ENCE 353 Solutions to Midterm 1

Question 1 (10 points): Support Reactions and Bending Moments in a connected Beam Structure.

Consider the multi-span beam structure shown in Figure 1.

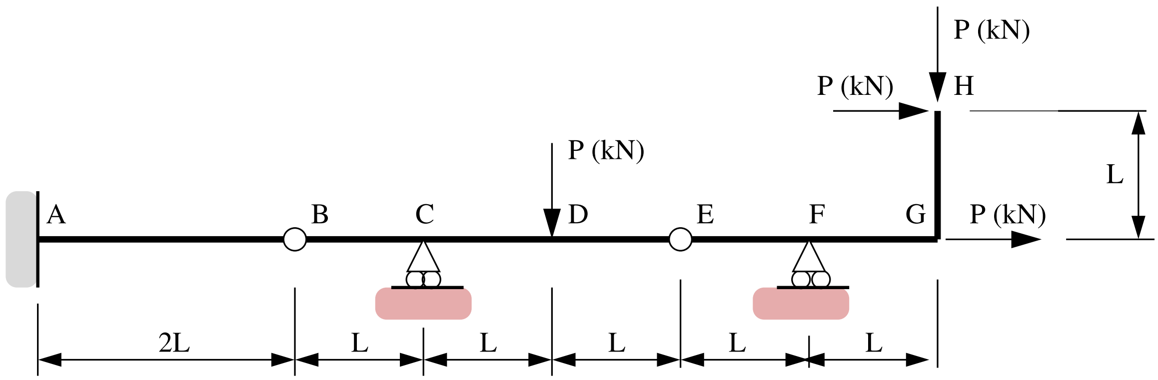


Figure 1. Front elevation view of multi-span beam structure.

The cantilever is fully-fixed to the wall at Point A. Points B and E are hinges. Horizontal and vertical point loads **P (kN)** are applied at points D, G and H.

Part [1a] (2 pts). Compute the degree of indeterminacy for the beam structure.

Sol'n: f = 5, r = 2 releases, hence:

$$\hat{i} = f - 3 - r = 5 - 3 - 2 = 0 \quad \longrightarrow \quad \text{statically determinate.} \tag{1}$$

Part [1b] (2 pts). Compute the vertical reaction forces at C and F.

Sol'n: Bending moment at E will be zero, hence:

$$\sum M_e = 0, \quad \longrightarrow \quad PL + 2PL - V_f L = 0, \quad \longrightarrow \quad V_f = 3P. \tag{2}$$

Summing forces in the vertical direction for E-F-G $\longrightarrow$ shear force across the hinge at E will be $2P$.

Next, take moments about B to find vertical reaction force at C.

$$\sum M_b = 0, \quad \longrightarrow \quad V_c L + 2P(3L) - 2PL = 0, \quad \longrightarrow \quad V_c = -4P. \tag{3}$$

Also, the shear force across the hinge at B will be $3P$.

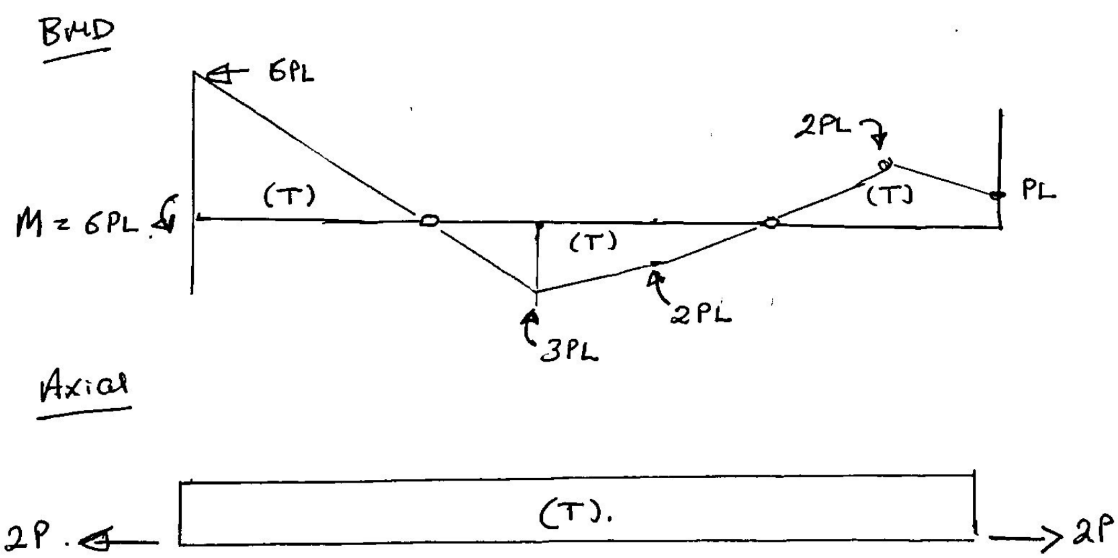
Part [1c] (2 pts). Show that the **total force** at hinge B is $\sqrt{13}$P.

