Cable Structures

Mark A. Austin

University of Maryland

austin@umd.edu

ENCE 353, Fall Semester 2025

October 19, 2025

Overview

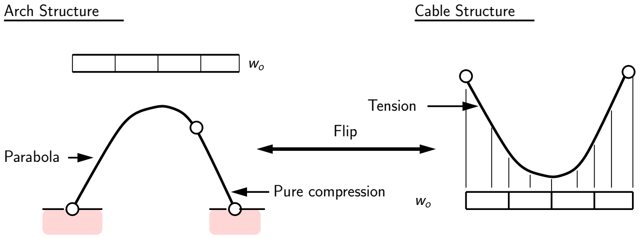
Cable Structures

Cable Structures

Cable

A cable is a flexible structure that cannot resist bending. The cable assumes a shape to carry the loads by tension alone.

Motivating Observation: Flip an arch upside down ...



cable elements carry loads in tension and form a parabolic shape.

Cable Structures vs Arch Structures

Arch Structures

Designer prescribes the shape of the arch, positions of hinges, loading patterns, and boundary conditions. Use analysis to find support reactions and distributions of bending moments, shear forces, axial forces.

Cable Structures

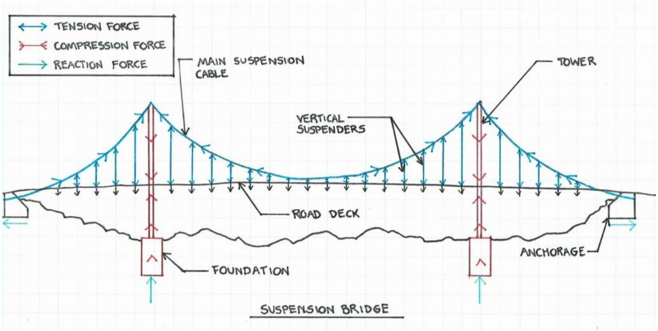
Designer prescribes the length of the cable, position of supports, and loading patterns. Shape of the cable system is determined by loads and physics (i.e., solution to the underlying differential equations).

Types of Cable Structure

Types of Cable Structure

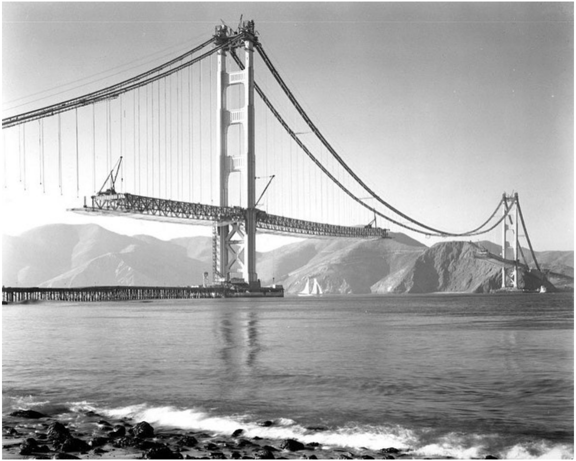
Suspension Structures

Characterized by a main cable suspended between two anchor points, with the load distributed along the length of the cable (e.g., suspension bridges and roofs).



Case Studies

Suspension: Construction of the Golden Gate Bridge (1935):



Case Studies

Suspension: Cable roof structure at Dulles Airport:

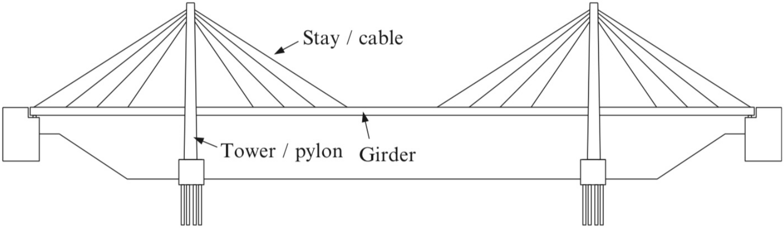
Case Studies

Suspension: Power lines ...

