Department of Civil and Environmental Engineering, Spring Semester, 2025

ENCE 353 Midterm 2, Open Notes and Open Book

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Exam Format and Grading. This take home midterm exam is open notes and open book. You need to comply with the university regulations for academic integrity.

There are three questions. Partial credit will be given for partially correct answers, so please show all your working.

Please see the **class web page for instructions on how to submit your exam paper**.

Question	Points	Score
1	15	
2	15	
3	10	
Total	40	

Question 1: 15 points

Moment-Area and Deflections. Consider the cantilevered beam structure shown in Figure 1.

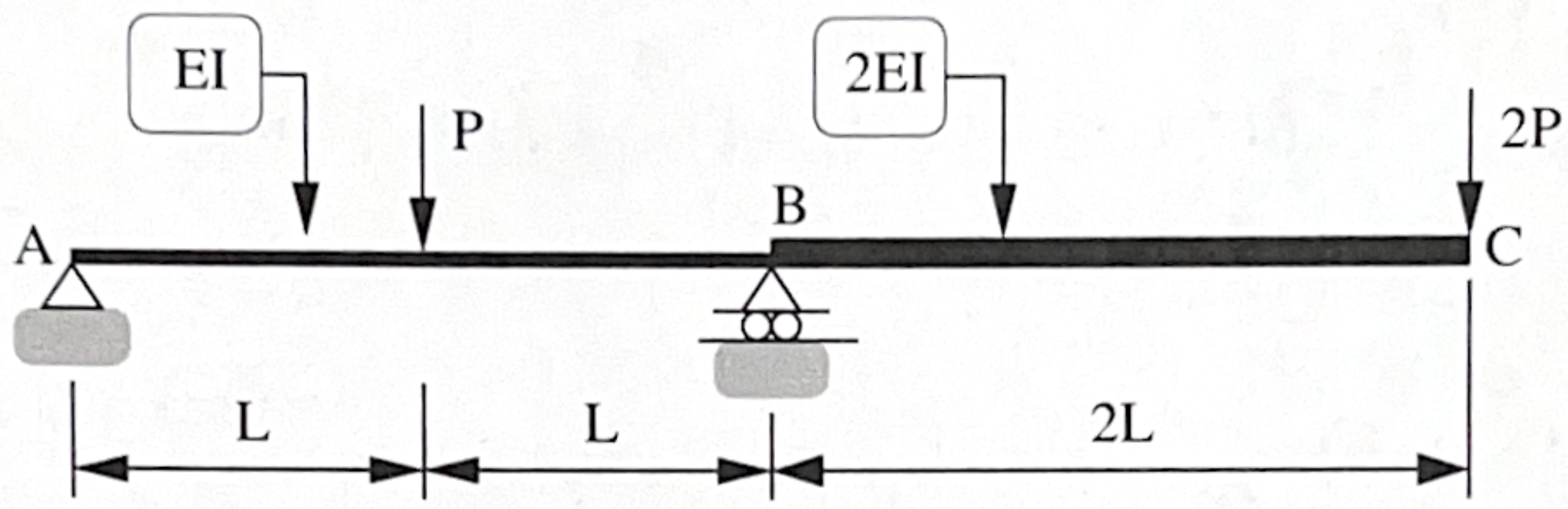
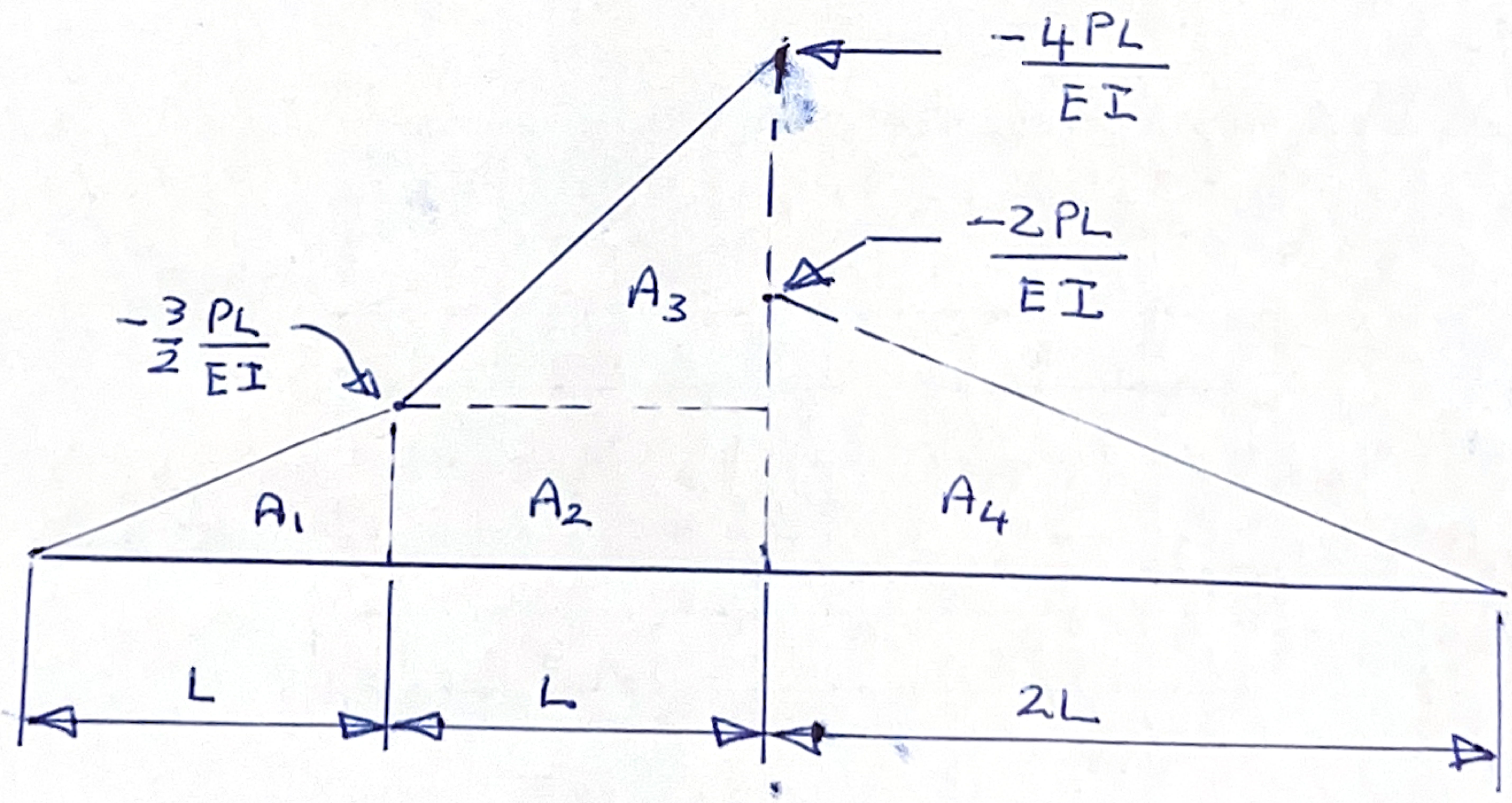


