Homework 5

Due: May 9, 2025

Question 1: 10 points

Figure 1 shows a simple three-bar truss. The bar elements have section properties EA throughout. Horizontal and vertical loads P are applied at node C.



Figure 1: Simple three-bar truss.

- [1a] (5 pts). Use the **method of virtual displacements** to compute the vertical reaction forces at nodes A and B. Show all of your working.
- [1b] (5 pts). Use the **method of virtual displacements** to compute the member forces AC and BC. Show all of your working.

Question 2: 10 points

The cantilevered beam structure shown in Figure 2 supports a uniformly distributed load w (N/m) between points C and D.



Figure 2: Front elevation view of a simple beam structure.

- [2a] (4 pts) Use the method of virtual displacements to compute formulae for the vertical reactions at A and C. Show all of your working.
- **[2b]** (6 pts) Use the method of **virtual displacements** to compute a formula for the bending moment at B. Show all of your working.

Question 3: 10 points

Consider the articulated cantilever beam structure shown in Figure 3.



Figure 3: Elevation view of articulated cantilever beam structure.

At Point A, the cantilever is fully fixed (no movement) to a wall. Point B is a hinge. Both members have cross section properties EI. A single point load \mathbf{P} (N) is applied at node D as shown in the figure.

[3a] (2 pts). Draw and label the bending moment diagram for this problem.

- [3b] (2 pts). Qualitatively sketch the deflected shape. Indicate regions of tension/compression, and any points where slope of the beam is discontinuous.
- [3c] (6 pts). Use the method of virtual forces to compute the vertical displacement and end rotation of the beam at D.

Show all of your working.

Question 4: 10 points

Figure 4 is a front elevation view of a simple truss that supports vertical loads at nodes C and D.



Figure 4: Front elevation view of a simple truss.

All of the truss members have cross section properties AE.

[4a] (5 pts). Compute the support reactions and distribution of forces throughout the structure.

[4b] (5 pts). Use the method of virtual forces to show that the total deflection at node C is:

$$\Delta = \frac{PL}{AE} \left[\frac{8\sqrt{10}}{3} \right]. \tag{1}$$

Question 5: 20 points

The T-shaped beam structure shown in Figure 5 has flexural stiffness EI throughout.



Figure 5: Front elevation view of a T-shaped beam.

[5a] (5 pts). Use the method of virtual displacements to compute the vertical reaction force at node A.[5b] (15 pts). Use the method of virtual forces to compute the flexibility matrix:

$$\begin{bmatrix} \triangle_{dx} \\ \triangle_{by} \\ \theta_A \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ M_3 \end{bmatrix}.$$
 (2)

Question 6: 10 points

Consider the supported cantilevered beam structure shown in Figure 6.

Use the principle of **virtual forces** to compute the two-by-two flexibility matrix connecting displacements at points B and C to applied loads P_b and P_c , i.e.,

$$\begin{bmatrix} \Delta_b \\ \Delta_c \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix} \begin{bmatrix} P_b \\ P_c \end{bmatrix}.$$
(3)



Figure 6: Front elevation view of a supported cantilevered beam structure.