Types of Cable Structure

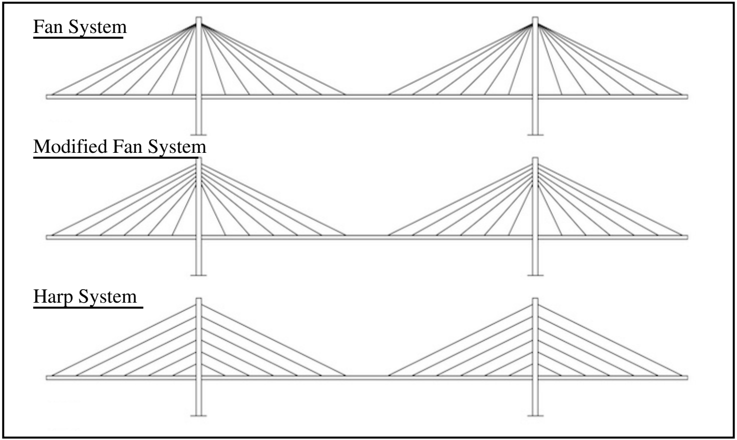
Cable-Stayed Structures

Cables are directly connected to the deck or structure, providing support and stability (e.g., cable-stayed bridges).



Types of Cable Structure

Types of Cable-Stayed Bridge:



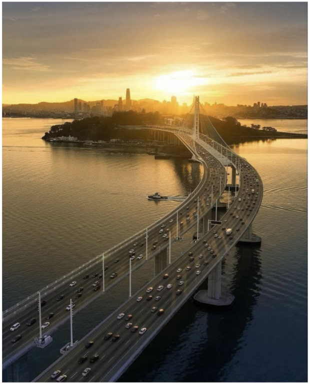
Case Studies

Cable-Stayed: Port Mann Bridge (Vancouver, Canada) (2012)

Construction: Kiewit Corporation, 2009-2015

Case Studies

Cable-Stayed: Oakland-Bay Bridge, East Span (2013)




Construction: Kiewit Corporation
Original budget: $250M; Final budget: $6.5B.

Types of Cable Structure

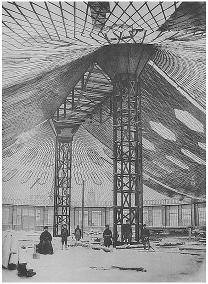
Tensile Structures

Cables serve as the primary load-bearing elements, often in combination with membranes or other materials (e.g., architectural designs for roofs and façades).

Benefits: Ideal for spanning large areas and for temporary (or weather protective) coverings.

Applications: Cable-stayed bridges, airport canopys, tensile canopies/awnings.

Constructed: 1895



Case Studies

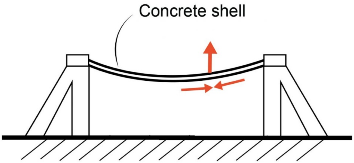
Tensile Structure: Denver International Airport

Case Studies

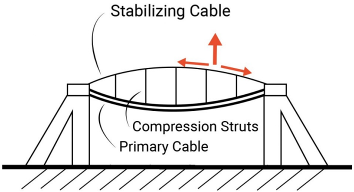
Tensile Structure: Brasilia National Stadium (2014)

Case Studies

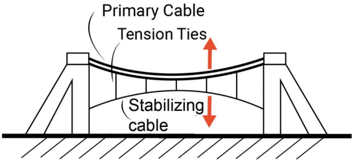
Suspension and Tension Working Together ...



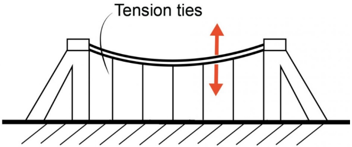
Stiffening through construction as an inverted arch (or shell)



Spreading against a cable with opposite curvature



Tensioning against a cable with opposite curvature



Fastening with transverse cables anchored

Case Studies

Tensile Structure: Suspension Bridge Structure in NZ.

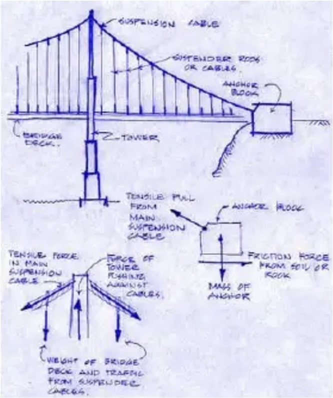
Analysis

Part 1: Ignore Self-Weight of Cable

Simplified Analysis of Cable Structures

Analysis Objectives:

- What are the forces in the cable structure?
- How will the cable shape change with different distributions of live load?
- What are the bending moments in the bridge deck?
- How should the cable be anchored to the foundation?
- What forces will occur in the supporting structure?