Figure 1: Front elevation view of a cantilevered beam structure.

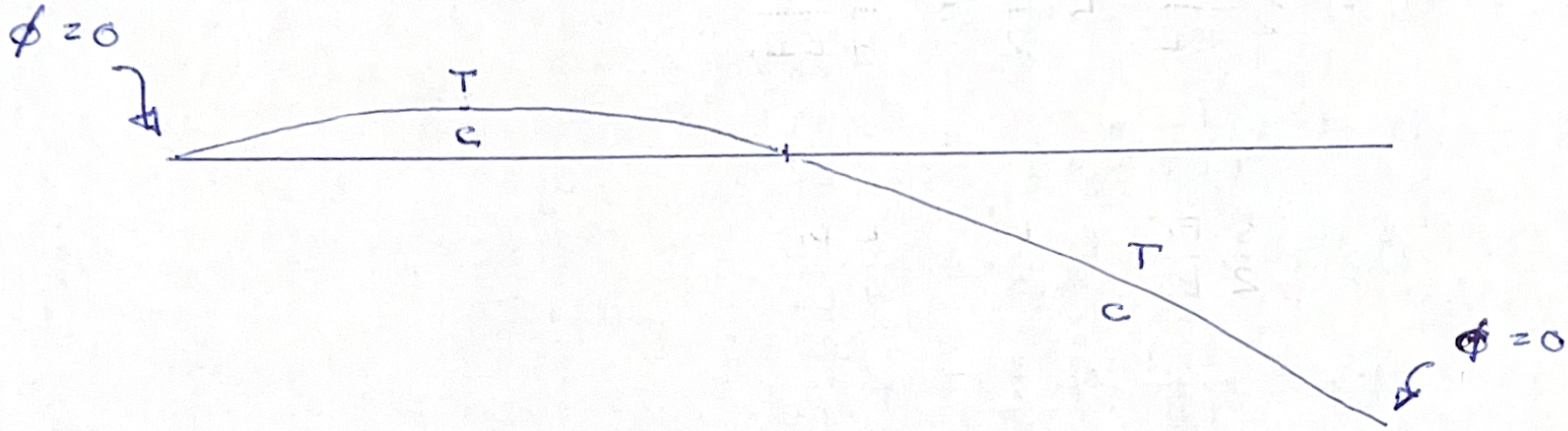
Notice that segments A-B and B-C have cross-sectional properties EI and 2EI, respectively.

