness coefficients.
- Use equilibrium of assembled members to find structural-level displacements and internal member forces.

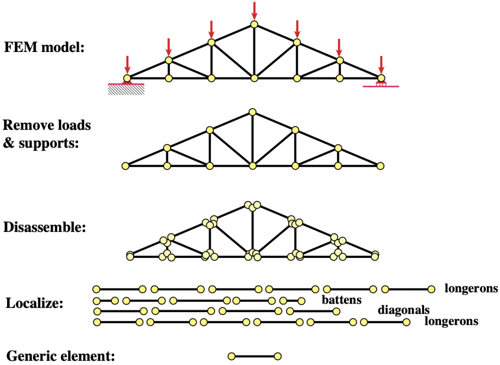
Computational Implementation:

- Easy to program on a computer.
- Lower memory requirments than for force method.

Direct Stiffness Method

Two-Step Procedure

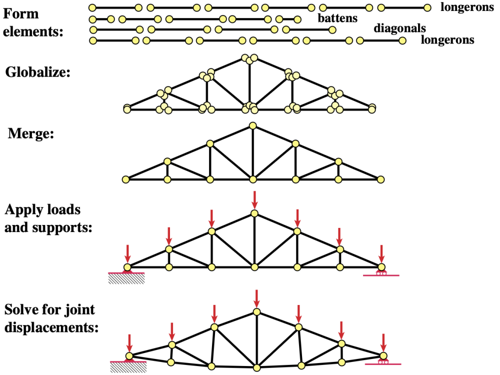
Decomposition: Pathway from structural model to generic elements.



Source: The Direct Stiffness Method, Book Chapter.

Two-Step Procedure

Assembly and Solution: Pathway from generic elements to structural assembly, deflections and member forces.



Source: The Direct Stiffness Method, Book Chapter.

Truss Member Behavior

Definition of Terms:

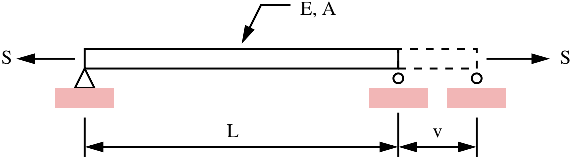
- R = External joint (structure) forces (and moments).
- r = External joint (structure) displacements (and rotations).
- S = Internal member end forces (and moments).
- v = Internal member end displacements (and rotations).
- a = Geometry transformation matrix.
- A = Cross-section area.
- E = Young's elastic modulus.

Stiffness and Flexibility:

- f = Member flexibility factors.
- k = Member stiffness factors.

Truss Member Behavior

1D Model of Truss Member Behavior:



Flexibility:

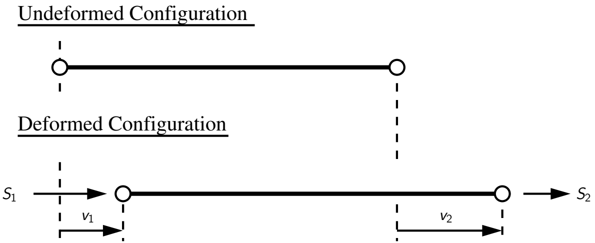
$$[v] = [f]\,[S] \quad \longrightarrow \quad [f] = \left[\frac{L}{AE}\right]. \tag{1}$$

Stiffness:

$$[S] = [k]\,[v] \quad \longrightarrow \quad [k] = \left[\frac{AE}{L}\right]. \tag{2}$$

Truss Member Behavior

Connect 1D Model of Truss Behavior to Two Joints:

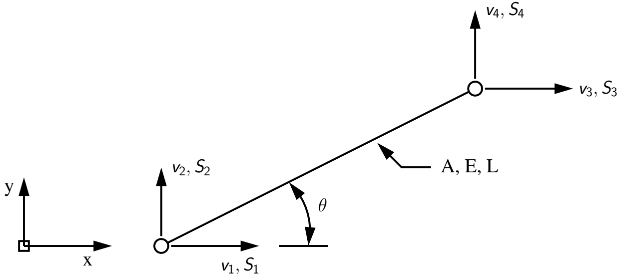


Stiffness Matrix: $[S] = [k]\,[v]$

$$\begin{bmatrix} S_1 \\ S_2 \end{bmatrix} = \frac{AE}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \tag{3}$$

Truss Member Behavior

Relate Member Displacements to Joint Displacements:



$$[v] = [a]\,[r] = \begin{bmatrix} -\cos(\theta) & -\sin(\theta) & \cos(\theta) & \sin(\theta) \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix}. \tag{4}$$

Truss Member Behavior

Similarly:

$$[S_i] = \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \end{bmatrix} = \left[a^T\right][S]\,. \tag{5}$$