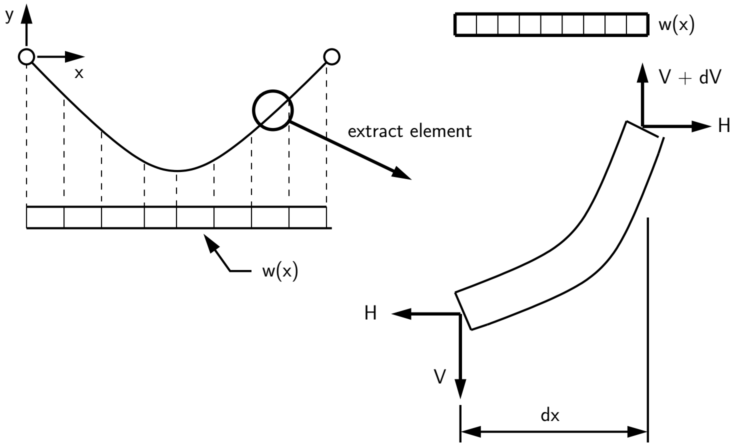
Simplified Analysis of Cable Structures

Key Points:

- Any section of the cable must be in equilibrium,
- Shape depends on loads,
- Relationship between loads and deflections is no longer linear. Superposition does not apply.

Simplified Analysis of Cable Structures

Equilibrium of a Cable Element. Any section of the cable element must be in equilibrium:



Simplified Analysis of Cable Structures

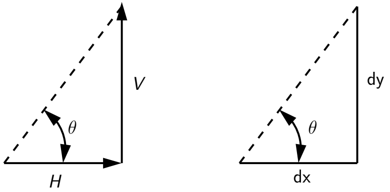
Analysis: Summing forces in the vertical direction:

$$\sum F_y = 0, \quad \longrightarrow \quad V + dV = V + w(x)dx \tag{1}$$

Hence,

$$\frac{dV}{dx} = w(x) \tag{2}$$

Next, notice that because cable can only carry loads in tension ...



Simplified Analysis of Cable Structures

Analysis: continued ...

hence,

$$V = H \tan\theta = H \frac{dy}{dx}. \tag{3}$$

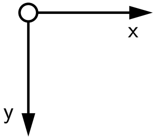
Combining equations 2 and 3:

$$\frac{d^2y}{dx^2} = \left[\frac{w(x)}{H} \right]. \tag{4}$$

Finally, integrate equation 4 and apply boundary conditions.

Simplified Analysis of Cable Structures

Note: If coordinates are defined:

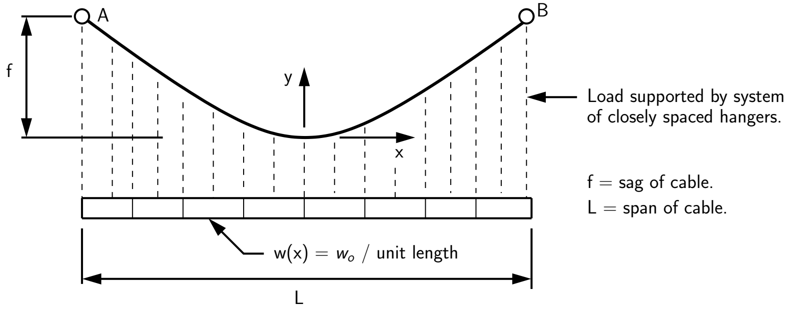


then we have:

$$\frac{d^2y}{dx^2} = \left[\frac{-w(x)}{H}\right]. \tag{5}$$

Simplified Analysis of Cable Structures

Example 1: Constant Uniform Horizontal Loading



Analysis: Cable profile is defined by:

$$\frac{d^2y}{dx^2} = \left[\frac{w_o}{H} \right]. \tag{6}$$

Simplified Analysis of Cable Structures

General Solution:

$$y(x) = \frac{w_o x^2}{2H} + C_1 x + C_2. \tag{7}$$

Apply Boundary Conditions:

- $y(0) = 0 \longrightarrow C_2 = 0$, and $\left.\frac{dy}{dx}\right|_{x=0} = 0 \longrightarrow C_1 = 0.$

Equation 7 simplifies to:

$$y(x) = \frac{w_o x^2}{2H} \quad \longleftarrow \quad \text{parabola!!} \tag{8}$$

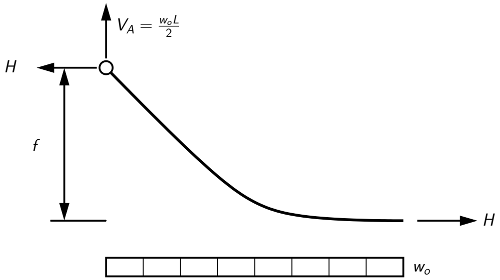
Also $y = f$ at $x = \pm \frac{L}{2}$, hence,

$$H = \frac{w_o L^2}{8f} \quad \longleftarrow \quad \text{horizontal component of cable force.} \tag{9}$$

Simplified Analysis of Cable Structures

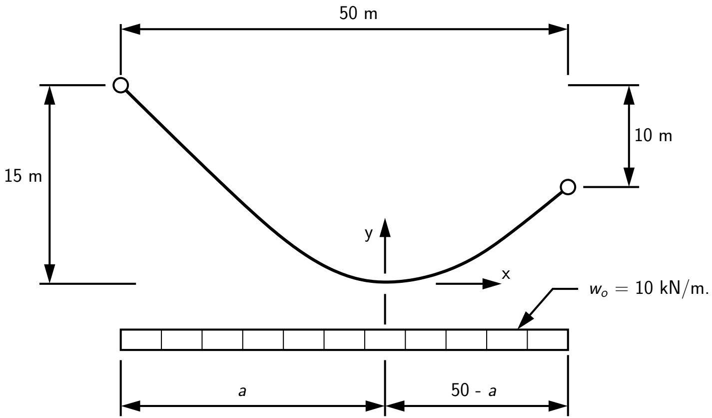
Points to note:

- We have positioned the coordinate system to simplify the math.
- H is constant along the structure.
- Equation 9 can be verified by looking at the equilibrium of half a structure:



Simplified Analysis of Cable Structures

Example 2: Find Location of Cable Profile Minimum



Simplified Analysis of Cable Structures

Analysis: From example 1, we know that the cable shape will be a parabola $y(x) = \mathrm{k}\ x^2$.

For the left- and right-hand sides:

$$15 = ka^2 \qquad \text{(left-hand side)} \tag{10}$$

$$5 = k(50-a)^2 \qquad \text{(right-hand side)} \tag{11}$$

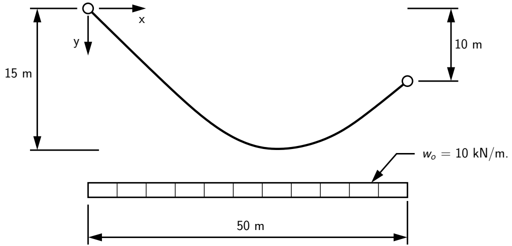
Divide equation 10 by 11:

$$\frac{15}{5} = \frac{a^2}{(50-a)^2} \quad \longrightarrow \quad \text{find a.} \tag{12}$$

Use statics to solve for vertical reaction forces and horizontal cable force.

Simplified Analysis of Cable Structures

Alternate Method: Suppose origin is at upper left-hand support:



Cable shape is given by:

$$\frac{d^2y}{dx^2} = \left[\frac{-w_o}{H}\right]. \qquad (13)$$

Boundary conditions are: y(0) = 0, y(50) = 10. Then use extra

Simplified Analysis of Cable Structures

information on max dip to find H.