[1a] (3 pts) Compute and draw the M(x)/EI diagram for the complete beam A-B-C.

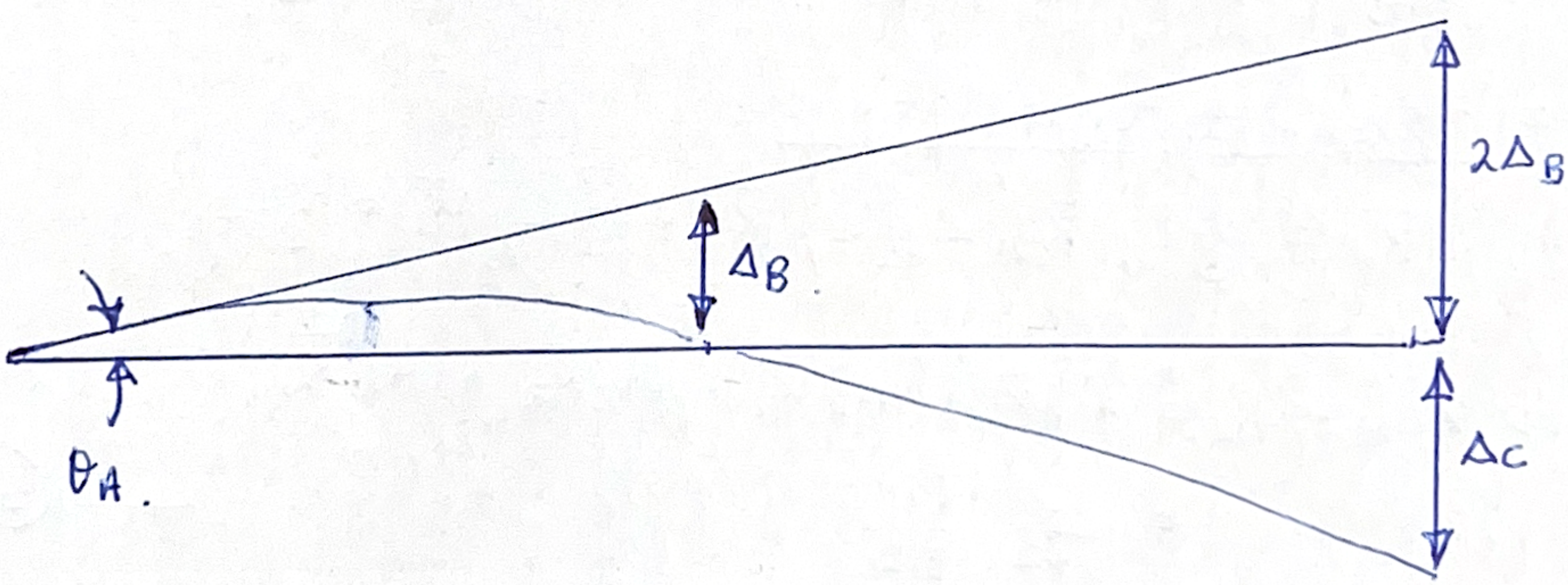
From equilibrium: $V_B = \frac{9}{2}P$, $V_A = -\frac{3}{2}P$.



[1b] (4 pts) Draw and label a diagram of the deflected shape. Clearly indicate on your diagram regions (or points) of the beam having zero curvature.



[1c] (4 pts) Draw and label a diagram showing how the rotation at A is related to the beam deflections at points B and C.



[1d] (4 pts) Use the method of moment-area to compute the vertical deflection of the beam at point C.

$$A_1 = \frac{3}{2}\frac{PL}{EI} \cdot L \cdot \frac{1}{2} = \frac{3}{4}\frac{PL^2}{EI}.$$

$$A_2 = \frac{3}{2}\frac{PL}{EI} \cdot L = \frac{3}{2}\frac{PL^2}{EI}$$

$$A_3 = \frac{5}{2}\frac{PL}{EI} \cdot L \cdot \frac{1}{2} = \frac{5}{4}\frac{PL^2}{EI}$$

$$A_4 = \frac{2PL}{EI} \cdot 2L \cdot \frac{1}{2} = 2\frac{PL^2}{EI}.$$

<u>For Δ_B calculation:</u>

$$\bar{x}_1 = \frac{4}{3}L, \quad \bar{x}_2 = \frac{L}{2}, \quad \bar{x}_3 = L/3.$$

$$\Rightarrow \Delta_B = A_1\bar{x}_1 + A_2\bar{x}_2 + A_3\bar{x}_3 = \frac{26}{12}\frac{PL^3}{EI}. \quad \text{— } (A)$$