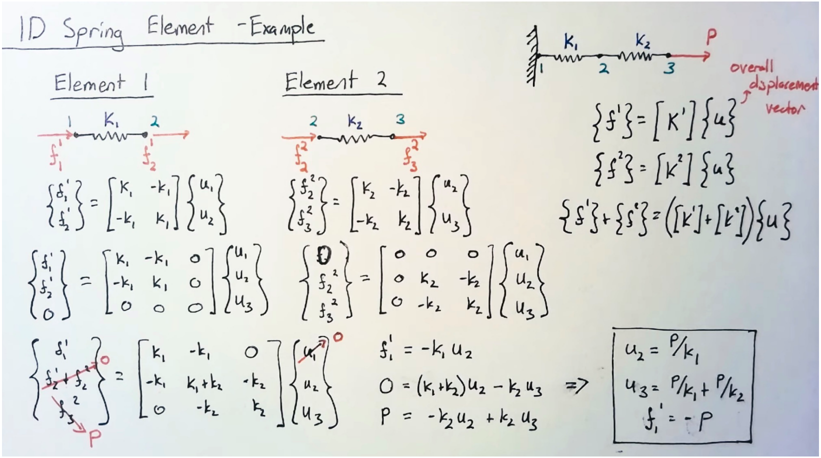
Hence:

$$\begin{aligned}[S_i] &= \left[a^T\right][k][a][v_i] \\ &= [k_i][v_i]\,.\end{aligned} \tag{6}$$

Here, $[k_i]$ is the (4×4) member stiffness matrix in terms of the member joint displacements and forces.

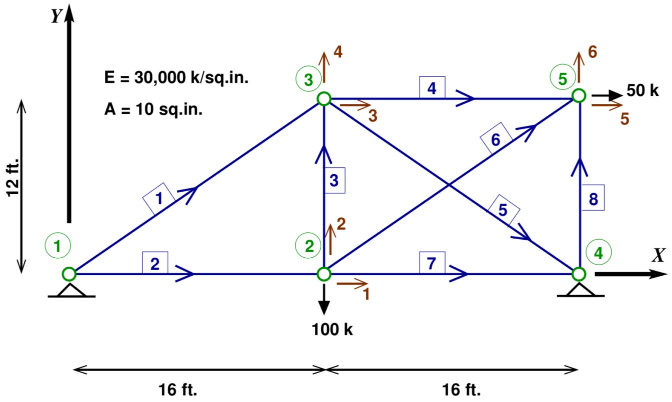
Structural Model Assembly

Example 1: Two-Spring Assembly



Structural Model Assembly

Example 2: Eight-bar Truss



Structural Model Assembly

Example 2: Eight-bar Truss

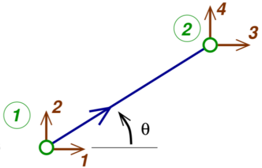
- 1 Global Joint Number
- 2 Bar Number
- Bar Direction
- 1 Structural Degree of Freedom
- Applied Force
- *1* Local Joint Number
- *1* Local Degree of Freedom
- Global Coordinate System

System Equation: {f} = [K]{d}

{d} : displacement vector
= { d1 d2 d3 d4 d5 d6 }'

{f} : load vector = { 0 –100 0 0 50 0 }'

Element Coordinates in Global Directions



Step-by-Step Procedure

1. Member Forces in terms of Internal Member Displacements

$$[S] = [k]\,[v] \tag{7}$$

2. Relate Member Displacements to Joint Displacements.

$$[v] = [a]\,[r] \tag{8}$$

3. Relate Member Forces to Joint Displacements.

$$[S] = [k]\,[a]\,[r] \tag{9}$$

4. Relate Joint Forces to Member Forces.

$$[R] = \left[a^T\right][S]\,. \tag{10}$$

Step-by-Step Procedure

5. Assemble Structure (Joint) Stiffness Matrix.

$$\begin{aligned} [R] &= \left[a^T\right][k][a][r] \\ &= [K][r] \end{aligned} \tag{11}$$