Note on Loading Patterns:

When w(x) = w_o (constant), integrate twice,

$$\frac{d^2y}{dx^2} = \left[\frac{w_o}{H}\right] \quad \longrightarrow \quad \text{quadratic cable shape.} \tag{14}$$

When w(x) = kx (linear function, integrate twice,

$$\frac{d^2y}{dx^2} = \left[\frac{kx}{H}\right] \quad \longrightarrow \quad \text{cubic cable shape.} \tag{15}$$

Analysis

Part 2: Include Self-Weight of Cable

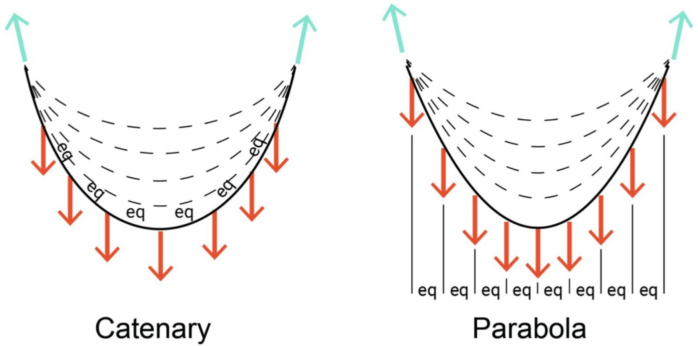
Analysis of Cable Hanging under its own Weight

Motivating Observation: Anyone who has walked across the Golden Gate Bridge knows that the cables are huge.

Self-weight of the cables matters!

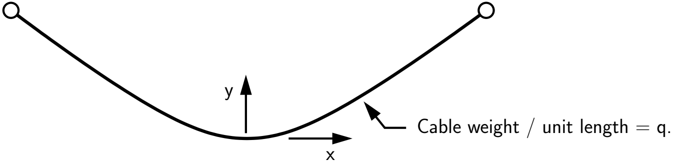
Analysis of Cable Hanging under its own Weight

Engineering Principle: A cable hanging under its own weight will form a catenary (similar but not the same as a parabola).



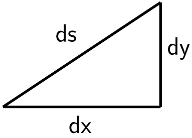
Analysis of Cable Hanging under its own Weight

Equations of Equilibrium:



Consider weight of a small element = q ds.

Weight per unit length in the horizontal direction, q_x is:



$$qds = q_x dx \quad \longrightarrow \quad q_x = q \left(\frac{ds}{dx} \right). \quad (16)$$

Analysis of Cable Hanging under its own Weight

From previous section, we also have:

$$\frac{d^2y}{dx^2} = \left(\frac{q_x}{H}\right). \qquad (17)$$

From geometry:

$$ds^2 = dx^2 + dy^2 \qquad (18)$$

Plugging 16 and 18 into 17:

$$\frac{d^2y}{dx^2} = \frac{q}{H}\left(\frac{ds}{dx}\right) = \frac{q}{H}\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{1/2}. \qquad (19)$$

Analysis of Cable Hanging under its own Weight

Mathematical Solution:

$$y(x) = \frac{1}{2c}\left[e^{cx} + e^{-cx} - 2\right] \tag{20}$$

where $c = \frac{q}{H}$.

Equation 20 is a catenary curve – mathematically it is a hyperbolic cosine function.

Note: From 20 we can expand e^{cx} and e^{-cx} as Taylor series:

$$e^{cx} = 1 + (cx) + \frac{(cx)^2}{2!} + \frac{(cx)^3}{3!} + \cdots \tag{21}$$

$$e^{-cx} = 1 - (cx) + \frac{(cx)^2}{2!} - \frac{(cx)^3}{3!} + \cdots \tag{22}$$

Analysis of Cable Hanging under its own Weight

Add equations 21 and 22:

$$e^{cx} + e^{-cx} = 0. \tag{23}$$

and simplify,

$$y(x) = \frac{q}{H}\left[\frac{x^2}{2} + \left(\frac{q}{H}\right)^2 \frac{x^4}{4!} + \cdots\right]. \tag{24}$$

Conclusion: Now recall from our simplified analysis:

$$H = \frac{qL^2}{8f} \longrightarrow \frac{q}{H} = \frac{8f}{L^2}. \tag{25}$$

Second order terms in 24 will be very small when $8f/L^2 \longrightarrow 0$, i.e., for large spans the catenary and parabola are almost the same!