<u>For $2\Delta_B + \Delta_C$ calculation:</u>

$$\bar{x}_1 = \frac{10}{3}L, \quad \bar{x}_2 = \frac{5}{2}L, \quad \bar{x}_3 = \frac{7}{3}L, \quad \bar{x}_4 = \frac{4}{3}L.$$

$$\Rightarrow 2\Delta_B + \Delta_C = A_1\bar{x}_1 + A_2\bar{x}_2 + A_3\bar{x}_3 + A_4\bar{x}_4$$

$$= \frac{142}{12}\frac{PL^3}{EI}. \quad \text{— } (B)$$

<u>Combining equations (A) & (B):</u>

$$\Delta_C = \left[\frac{142 - 52}{12}\right]\frac{PL^3}{EI} = \frac{15}{2}\frac{PL^3}{EI}.$$

Question 2: 15 points

Moment-Area and Rotational Deflections. The simple beam shown in Figure 2 has length L and uniform section properties EI. A point load P is applied at distance a from the left-hand support.

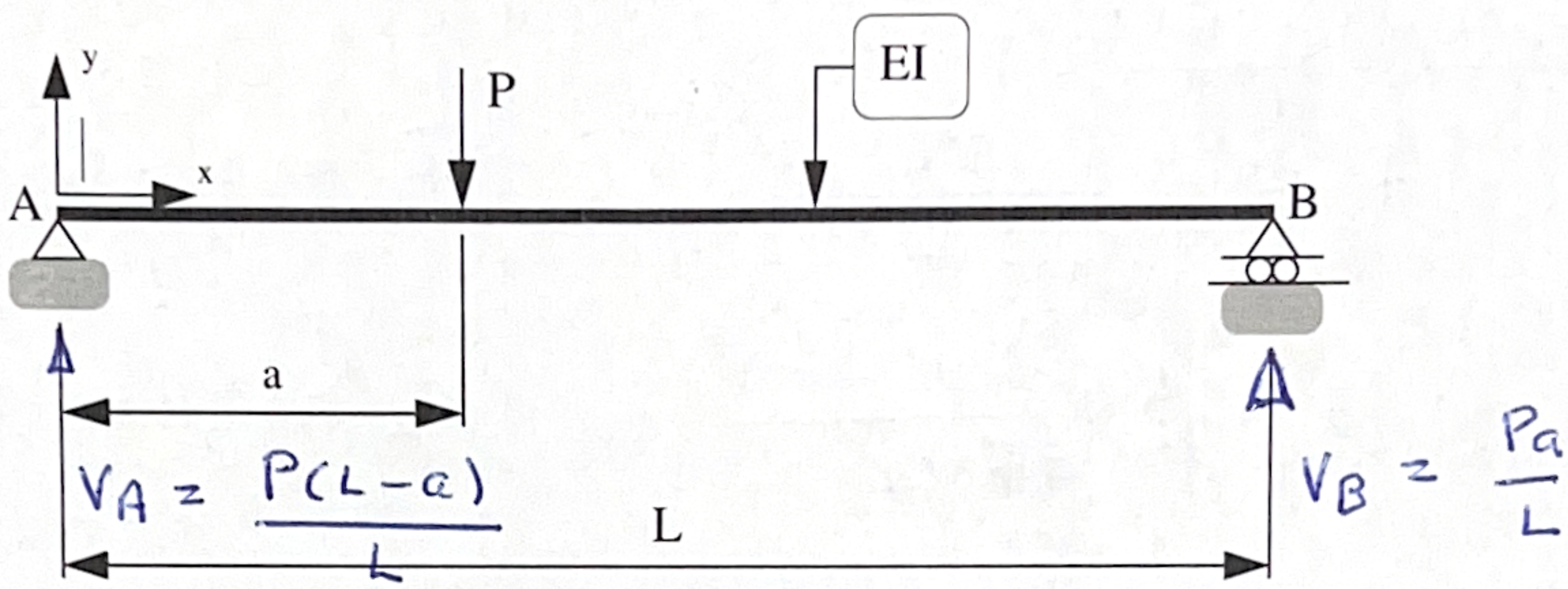
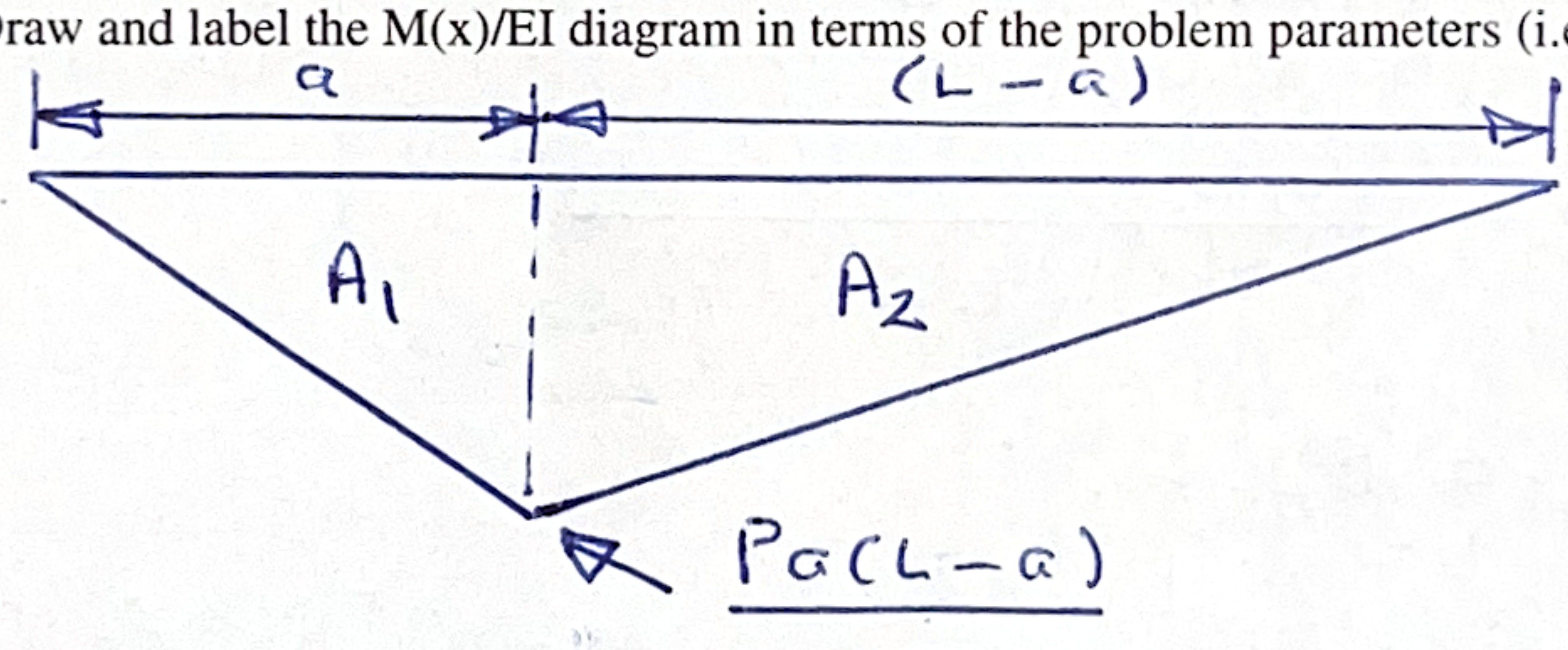