6. Solve Matrix Equations for Joint Displacements, i.e.,

$$[K][r] = [R]. \tag{12}$$

7. Obtain Member Forces from Joint Displacements,

$$[S] = [k][a][r] \tag{13}$$

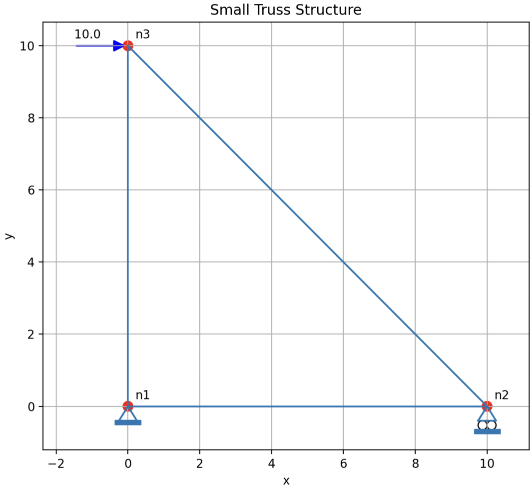
Example 1

Example 1

Analysis of Three-Bar Truss

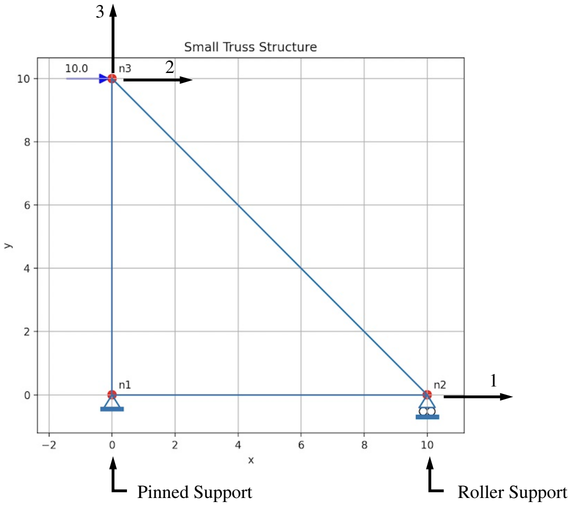
Example 1: Analysis of Three-Bar Truss

Geometry and External Loads



Example 1: Analysis of Three-Bar Truss

Boundary Conditions and Global DOF



Example 1: Manual Specification of Three-Bar Truss

Specification: Nodes and Elements

```
Nodes
====================================
Node: n1
Coordinate: (x,y) = ( 0.00, 0.00 )
DOF: (x,y) = [False, False]
Map LDOF-to-GDOF: (x,y) = [0, 0]

Node: n2
Coordinate: (x,y) = ( 10.00, 0.00 )
DOF: (x,y) = [True, False]
Map LDOF-to-GDOF: (x,y) = [0, 0]

Node: n3
Coordinate: (x,y) = ( 0.00, 10.00 )
DOF: (x,y) = [True, True]
Map LDOF-to-GDOF: (x,y) = [0, 0]
====================================
```

```
Elements
================================
Bar2D: e01: Nodes (n1, n2)
Length: L = 10.000000
Young's Modulus:    E = 1.000000
Cross Section Area: A = 1.000000

Bar2D: e02: Nodes (n1, n3)
Length: L = 10.000000 ...
Young's Modulus:    E = 1.000000
Cross Section Area: A = 1.000000

Bar2D: e03: Nodes (n2, n3)
Length: L = 14.142136
Young's Modulus:    E = 1.000000
Cross Section Area: A = 1.000000
================================
```

Example 1: Analysis of Three-Bar Truss

Boundary Conditions

```
Node n1: (x,y) pin support: dof free to move    --> [ False, False ]
Node n2: (x,y) roller support: dof free to move --> [  True, False ]
Node n3: (x,y) dof free to move                 --> [  True,  True ]
```

Nodal Loads

```
Node Load: n3 ...
Loads: [ Fx, Fy ] = [ 10.00,  0.00 ]
```

Model Compilation:

```
====================================================== ...
No nodes      = 3
No elements   = 3
No node loads = 1
```

Example 1: Analysis of Three-Bar Truss

Model Compilation: continued:

```
Node: n1
Coordinates: (x,y) = (   1.00,    0.00)
DOF: (x,y) = [ False, False ]
Map: LDOF->GDOF: [0, 0] ...
Node: n2
Coordinates: (x,y) = (   2.00,   10.00)
DOF: (x,y) = [ True, False ]
Map: LDOF->GDOF: [1, 0]
Node: n3
Coordinates: (x,y) = (   3.00,    0.00)
DOF: (x,y) = [ True, True ]
Map: LDOF->GDOF: [2, 3]

Element: e01, Element: e02, Element: e03

Node Load: n3
Loads: [ Fx, Fy ] = [ 10.00, 0.00 ]
======================================================
```

Example 1: Analysis of Three-Bar Truss

LocalDOF-to-GlobalDOF Mapping:

Matrix: Destination Array

row/col	1	2	3	4
1	0.00000e+00	0.00000e+00	1.00000e+00	0.00000e+00
2	0.00000e+00	0.00000e+00	2.00000e+00	3.00000e+00
3	1.00000e+00	0.00000e+00	2.00000e+00	3.00000e+00

Key Points:

- Element 1 dofs map to gdof 1.
- Element 2 dofs map to gdofs 2 and 3.
- Element 3 dofs map to gdofs 1, 2, and 3.

Example 1: Analysis of Three-Bar Truss

Element Stiffness Matrices: Bar 1

```
Bar 1 stiffness (local coordinates):
====================================

Matrix: Element Stiffness (local coordinates)
row/col              1             2
   1       1.00000e-01  -1.00000e-01
   2      -1.00000e-01   1.00000e-01

Bar 1 stiffness (joint coordinates):
====================================

Matrix: Element Stiffness (joint coordinates)
row/col              1             2             3             4
   1       1.00000e-01   0.00000e+00  -1.00000e-01  -0.00000e+00
   2       0.00000e+00   0.00000e+00  -0.00000e+00  -0.00000e+00
   3      -1.00000e-01   0.00000e+00   1.00000e-01   0.00000e+00
   4      -0.00000e+00  -0.00000e+00   0.00000e+00   0.00000e+00
```

Example 1: Analysis of Three-Bar Truss

Element Stiffness Matrices: Bar 2

```
Bar 2 stiffness (local coordinates):
====================================

Matrix: Element Stiffness (local coordinates)
row/col              1             2
   1       1.00000e-01  -1.00000e-01
   2      -1.00000e-01   1.00000e-01

Bar 2 stiffness (joint coordinates):
====================================

Matrix: Element Stiffness (joint coordinates)
row/col              1             2             3             4
   1       0.00000e+00   0.00000e+00  -0.00000e+00  -0.00000e+00
   2       0.00000e+00   1.00000e-01  -0.00000e+00  -1.00000e-01
   3      -0.00000e+00   0.00000e+00   0.00000e+00   0.00000e+00
   4      -0.00000e+00  -1.00000e-01   0.00000e+00   1.00000e-01
```