Sol'n: Axial force at B = 2P, shear force at B = 3P, hence:

$$\text{Total force} = \left[2^2 + 3^2\right]^{1/2} = \sqrt{13}P. \tag{4}$$

Part [1d] (4 pts). Draw and label diagrams showing how the **bending moment** and **axial force** vary along the beam, nodes A through G. Clearly indicate on your bending moment diagram, regions that are in tension/compression.

Sol'n: ...



Question 2 (10 points): Method of Sections.

The cantilevered frame structure carries vertical loads P kN at notes D, F, H and I. Use the **method of sections** to determine the forces in members a, b and c, as a function of the problem parameters L and P.

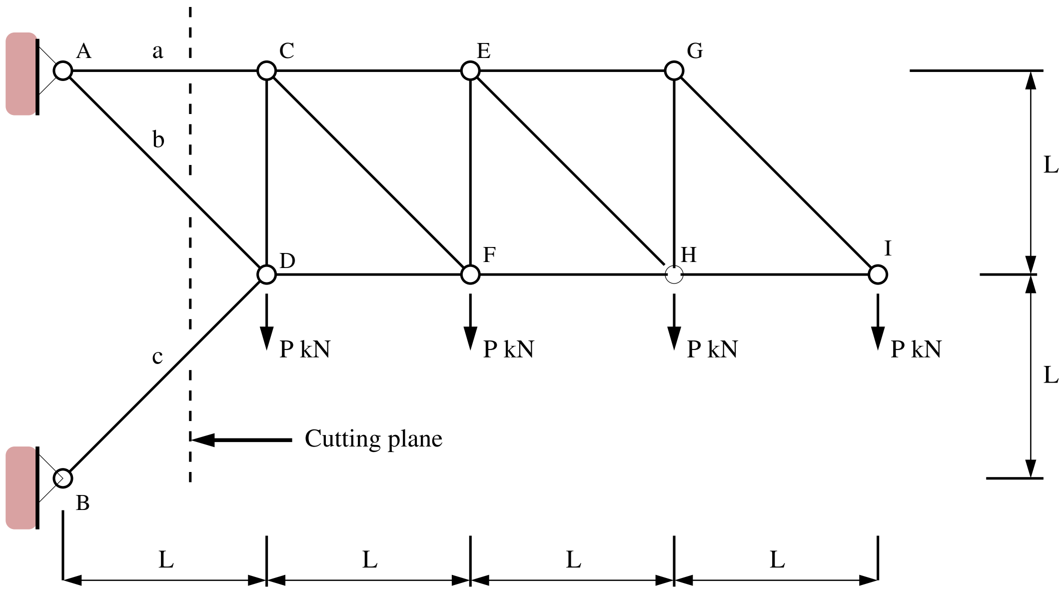


Figure 2. Cantilevered frame structure.

State if the members are in tension or compression.

Sol'n: We begin by cutting the structure as shown in Figure 2.

Taking moments about D for the right-hand substructure gives:

$$\sum M_D = 0 \qquad PL + 2PL + 3PL = LF_{ac} \longrightarrow F_{ac} = 6P(T). \tag{5}$$

Taking moments about A for the complete structure gives:

$$\sum M_A = 0 \qquad PL + 2PL + 3PL + 4PL = 2LH_b \longrightarrow H_b = 5P. \tag{6}$$

Next, observe that the orientation of element B-D is $\pi/4$ radians. Hence, the horizontal and vertical components of reaction force at B need to be equal, i.e., V_b = 5P and $F_{bd} = -5\sqrt{2}P(C)$.

Looking at the equilibrium of the structure at A, the horizontal and vertical reaction forces are: V_a = -P and H_a = -5P.

Summing forces in the vertical direction at A, member force $F_{ad} = -\sqrt{2}P(C)$.

Summary of Results: $F_{ac} = 6P(T)$, $F_{bd} = -5\sqrt{2}P(C)$, $F_{ad} = -\sqrt{2}P(C)$.