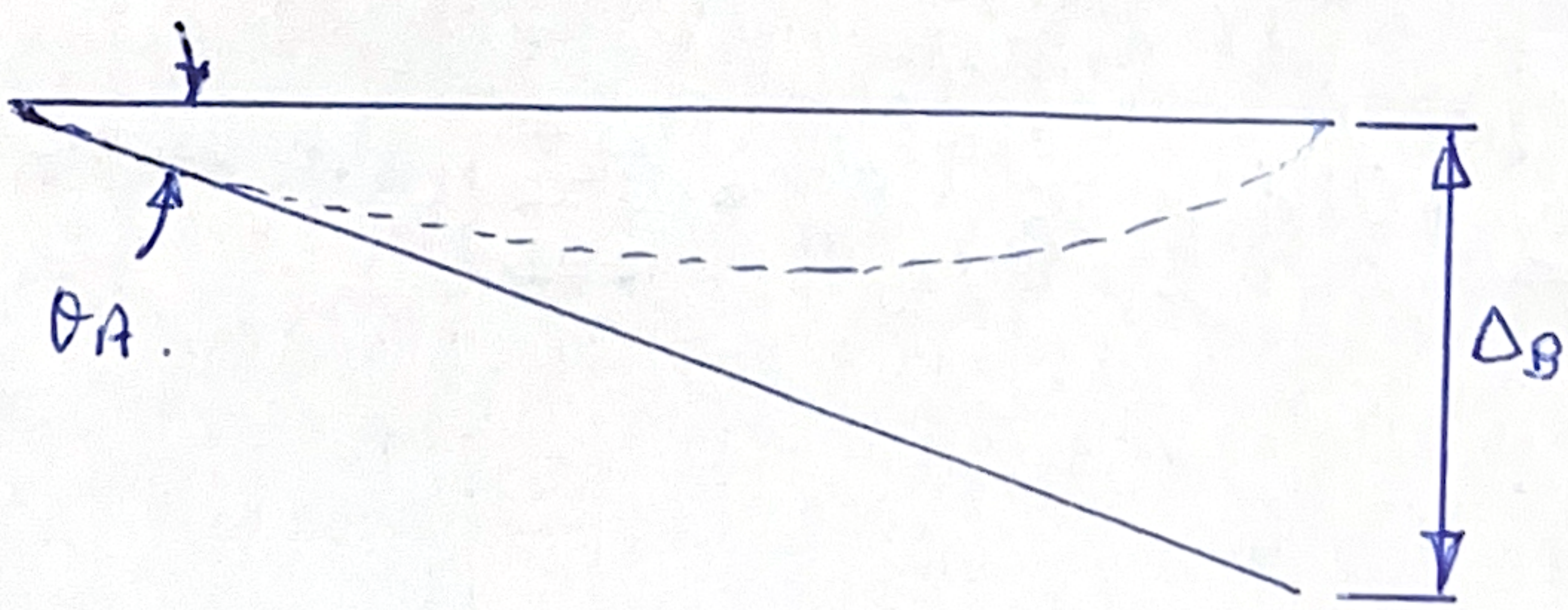
Figure 2: Front elevation view of a simple beam structure.

