ENCE 353 Introduction to Structural Analysis,

Spring Semester, 2025

Homework 3

(Due: April 04, 2025)

Question 1: 10 points. The three-pin parabolic arch shown in Figure 1 has a profile shape,

$$y(x) = \left[\frac{4f}{l^2}\right] x \left(l - x\right). \tag{1}$$

where f = 4m and l = 16m.



Figure 1: Elevation view of a parabolic three-pin arch.

Questions:

[1a] Calculate the horizontal and vertical components of reaction force at A and B.

[1b] Calculate the internal forces (i.e., shear, moment and axial forces) at point E.

[1c] Draw the bending moment diagram.

Question 2: 10 points

Figure 2 shows an elevation view of a pre-fabricated steel building frame that is subject to a variety of snow and wind loadings.



Figure 2: Elevation view of pre-fabricated steel building frame subject to snow and wind loadings.

Assuming that the frames are spaced at 20 ft centers, and that the foundation-level supports and roof apex are pinned (i.e., the frame can be modeled as a three-pinned arch), compute the vertical and horizontal reactions at the base supports.

Question 3: 15 points

Analysis of a Three-Pinned Parabolic Arch. This question is inspired by the St. Louis Gateway Arch shown on the class web page. We will compute the vertical and horizontal support reactions due to self-weight the arch alone, and explore the validity of approximations in the analysis along the way.

Since the mathematical details of this problem are a bit complicated, I suggest that you use **Wolfram Alpha** (see: https://www.wolframalpha.com) for the integration, and read the web page output carefully for hints on suitable simplifications. Alternatively, **Microsoft Copilot** with links to ChaptGPT 4 might (??) also work as well.

Problem Setup. Figure 3 is a front elevation view of a three-pinned parabolic arch that has a profile: $y(x) = kx^2$.



Figure 3: Front elevation view of a three-pinned parabolic arch.

The arch has height h, span L, and has self-weight W_o (N/m) along its profile. Points A, B and C are pins.

[3a] (3 pts) Starting from first principles of geometry, show that the equivalent loading measured in the horizontal direction is:

$$w(x) = W_o \left[1 + 4k^2 x^2 \right]^{1/2}.$$
(2)

Show all of your working:

[3b] (3 pts) Show that an approximate value of V_A is:

$$V_A \approx \frac{W_o L}{2} \left[1 + \frac{8}{3} \left(\frac{h}{L} \right)^2 \right]. \tag{3}$$

Notice that when h/L = 0, the arch becomes a straight beam and $V_A = \frac{W_o L}{2}$.

[3c] (3 pts) Using Wolfram Alpha, or otherwise, derive a formula for the moments about C due to self-weight of the arch alone, i.e.,

$$\int_0^{L/2} w(x) x dx. \tag{4}$$

All reasonable answers will be accepted.

- **[3d]** (3 pts) With equations 3 and 4 in place, write down and label the equation you would solve to compute the horizontal reaction force at A.
- [3e] (3 pts) Now suppose that equation 3 is applied to the St. Louis Gateway Arch profile (see pic on class web page), where h/L = 1.

Does the computed value for V_A seem reasonable to you, or not? And if not, how you would correct the analysis? Either way, justify your answer.

Question 4: 10 points

The cable structure shown in Figure 4 carries a uniform load w_o N/m along its entire length.



Figure 4: Elevation view of a pedestrian swing bridge.

[4a] Starting from first principles (i.e., the differential equation), show that cable profile is given by the equation

$$y(x) = \frac{w_o x^2}{2H} + \left(1 - \frac{15w_o}{H}\right)x.$$
 (5)

Now let us assume that the minumum value of the cable profile occurs at x = 10.

[4b] Show that the horizontal cable force is:

$$H = 5w_o. (6)$$

[4c] Derive a simple expression for the maximum tensile force in the cable.

Question 5: 10 points

The cable structure shown in Figure carries a triangular load that is zero at the left-hand support and increases to w_o N/m at the right-hand support.



[5a] Starting from first principles (i.e., the differential equation), show that cable profile is given by the equation

$$y(x) = \frac{w_o x^3}{180H} + \left(1 - \frac{5w_o}{H}\right)x.$$
(7)

Now let us assume that the minumum value of the cable profile occurs at x = 10.

[5b] Show that the horizontal cable force is:

$$H = \frac{20w_o}{6}.$$
(8)

[5c] Draw and label a diagram showing the horizontal and vertical components of reaction force at the leftand right-hand cable supports.