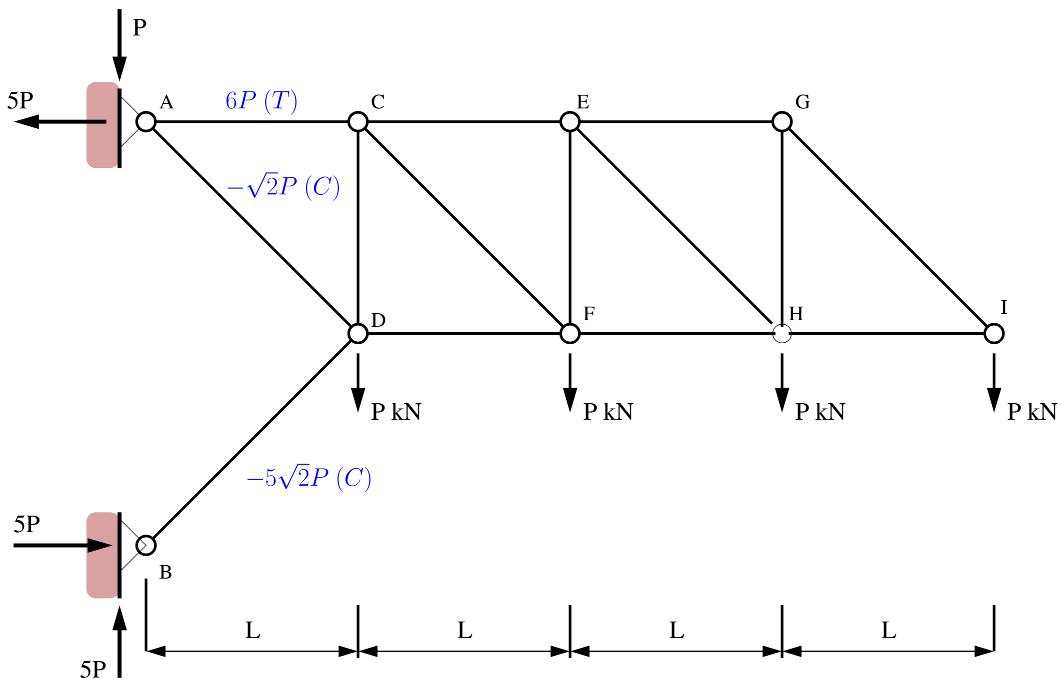


Figure 3. Critical forces in cantilevered frame structure.

Question 3 (20 points): Analysis of a Zigzag Truss Structure.

Consider the zigzag truss structure shown in Figure 4.

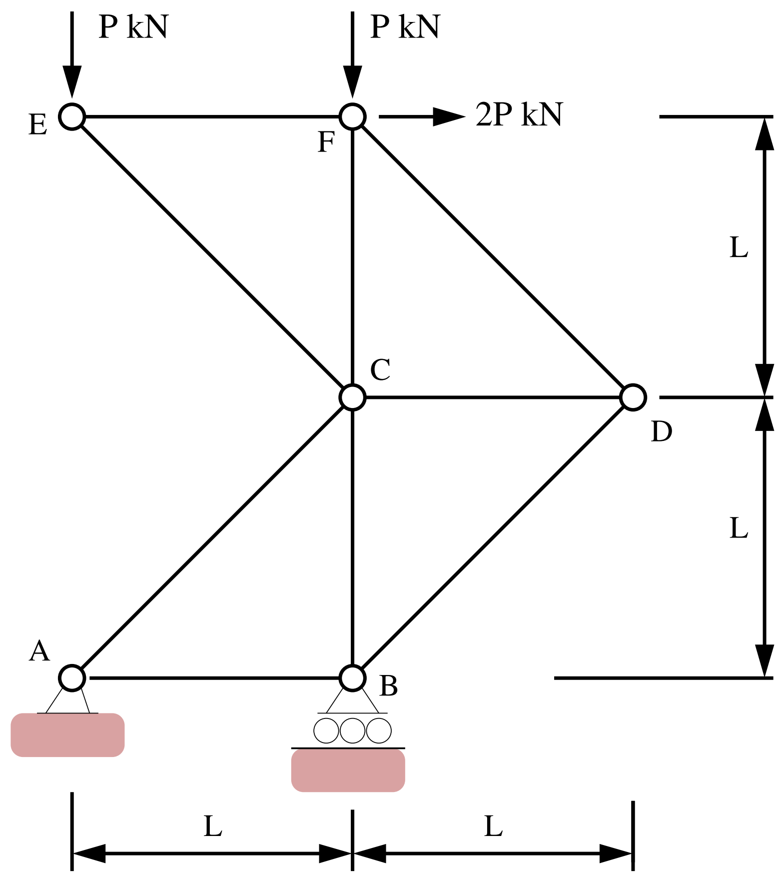


Figure 4. Elevation view of a crooked truss structure.