[2a] (3 pts) Draw and label the M(x)/EI diagram in terms of the problem parameters (i.e., P, EI, L and a).

$\phi = \frac{M}{EI}$



Deflection:



[2b] (8 pts) Use the **method of moment-area** to show that the beam rotation at A is:

$$\theta_A = \left[\frac{P}{6EI}\right]\left[\frac{a\,(L-a)\,(2L-a)}{L}\right]. \tag{1}$$

From page 5:

$$A_1 = \frac{1}{2}\frac{Pa^2(L-a)}{EI\ L}, \quad \bar{x}_1 = L - \frac{2}{3}a$$

$$A_2 = \frac{1}{2}\frac{Pa(L-a)^2}{EI\ L}, \quad x_2 = \frac{2}{3}(L-a).$$

Apply moment area:

$$\Delta_B = A_1\bar{x}_1 + A_2\bar{x}_2$$

$$= \frac{Pa(L-a)(2L-a)}{6EI}$$

$$= \theta_A \cdot L.$$

$$\Rightarrow \theta_A = \left[\frac{P}{6EI}\right]\left[\frac{a(L-a)(2L-a)}{L}\right] \checkmark \text{ --- } C$$

Note: If $a = L/2$, $\theta_A = \frac{PL^2}{16EI}$, it works!!

[2c] (4 pts) Show that the **maximum value** of beam rotation at A occurs when:

$$a = L\left[1 - \frac{\sqrt{12}}{6}\right]. \tag{2}$$

From equation (c):

$$\left[\frac{6EIL}{P}\right]\theta_A = a(L-a)(2L-a).$$

$$= a(2L^2 - 3La + a^2).$$

At maximum value:

$$\frac{d\theta_A}{da} = 0 \Rightarrow 2L^2 - 6La + 3a^2 = 0,$$ which is a quadratic in a.

$$3a^2 - 6La + 2L^2 = 0$$

$$\Rightarrow a = \frac{6L \pm \sqrt{36L^2 - 24L^2}}{6}$$

$$= \frac{6L \pm \sqrt{12}L}{6}$$

Choose root: $a < L$.

$$= \frac{6L - \sqrt{12}L}{6} = \left[1 - \frac{\sqrt{12}}{6}\right]L.$$

$$\doteq 0.422L.$$

Note: $a = 0.422L \rightarrow \theta_A = 0.0641\frac{PL^2}{EI}.$

$a = 0.5L \rightarrow \theta_A = 0.0625\frac{PL^2}{EI}$

It works!

Question 3: 10 points

Simple Three-Pinned Arch. Figure 3 is a front elevation view of a simple three-pinned arch. Vertical and horizontal loads 2P are applied at nodes B and D.

