Cable Structures

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Overview

1. Cable Structures
   - Definition and Motivation
   - Cable Structures vs Arch Structures
2. Types of Cable Structure
   - Suspension Structure, Cable-Stayed Structure, Tension Structure
   - Case Studies
3. Analysis (Part 1: Ignore Self-Weight of Cable)
   - Equations of Equilibrium
4. Analysis (Part 2: Include Self-Weight of Cable)
   - Equations of Equilibrium

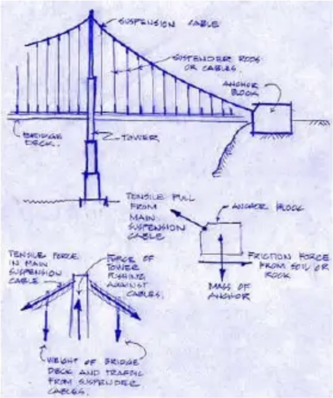
Analysis

Part 1: Ignore Self-Weight of Cable

Simplified Analysis of Cable Structures

Analysis Objectives:

- What are the forces in the cable structure?
- How will the cable shape change with different distributions of live load?
- What are the bending moments in the bridge deck?
- How should the cable be anchored to the foundation?
- What forces will occur in the supporting structure?



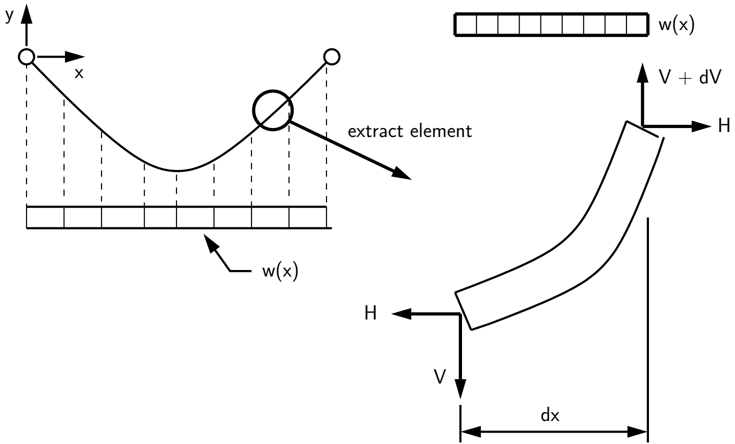
Simplified Analysis of Cable Structures

Key Points:

- Any section of the cable must be in equilibrium,
- Shape depends on loads,
- Relationship between loads and deflections is no longer linear. Superposition does not apply.

Simplified Analysis of Cable Structures

Equilibrium of a Cable Element. Any section of the cable element must be in equilibrium:



Simplified Analysis of Cable Structures

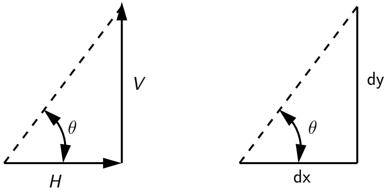
Analysis: Summing forces in the vertical direction:

$$\sum F_y = 0, \quad \longrightarrow \quad V + dV = V + w(x)dx \tag{1}$$

Hence,

$$\frac{dV}{dx} = w(x) \tag{2}$$

Next, notice that because cable can only carry loads in tension ...



Simplified Analysis of Cable Structures

Analysis: continued ...

hence,

$$V = H \tan\theta = H\frac{dy}{dx}. \tag{3}$$

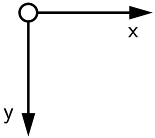
Combining equations 2 and 3:

$$\frac{d^2y}{dx^2} = \left[\frac{w(x)}{H}\right]. \tag{4}$$

Finally, integrate equation 4 and apply boundary conditions.

Simplified Analysis of Cable Structures

Note: If coordinates are defined:

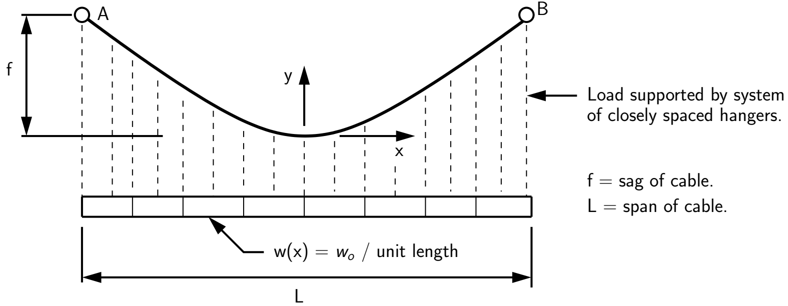


then we have:

$$\frac{d^2y}{dx^2} = \left[\frac{-w(x)}{H}\right]. \tag{5}$$

Simplified Analysis of Cable Structures

Example 1: Constant Uniform Horizontal Loading



f = sag of cable.