Example 1: Analysis of Three-Bar Truss

Element Stiffness Matrices: Bar 3

```
Bar 3 stiffness (local coordinates):
====================================

Matrix: Element Stiffness (local coordinates)
row/col              1             2
   1       7.07107e-02  -7.07107e-02
   2      -7.07107e-02   7.07107e-02

Bar 3 stiffness (joint coordinates):
====================================

Matrix: Element Stiffness (joint coordinates)
row/col              1             2             3             4
   1       3.53553e-02  -3.53553e-02  -3.53553e-02   3.53553e-02
   2      -3.53553e-02   3.53553e-02   3.53553e-02  -3.53553e-02
   3      -3.53553e-02  -3.53553e-02   3.53553e-02  -3.53553e-02
   4       3.53553e-02  -3.53553e-02  -3.53553e-02   3.53553e-02
```

Example 1: Analysis of Three-Bar Truss

Destination Array:

```
Matrix: Destination Array
row/col              1             2             3             4
   1       0.00000e+00   0.00000e+00   1.00000e+00   0.00000e+00
   2       0.00000e+00   0.00000e+00   2.00000e+00   3.00000e+00
   3       1.00000e+00   0.00000e+00   2.00000e+00   3.00000e+00
```

Step-by-Step Assembly of Global Stiffness Matrix:

```
for ielmt in range( len( self.elements ) ):        # <-- Loop over elements.

    for ij in range ( len( destination[0] ) ):     # <-- Add element-level
        for ik in range ( len( destination[0] ) ): #     components to
            dmap1 = destination[ielmt][ij];        #     global stiffness.
            dmap2 = destination[ielmt][ik];
            if( dmap1 > 0 and dmap2 > 0):
                stiffness[ int(dmap1)-1 ][ int(dmap2)-1 ] += ke[ij][ik];
```

Example 1: Analysis of Three-Bar Truss

Step 1: Add Elmt 1 to Global Stiffness Matrix ....

```
Matrix: Global Stiffness Matrix (incremental)
row/col              1             2             3
   1       1.00000e-01   0.00000e+00   0.00000e+00
   2       0.00000e+00   0.00000e+00   0.00000e+00
   3       0.00000e+00   0.00000e+00   0.00000e+00
```

Step 2: Add Elmt 2 to Global Stiffness Matrix ....

```
Matrix: Global Stiffness Matrix (incremental)
row/col              1             2             3
   1       1.00000e-01   0.00000e+00   0.00000e+00
   2       0.00000e+00   0.00000e+00   0.00000e+00
   3       0.00000e+00   0.00000e+00   1.00000e-01
```

Example 1: Analysis of Three-Bar Truss

Step 3: Add Elmt 3 to Global Stiffness Matrix ....

```
Matrix: Global Stiffness Matrix (incremental)
row/col              1             2             3
   1       1.35355e-01  -3.53553e-02   3.53553e-02
   2      -3.53553e-02   3.53553e-02  -3.53553e-02
   3       3.53553e-02  -3.53553e-02   1.35355e-01
```

Global Stiffness Matrix

```
Matrix: GlobalStiffness: stiff01
row/col              1             2             3
   1       1.35355e-01  -3.53553e-02   3.53553e-02
   2      -3.53553e-02   3.53553e-02  -3.53553e-02
   3       3.53553e-02  -3.53553e-02   1.35355e-01
```

Example 1: Analysis of Three-Bar Truss

Assembly of External Load Vector:

```
Matrix: Load Vector: eload01
row/col                 1
   1        0.00000e+00
   2        1.00000e+01
   3        0.00000e+00
```

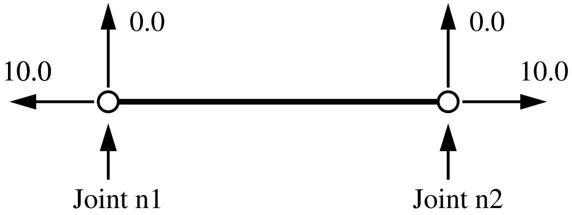
Solution of Global Displacements:

```
Matrix: Displacement Vector (displ01)
row/col                 1
   1        1.00000e+02
   2        4.82843e+02
   3        1.00000e+02
```

Example 1: Analysis of Three-Bar Truss

Elmt 1: Joint Displacements and Forces:

```
Matrix: Elmt (joint displacements [r]):  Matrix: Elmt (joint forces):
row/col                  1               row/col                  1
   1        0.00000e+00                     1        -1.00000e+01
   2        0.00000e+00                     2         0.00000e+00
   3        1.00000e+02                     3         1.00000e+01
   4        0.00000e+00                     4         0.00000e+00
```



Example 1: Analysis of Three-Bar Truss

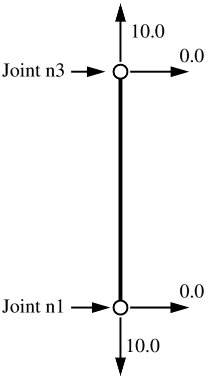
Elmt 1: Member Displacements and Forces:

```
Matrix: Geometry transformation ([a]):
row/col              1             2             3             4
   1       1.00000e+00   0.00000e+00   0.00000e+00   0.00000e+00
   2       0.00000e+00   0.00000e+00   1.00000e+00   0.00000e+00

Matrix: Member displacements ([v] = [a][r]):
row/col              1
   1       0.00000e+00
   2       1.00000e+02

Matrix: Member forces ([S] = [k][v]):
row/col              1
   1      -1.00000e+01
   2       1.00000e+01

--- Axial force = 10.000000 (T) ...
```