Horizontal and vertical loads **P** (kN) and **2P** (kN) are applied at nodes E and F as shown in the figure.

Part [3a] (3 pts). Compute the degree on statical indeterminacy for this system.

Sol'n: m = 9, r = 3, j = 6. Hence,

$$m + r - 2j = 9 + 3 - 2(6) = 0, \quad \longrightarrow \quad \text{statically determinate.} \qquad (7)$$

Part [3b] (4 pts). Compute the **total support reactions** at A and B.

Sol'n: Taking moments about A,

$$\sum M_A = 0, \quad PL + 2P(2L) - V_B L = 0, \longrightarrow V_B = 5P. \tag{8}$$

Sum forces in the vertical direction:

$$\sum V = 0, \quad V_A + V_B - 2P = 0, \longrightarrow V_A = -3P. \tag{9}$$

Sum forces in the horizontal direction:

$$\sum H = 0, \quad H_A + 2P = 0, \longrightarrow H_A = -2P. \tag{10}$$

Total support reaction at A = $\left[V_A^2 + H_A^2\right]^{1/2} = \sqrt{13}$ kN. Total support reaction at B = 5 kN.

Part [3c] (8 pts). Using the method of joints (or otherwise) compute the distribution of tension and compression forces throughout the structure. Show all of your working.

Joint A:

$$\sum V = 0, \quad V_A + \frac{F_{ac}}{\sqrt{2}} = 0, \longrightarrow F_{ac} = 3\sqrt{2}P\ (T). \tag{11}$$

$$\sum H = 0, \quad H_A + \frac{F_{ac}}{\sqrt{2}} + F_{ab} = 0, \longrightarrow F_{ab} = -P\ (C). \tag{12}$$

Joint E:

$$\sum V = 0, \quad -\frac{F_{ce}}{\sqrt{2}} - P = 0, \longrightarrow F_{ce} = -\sqrt{2}P\ (C). \tag{13}$$

$$\sum H = 0, \quad \frac{F_{ce}}{\sqrt{2}} + F_{ef} = 0, \longrightarrow F_{ef} = P\ (T). \tag{14}$$

Joint B:

$$\sum H = 0, \quad \frac{F_{bd}}{\sqrt{2}} - F_{ab} = 0, \longrightarrow F_{bd} = -\sqrt{2}P\ (C). \tag{15}$$

$$\sum V = 0, \quad \frac{F_{bd}}{\sqrt{2}} + F_{bc} + V_B = 0, \longrightarrow F_{bc} = -4P\ (C). \tag{16}$$

Joint D:

$$\sum V = 0, \quad \frac{F_{bd}}{\sqrt{2}} - \frac{F_{df}}{\sqrt{2}} = 0, \longrightarrow F_{df} = -\sqrt{2}P\ (C). \tag{17}$$

$$\sum H = 0, \quad F_{cd} + \frac{F_{bd}}{\sqrt{2}} + \frac{F_{df}}{\sqrt{2}} = 0, \longrightarrow F_{cd} = 2P\ (T). \tag{18}$$

Joint F:

$$\sum H = 0, \quad -F_{ef} + \frac{F_{df}}{\sqrt{2}} + 2 = 0. \tag{19}$$

$$\sum V = 0, \quad -F_{cf} - \frac{F_{df}}{\sqrt{2}} - P = 0, \longrightarrow F_{cf} = 0. \tag{20}$$

Joint C: Can use this joint to verify the truss is in equilibrium.

Part [3d] (5 pts). Now suppose that the maximum tensile force any member can support is 10 kN, and that the maximum allowable compressive force is:

$$P_{ci} = 5\left(\frac{L}{L_i}\right)^2 kN, \tag{21}$$

where L_i is the length of the i-th element, and P_{ci} is the maximum allowable compressive force of the i-th element before buckling. Determine the maximum value of P (kN) that the tower can safely carry.

Sol'n: From the analysis: (a) Maximum tensile force is: $3\sqrt{2}P$ (T), (b) Maximum compressive force is: $-4P$ (C).

Tension constraint in F_{ac}:

$$3\sqrt{2}P \leq 10\text{kN} \quad \longrightarrow \quad P \leq 2.35\text{kN} \tag{22}$$

Compression constraint in F_{bc}:

$$4P \leq 5\text{kN} \quad \longrightarrow \quad P \leq 1.25\text{kN} \tag{23}$$

The most restrictive constraint is buckling of element BC. The structure will fail once P exceeds 1.25 kN.

Summary: Distribution of tensile, compressive, and zero-force members: