Cable Structures

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Overview

Analysis

Part 2: Include Self-Weight of Cable

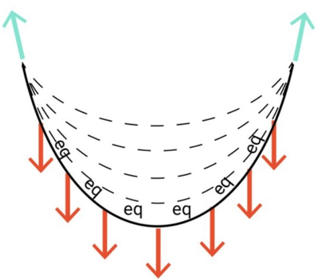
Analysis of Cable Hanging under its own Weight

Motivating Observation: Anyone who has walked across the Golden Gate Bridge knows that the cables are huge.

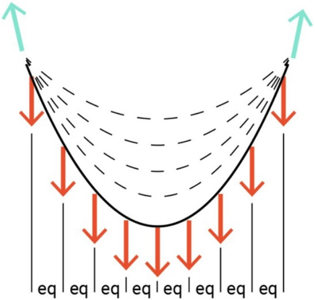
Self-weight of the cables matters!

Analysis of Cable Hanging under its own Weight

Engineering Principle: A cable hanging under its own weight will form a catenary (similar but not the same as a parabola).



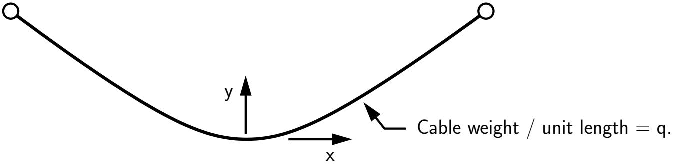
Catenary



Parabola

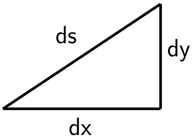
Analysis of Cable Hanging under its own Weight

Equations of Equilibrium:



Consider weight of a small element = q ds.

Weight per unit length in the horizontal direction, q_x is:



$$qds = q_x dx \quad \longrightarrow \quad q_x = q\left(\frac{ds}{dx}\right). \quad (16)$$

Analysis of Cable Hanging under its own Weight

From previous section, we also have:

$$\frac{d^2y}{dx^2} = \left(\frac{q_x}{H}\right). \tag{17}$$

From geometry:

$$ds^2 = dx^2 + dy^2 \tag{18}$$

Plugging 16 and 18 into 17:

$$\frac{d^2y}{dx^2} = \frac{q}{H}\left(\frac{ds}{dx}\right) = \frac{q}{H}\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{1/2}. \tag{19}$$

Analysis of Cable Hanging under its own Weight

Mathematical Solution:

$$y(x) = \frac{1}{2c}\left[e^{cx} + e^{-cx} - 2\right] \tag{20}$$

where $c = \frac{q}{H}$.

Equation 20 is a catenary curve – mathematically it is a hyperbolic cosine function.

Note: From 20 we can expand e^{cx} and e^{-cx} as Taylor series:

$$e^{cx} = 1 + (cx) + \frac{(cx)^2}{2!} + \frac{(cx)^3}{3!} + \cdots \tag{21}$$

$$e^{-cx} = 1 - (cx) + \frac{(cx)^2}{2!} - \frac{(cx)^3}{3!} + \cdots \tag{22}$$

Analysis of Cable Hanging under its own Weight

Add equations 21 and 22:

$$e^{cx} + e^{-cx} = 0. \tag{23}$$

and simplify,

$$y(x) = \frac{q}{H}\left[\frac{x^2}{2} + \left(\frac{q}{H}\right)^2 \frac{x^4}{4!} + \cdots\right]. \tag{24}$$

Conclusion: Now recall from our simplified analysis:

$$H = \frac{qL^2}{8f} \longrightarrow \frac{q}{H} = \frac{8f}{L^2}. \tag{25}$$

Second order terms in 24 will be very small when $8f/L^2 \longrightarrow 0$, i.e., for large spans the catenary and parabola are almost the same!