Example 1: Analysis of Three-Bar Truss

Elmt 2: Joint Displacements and Forces:

```
Matrix: Elmt (joint displacements [r]):
row/col                 1
   1         0.00000e+00
   2         0.00000e+00
   3         4.82843e+02
   4         1.00000e+02

Matrix: Elmt (joint forces):
row/col                 1
   1         0.00000e+00
   2        -1.00000e+01
   3         0.00000e+00
   4         1.00000e+01
```



Example 1: Analysis of Three-Bar Truss

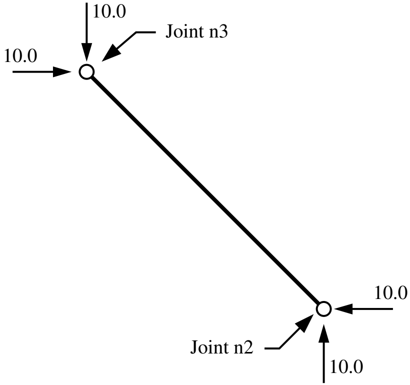
Elmt 2: Member Displacements and Forces:

```
Matrix: Geometry transformation ([a]):
row/col             1             2             3             4
   1      0.00000e+00   1.00000e+00   0.00000e+00   0.00000e+00
   2      0.00000e+00   0.00000e+00   0.00000e+00   1.00000e+00

Matrix: Member displacements ([v] = [a][r]):
row/col             1
   1      0.00000e+00
   2      1.00000e+02

Matrix: Member forces ([S] = [k][v]):
row/col             1
   1     -1.00000e+01
   2      1.00000e+01

--- Axial force = 10.000000 (T) ...
```

Example 1: Analysis of Three-Bar Truss

Elmt 3: Joint Displacements and Forces:

```
Matrix: Elmt (joint displacements [r]):
row/col                  1
   1          1.00000e+02
   2          0.00000e+00
   3          4.82843e+02
   4          1.00000e+02

Matrix: Elmt (joint forces):
row/col                  1
   1         -1.00000e+01
   2          1.00000e+01
   3          1.00000e+01
   4         -1.00000e+01
```



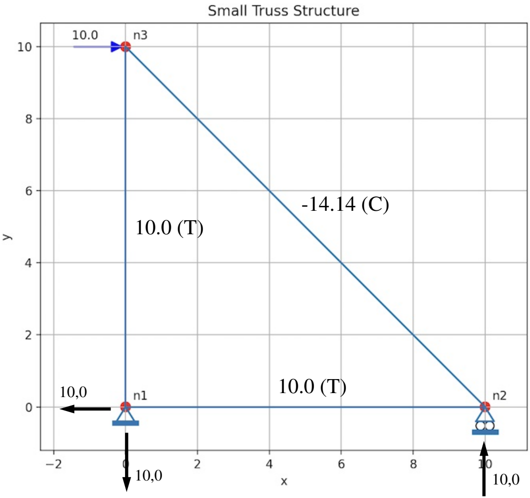
Example 1: Analysis of Three-Bar Truss

Elmt 3: Member Displacements and Forces:

```
Matrix: Geometry transformation ([a]):
row/col              1              2              3              4
   1      -7.07107e-01   7.07107e-01   0.00000e+00   0.00000e+00
   2       0.00000e+00   0.00000e+00  -7.07107e-01   7.07107e-01

Matrix: Member displacements ([v] = [a][r]):
row/col              1
   1      -7.07107e+01
   2      -2.70711e+02

Matrix: Member forces ([S] = [k][v]):
row/col              1
   1       1.41421e+01
   2      -1.41421e+01

--- Axial force = -14.142136 (C) ...
```

Example 1: Analysis of Three-Bar Truss

Summary of Member Forces and Reactions



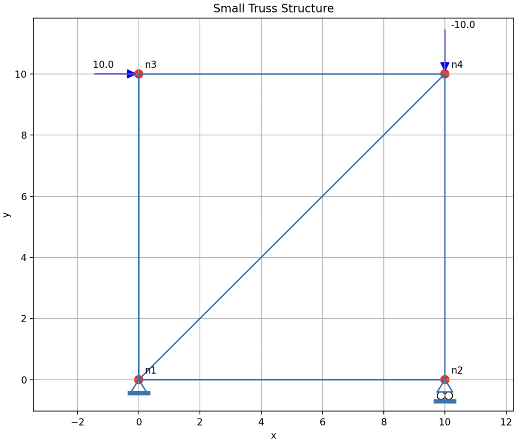
Example 2

Example 2

Analysis of Five-Bar Truss

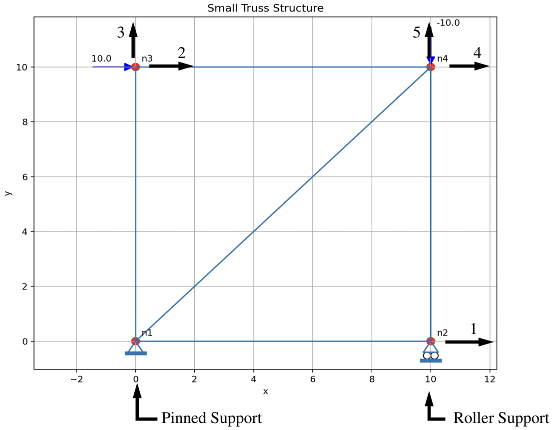
Example 2: Analysis of Five-Bar Truss

Geometry and External Loads



Example 2: Analysis of Five-Bar Truss

Boundary Conditions and Global DOF



Example 2: Analysis of Five-Bar Truss

Preliminary Observations:

1. Node n2 is a roller oriented horizontally – thus, we expect that the bar element connecting nodes n1 and n2 will be a zero-force member.
2. Taking moments about A indicates that the reaction force at B will be 20 acting upwards.
3. Summing forces in the vertical direction for the whole structure – the reaction force at A will be 10 acting downwards.
4. Support A will also have a horizontal reaction force acting right-to-left.