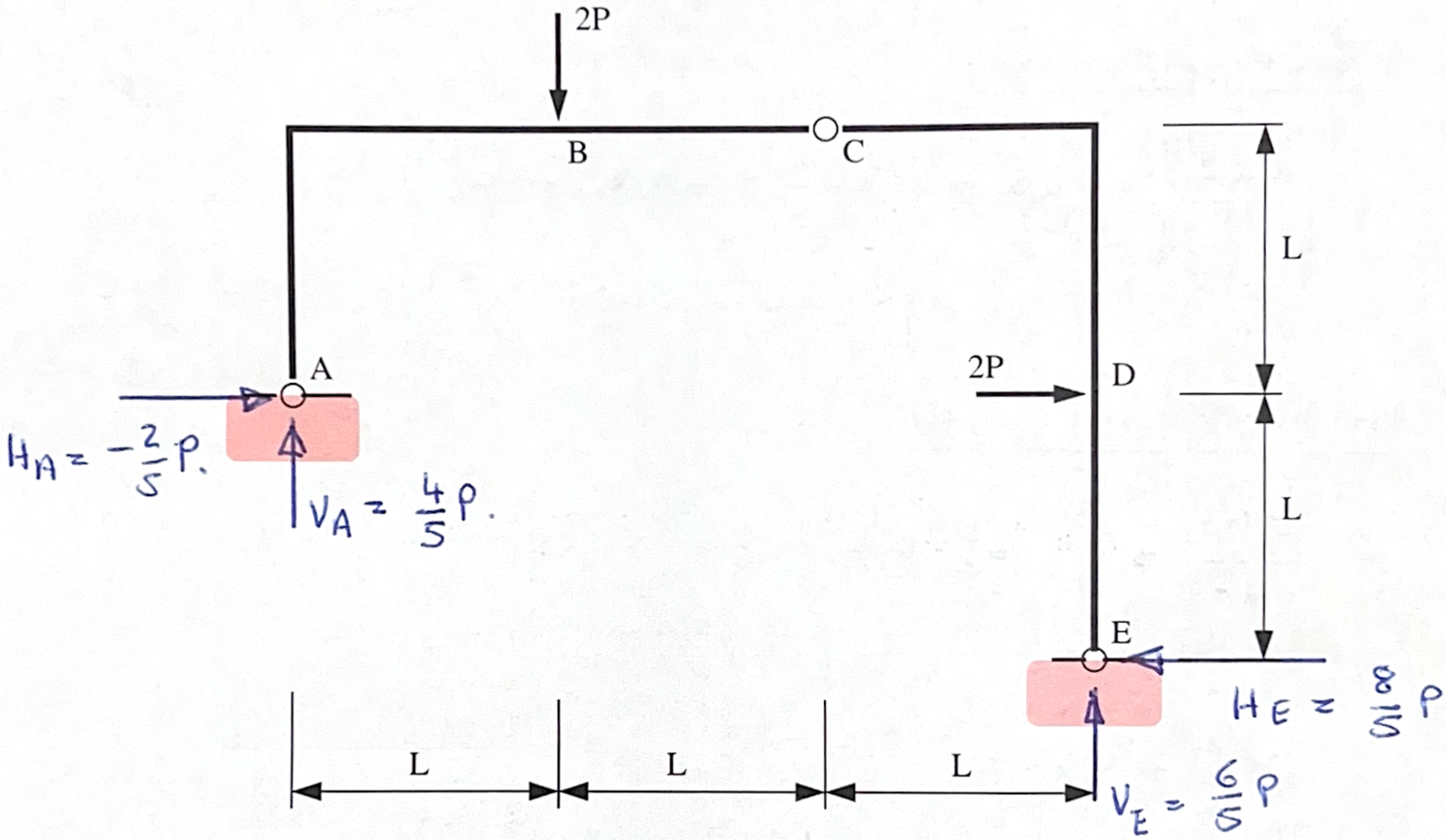


Figure 3: Front elevation view of a simple three-pinned arch.

[3a] (4 pts) Compute the vertical and horizontal components of reaction force at supports A and E as a function of P.