L = span of cable.

Analysis: Cable profile is defined by:

$$\frac{d^2y}{dx^2} = \left[\frac{w_o}{H}\right]. \tag{6}$$

Simplified Analysis of Cable Structures

General Solution:

$$y(x) = \frac{w_o x^2}{2H} + C_1 x + C_2. \tag{7}$$

Apply Boundary Conditions:

- y(0) = 0 $\longrightarrow C_2 = 0$, and $\left.\frac{dy}{dx}\right|_{x=0} = 0 \longrightarrow C_1 = 0.$

Equation 7 simplifies to:

$$y(x) = \frac{w_o x^2}{2H} \quad \longleftarrow \quad \text{parabola!!} \tag{8}$$

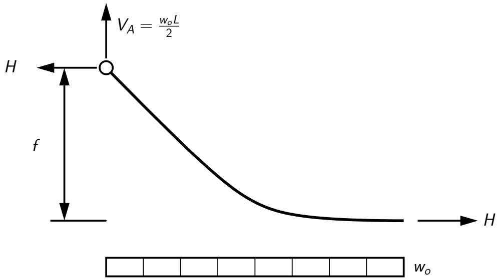
Also y = f at x = $\pm\ \frac{L}{2}$, hence,

$$H = \frac{w_o L^2}{8f} \quad \longleftarrow \quad \text{horizontal component of cable force.} \tag{9}$$

Simplified Analysis of Cable Structures

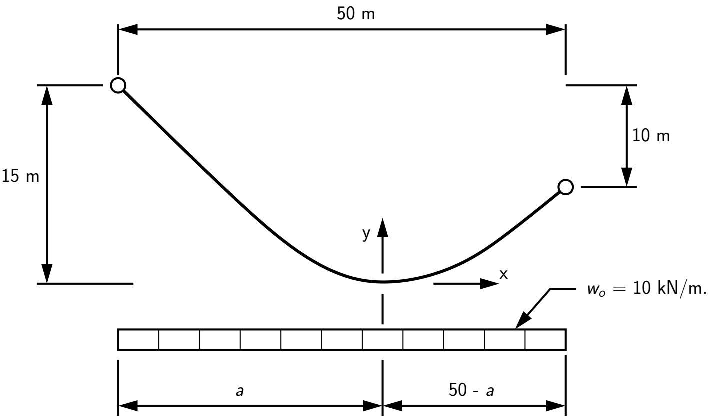
Points to note:

- We have positioned the coordinate system to simplify the math.
- H is constant along the structure.
- Equation 9 can be verified by looking at the equilibrium of half a structure:



Simplified Analysis of Cable Structures

Example 2: Find Location of Cable Profile Minimum



Simplified Analysis of Cable Structures

Analysis: From example 1, we know that the cable shape will be a parabola $y(x) = \mathrm{k}\ x^2$.

For the left- and right-hand sides:

$$15 = ka^2 \qquad \text{(left-hand side)} \tag{10}$$

$$5 = k(50 - a)^2 \qquad \text{(right-hand side)} \tag{11}$$

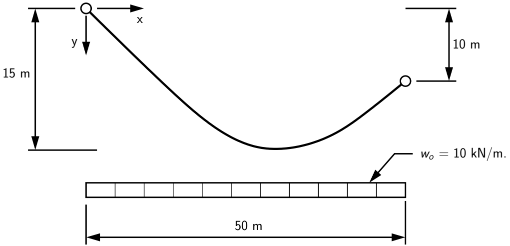
Divide equation 10 by 11:

$$\frac{15}{5} = \frac{a^2}{(50 - a)^2} \quad \longrightarrow \quad \text{find a.} \tag{12}$$

Use statics to solve for vertical reaction forces and horizontal cable force.

Simplified Analysis of Cable Structures

Alternate Method: Suppose origin is at upper left-hand support:



Cable shape is given by:

$$\frac{d^2y}{dx^2} = \left[\frac{-w_o}{H}\right]. \tag{13}$$

Boundary conditions are: y(0) = 0, y(50) = 10. Then use extra

Simplified Analysis of Cable Structures

information on max dip to find H.

Note on Loading Patterns:

When w(x) = w_o (constant), integrate twice,

$$\frac{d^2y}{dx^2} = \left[\frac{w_o}{H}\right] \quad \longrightarrow \quad \text{quadratic cable shape.} \tag{14}$$

When w(x) = kx (linear function, integrate twice,

$$\frac{d^2y}{dx^2} = \left[\frac{kx}{H}\right] \quad \longrightarrow \quad \text{cubic cable shape.} \tag{15}$$