Example 2: Analysis of Five-Bar Truss

Specification: Nodes and Elements

```
Nodes
====================================
Node: n1, (x,y) = ( 0.00,  0.00 )
DOF: (x,y) = [ False, False ]
Map LDOF-to-GDOF: (x,y) = [0, 0]

Node: n2, (x,y) = ( 10.00,  0.00 )
DOF: (x,y) = [ True, False ] kk
Map LDOF-to-GDOF: (x,y) = [1, 0]

Node: n3, (x,y) = (  0.00, 10.00 )
DOF: (x,y) = [ True, True ]
Map LDOF-to-GDOF: (x,y) = [2, 3]

Node: n4, (x,y) = ( 10.00, 10.00 )
DOF: (x,y) = [ True, True ]
Map LDOF-to-GDOF: (x,y) = [4, 5]
```

```
Elements (E = A = 1.0)
================================
Bar2D: e01: Nodes (n1, n2)
Length L = 10.00000

Bar2D: e02: Nodes (n1, n3)
Length L = 10.00000

Bar2D: e03: Nodes (n2, n4)
Length L = 10.0000

Bar2D: e04: Nodes (n3, n4)
Length L = 10.0000

Bar2D: e05: Nodes (n1, n4)
Length L = 14.142136
```

Example 2: Analysis of Five-Bar Truss

Boundary Conditions

```
Node n1: (x,y) pin support: dof free to move    --> [ False, False ]
Node n2: (x,y) roller support: dof free to move --> [  True, False ]
Node n3: (x,y) dof free to move                 --> [  True,  True ]
Node n4: (x,y) dof free to move                 --> [  True,  True ]
```

Nodal Loads

```
Node Load: n3
Loads: [ Fx, Fy ] = [ 10.00,   0.00 ]
Node Load: n4
Loads: [ Fx, Fy ] = [  0.00, -10.00 ]
```

Model Compilation:

```
================================================================
No nodes      = 4
No elements   = 5
No node loads = 2
```

Example 2: Analysis of Five-Bar Truss

Model Compilation: Continued ...

```
Node: n1
Coordinates: (x,y) = (  0.00,   0.00)
DOF: (x,y) = [ False, False ]
Map: LDOF->GDOF: [0, 0]
Node: n2
Coordinates: (x,y) = ( 10.00,   0.00)
DOF: (x,y) = [ True, False ]
Map: LDOF->GDOF: [1, 0]
Node: n3
Coordinates: (x,y) = (  0.00,  10.00)
DOF: (x,y) = [ True, True ]
Map: LDOF->GDOF: [2, 3]
Node: n4
Coordinates: (x,y) = ( 10.00,  10.00)
DOF: (x,y) = [ True, True ]
Map: LDOF->GDOF: [4, 5]

Element: e01, Element: e02, Element: e03, Element: e04, Element: e05
```

Example 2: Analysis of Five-Bar Truss

Model Compilation: Continued ...

```
Node Load: n3 ...
Loads: [ Fx, Fy ] = [ 10.00, 0.00 ]
Node Load: n4 ...
Loads: [ Fx, Fy ] = [ 0.00, -10.00 ]

=====================================================
```

Destination Array

Matrix: Destination Array

row/col	1	2	3	4
1	0.00000e+00	0.00000e+00	1.00000e+00	0.00000e+00
2	0.00000e+00	0.00000e+00	2.00000e+00	3.00000e+00
3	1.00000e+00	0.00000e+00	4.00000e+00	5.00000e+00
4	2.00000e+00	3.00000e+00	4.00000e+00	5.00000e+00
5	0.00000e+00	0.00000e+00	4.00000e+00	5.00000e+00

Example 2: Analysis of Five-Bar Truss

Global Stiffness Matrix

Matrix: GlobalStiffness: stiff01

row/col	1	2	3	4	5
1	1.00000e-01	0.00000e+00	0.00000e+00	0.00000e+00	0.00000e+00
2	0.00000e+00	1.00000e-01	0.00000e+00	-1.00000e-01	0.00000e+00
3	0.00000e+00	0.00000e+00	1.00000e-01	0.00000e+00	0.00000e+00
4	0.00000e+00	-1.00000e-01	0.00000e+00	1.35355e-01	3.53553e-02
5	0.00000e+00	0.00000e+00	0.00000e+00	3.53553e-02	1.35355e-01-

External Load Vector

Matrix: Load Vector: eload01

row/col	1
1	0.00000e+00
2	1.00000e+01
3	0.00000e+00
4	0.00000e+00
5	-1.00000e+01

Example 2: Analysis of Five-Bar Truss

Global Displacement Vector

```
Matrix: Displacement Vector (displ01)
row/col                 1
   1         0.00000e+00
   2         5.82843e+02
   3         0.00000e+00
   4         4.82843e+02
   5        -2.00000e+02
```

Example 2: Analysis of Five-Bar Truss

Element-Level Displacements and Member Forces

Elmt 1: Connects nodes n1 and n2

```
Matrix: Member displ ([v] = [a][r]):  Matrix: Member forces ([S] = [k][v]):
row/col                1               row/col                1
   1        0.00000e+00                   1        0.00000e+00
   2        0.00000e+00                   2        0.00000e+00

---> Axial force = 0.000000 (T) ...
```

Elmt 2: Connects nodes n1 and n3

```
Matrix: Member displ ([v] = [a][r]):   Matrix: Member forces ([S] = [k][v]):
row/col                1                row/col                1
   1        0.00000e+00                    1        0.00000e+00
   2        0.00000e+00                    2        0.00000e+00

---> Axial force = 0.000000 (T) ...
```

Example 2: Analysis of Five-Bar Truss

Elmt 3: Connects nodes n2 and n4

```
Matrix: Member displ ([v] = [a][r]):   Matrix: Member forces ([S] = [k][v]):
row/col                  1             row/col                  1
   1        0.00000e+00                   1        2.00000e+01
   2       -2.00000e+02                   2       -2.00000e+01

---> Axial force = -20.000000 (C) ...
```

Elmt 4: Connects nodes n3 and n4

```
Matrix: Member displ ([v] = [a][r]):   Matrix: Member forces ([S] = [k][v]):
row/col                  1             row/col                  1
   1        5.82843e+02                   1        1.00000e+01
   2        4.82843e+02                   2       -1.00000e+01

---> Axial force = -10.000000 (C) ...
```

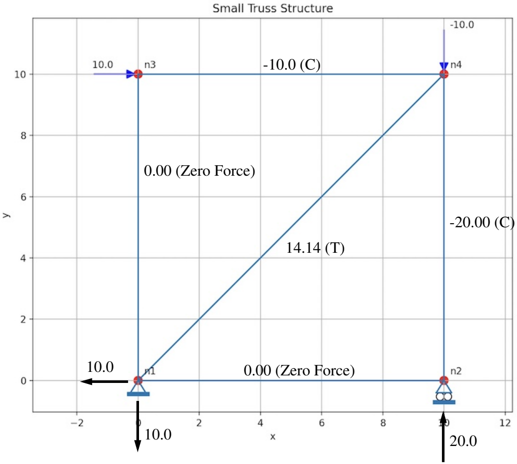
Example 2: Analysis of Five-Bar Truss

Elmt 5: Connects nodes n1 and n4

```
Matrix: Member displ ([v] = [a][r]):   Matrix: Member forces ([S] = [k][v]):
row/col                  1             row/col                  1
   1        0.00000e+00                   1        -1.41421e+01
   2        2.00000e+02                   2         1.41421e+01

---> Axial force = 14.142136 (T) ...
```

Example 2: Analysis of Five-Bar Truss

Summary of Member Forces and Support Reactions



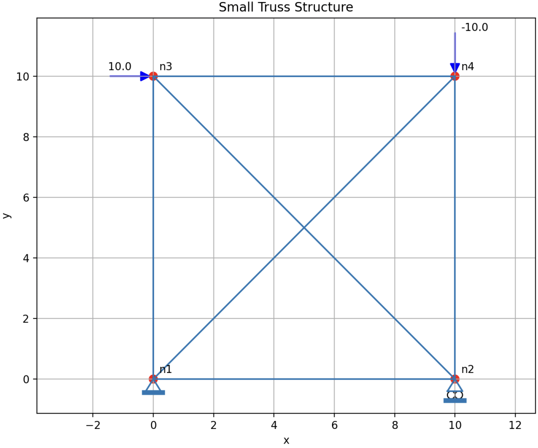
Example 3

Example 3

Analysis of Six-Bar Truss

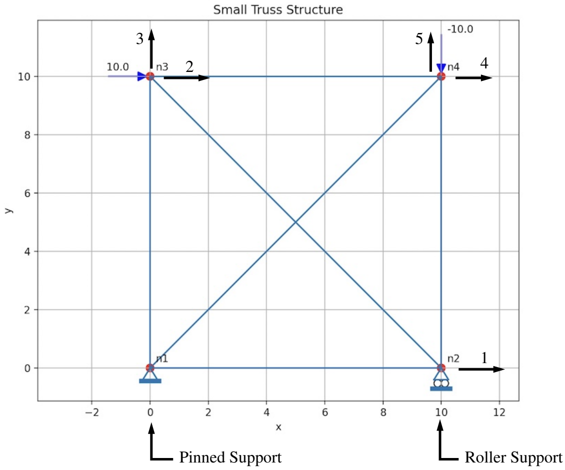
Example 3: Analysis of Six-Bar Truss

Geometry and External Loads



Example 3: Analysis of Six-Bar Truss

Boundary Conditions and Global DOF



Example 3: Analysis of Six-Bar Truss

Specification: Nodes and Elements

```
Nodes
====================================
Node: n1, (x,y) = ( 0.00,  0.00 )
DOF: (x,y) = [ False, False ]
Map LDOF-to-GDOF: (x,y) = [0, 0]

Node: n2, (x,y) = ( 10.00,  0.00 )
DOF: (x,y) = [ True, False ] kk
Map LDOF-to-GDOF: (x,y) = [1, 0]

Node: n3, (x,y) = (  0.00, 10.00 )
DOF: (x,y) = [ True, True ]
Map LDOF-to-GDOF: (x,y) = [2, 3]

Node: n4, (x,y) = ( 10.00, 10.00 )
DOF: (x,y) = [ True, True ]
Map LDOF-to-GDOF: (x,y) = [4, 5]
```

```
Elements (E = A = 1.0)
================================
Bar2D: e01: Nodes (n1, n2)
Length L = 10.00000

Bar2D: e02: Nodes (n1, n3)
Length L = 10.00000

Bar2D: e03: Nodes (n2, n4)
Length L = 10.0000

Bar2D: e04: Nodes (n3, n4)
Length L = 10.0000

Bar2D: e05: Nodes (n1, n4)
Length L = 14.142136

Bar2D: e06: Nodes (n2, n3)
Length L = 14.142136
```

Example 3: Analysis of Six-Bar Truss

Boundary Conditions

```
Node n1: (x,y) pin support: dof free to move    --> [ False, False ]
Node n2: (x,y) roller support: dof free to move --> [  True, False ]
Node n3: (x,y) dof free to move                 --> [  True,  True ]
Node n4: (x,y) dof free to move                 --> [  True,  True ]
```

Nodal Loads

```
Node Load: n3
Loads: [ Fx, Fy ] = [ 10.00,   0.00 ]
Node Load: n4
Loads: [ Fx, Fy ] = [  0.00, -10.00 ]
```

Example 3: Analysis of Six-Bar Truss

Destination Array

Matrix: Destination Array

row/col	1	2	3	4
1	0.00000e+00	0.00000e+00	1.00000e+00	0.00000e+00
2	0.00000e+00	0.00000e+00	2.00000e+00	3.00000e+00
3	1.00000e+00	0.00000e+00	4.00000e+00	5.00000e+00
4	2.00000e+00	3.00000e+00	4.00000e+00	5.00000e+00
5	0.00000e+00	0.00000e+00	4.00000e+00	5.00000e+00
6	1.00000e+00	0.00000e+00	2.00000e+00	3.00000e+00

Example 3: Analysis of Six-Bar Truss

Global Stiffness Matrix

Matrix: GlobalStiffness: stiff01

row/col	1	2	3	4	5
1	1.35355e-01	-3.53553e-02	3.53553e-02	0.00000e+00	0.00000e+00
2	-3.53553e-02	1.35355e-01	-3.53553e-02	-1.00000e-01	0.00000e+00
3	3.53553e-02	-3.53553e-02	1.35355e-01	0.00000e+00	0.00000e+00
4	0.00000e+00	-1.00000e-01	0.00000e+00	1.35355e-01	3.53553e-02
5	0.00000e+00	0.00000e+00	0.00000e+00	3.53553e-02	1.35355e-01

External Load Vector

Matrix: Load Vector: eload01

row/col	1
1	0.00000e+00
2	1.00000e+01
3	0.00000e+00
4	0.00000e+00
5	-1.00000e+01

Example 3: Analysis of Six-Bar Truss

Global Displacements

```
Matrix: Displacement Vector (displ01)
row/col                  1
   1        6.03553e+01
   2        2.91421e+02
   3        6.03553e+01
   4        2.51777e+02
   5       -1.39645e+02
```

Example 3: Analysis of Six-Bar Truss

Element-Level Displacements and Member Forces

Elmt 1: Connects nodes n1 and n2

```
Matrix: Member displ ([v] = [a][r]):  Matrix: Member forces: ([S] = [k][v]):
row/col                1              row/col                1
   1        0.00000e+00                  1        -6.03553e+00
   2        6.03553e+01                  2         6.03553e+00

---> Axial force = 6.035534 (T) ...
```

Elmt 2: Connects nodes n1 and n3

```
Matrix: Member displ ([v] = [a][r]):  Matrix: Member forces: ([S] = [k][v]):
row/col                1              row/col                1
   1        0.00000e+00                  1        -6.03553e+00
   2        6.03553e+01                  2         6.03553e+00

---> Axial force = 6.03 (T) ...
```

Example 3: Analysis of Six-Bar Truss

Elmt 3: Connects nodes n2 and n4

```
Matrix: Member displ ([v] = [a][r]):   Matrix: Member forces: ([S] = [k][v]):
row/col                 1              row/col                 1
   1          0.00000e+00                 1          1.39645e+01
   2         -1.39645e+02                 2         -1.39645e+01

---> Axial force = -13.96 (C) ...
```

Elmt 4: Connects nodes n3 and n4

```
Matrix: Member displ ([v] = [a][r]):   Matrix: Member forces: ([S] = [k][v]):
row/col                 1              row/col                 1
   1          2.91421e+02                 1          3.96447e+00
   2          2.51777e+02                 2         -3.96447e+00

---> Axial force = -3.96 (C) ...
```

Example 3: Analysis of Six-Bar Truss

Elmt 5: Connects nodes n1 and n4

```
Matrix: Member displ ([v] = [a][r]):   Matrix: Member forces: ([S] = [k][v]):
row/col                1               row/col                1
   1         0.00000e+00                  1         -5.60660e+00
   2         7.92893e+01                  2          5.60660e+00

---> Axial force = 5.60 (T) ...
```

Elmt 6: Connects nodes n2 and n3

```
Matrix: Member displ ([v] = [a][r]):   Matrix: Member forces: ([S] = [k][v]):
row/col                1               row/col                1
   1        -4.26777e+01                  1          8.53553e+00
   2        -1.63388e+02                  2         -8.53553e+00

---> Axial force = -8.53 (C) ...
```

Example 3: Analysis of Six-Bar Truss

Summary of Member Forces and Support Reactions