$\sum H = 0 \longrightarrow -H_A + H_E = 2P.$ —— A

$\sum V = 0 \longrightarrow V_A + V_E = 2P$ —— B

$\sum M_C$ for RHS:

$2PL + V_E \cdot L = H_E\, 2P \cdot L.$ —— C

Queation 3a continued:

$\sum_{1}^{\to} M_C = 0$ for LHS:

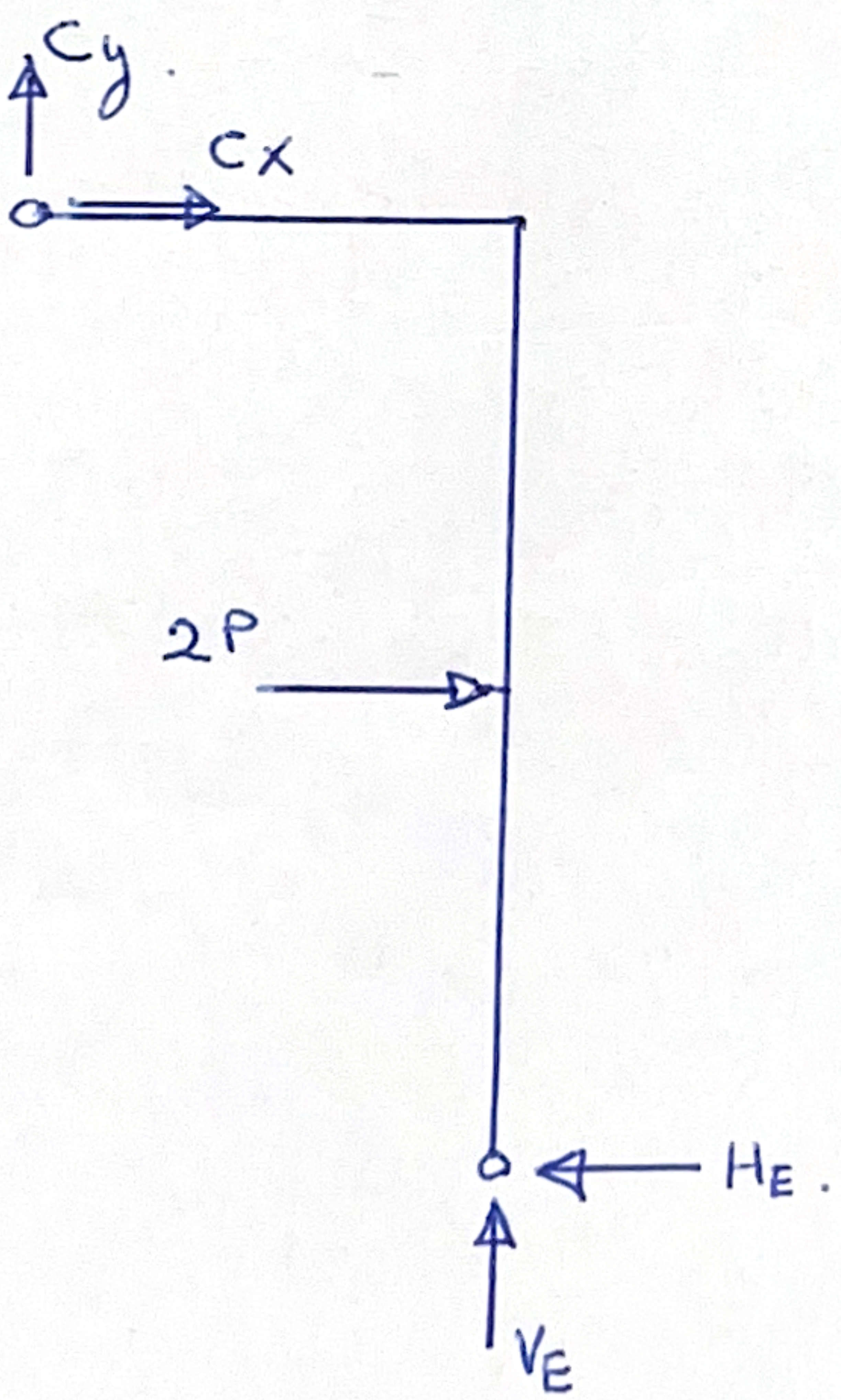
$$2PL + H_A L = V_A \cdot 2L$$

$$\Rightarrow 2P + H_A = 2V_A. \quad \text{—— (D)}$$

Solving equations (A) – (D):

$$V_A = \frac{4}{5}P, \quad V_E = \frac{6}{5}P, \quad H_A = -\frac{2}{5}P, \quad H_E = \frac{8}{5}P. \quad \text{—— (E)}$$

[3b] (3 pts) Compute and axial and shear forces transfered across the hinge at C. You can annotate Figure 3 if you think it will help to explain your solution.



$$\sum_{1}^{\uparrow} F_y = 0 \quad C_y + V_e = 0$$

$$\Rightarrow C_y = -\frac{6}{5}P. \quad \text{—— (F)}$$

$$\sum_{1}^{\to} F_x = 0 \quad C_x + 2P = H_E$$

$$\Rightarrow C_x = -\frac{2}{5}P. \quad \text{—— (G)}$$

[3c] (3 pts) Draw and label the bending moment diagram.