Solutions to Homework 1

Question 1: 20 points.

Consider the combined multi-span beam/truss structure shown in Figure 1.

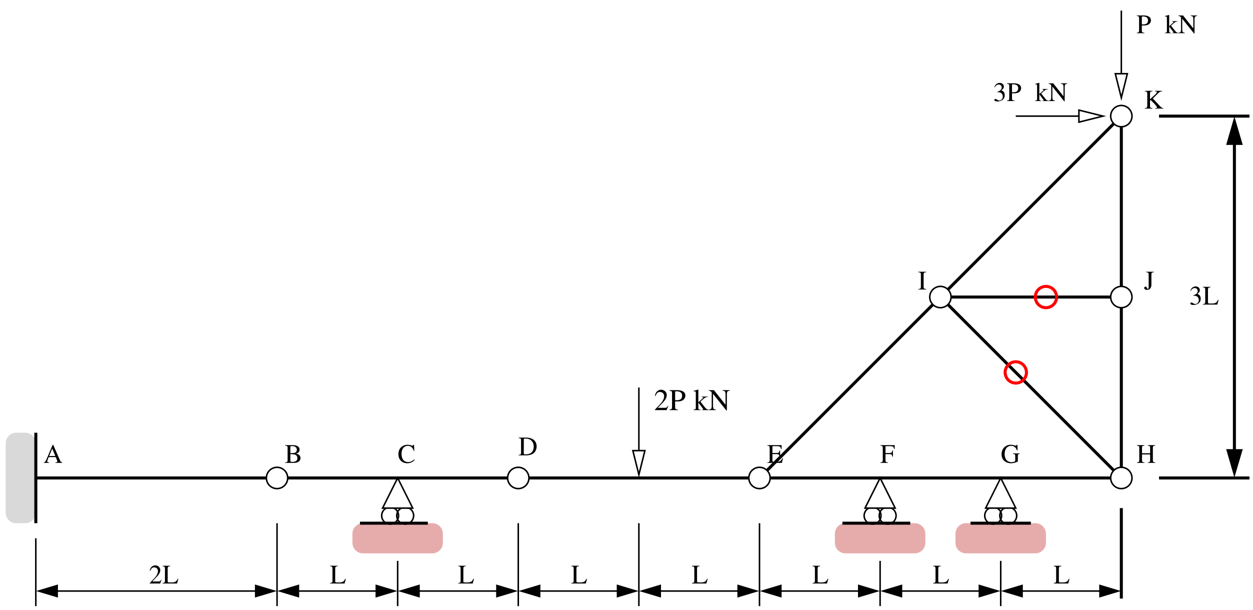


Figure 1: Front elevation view of multi-span beam structure.

The cantilever is fully-fixed to the wall at Point A. Points B, D, E and H are hinges. Horizontal and vertical point loads **3P** (kN) and **P** (kN) are applied to the truss as shown in Figure 1.

Part [1a]. Compute the degree of indeterminacy for the articulated beam structure (A-B-C-D-E-F-G-H).

Sol'n: Here f = 6 and r = 3. Hence, $\hat{i}$ = f - 3 - r = 0. It's statically determinate.

Part [1b]. Identify the zero-force members in the truss structure.

Sol'n: See red dots on Figure 1.

Part [1c]. Compute the distribution of forces throughout the truss structure. Draw a diagram summarizing your results.

Sol'n: Remove zero-force members from the truss, then consider equilibrium at node K. This give:

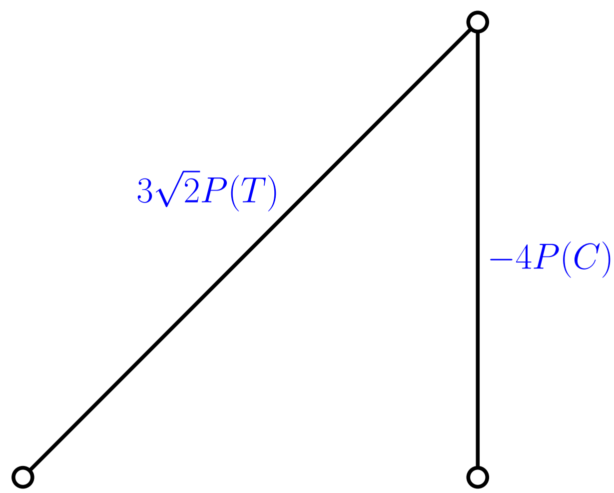
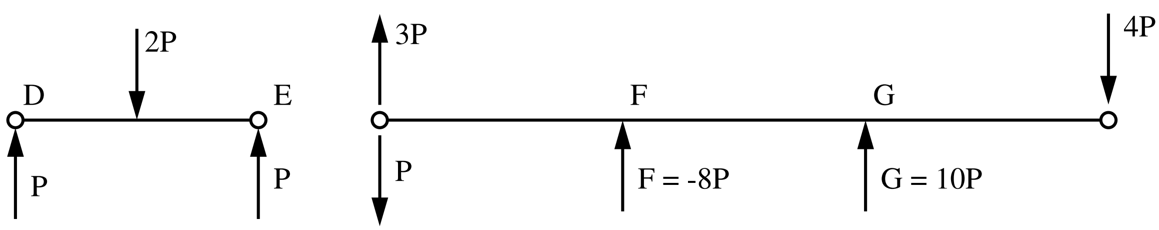


Figure 2: Forces acting on truss structure

Part [1d]. Compute the vertical reaction forces at nodes F and G.

Sol'n: Isolate substructures D-E and E-F-G-H, and consider equilibrium.



Taking moments about E for the right-most substructure:

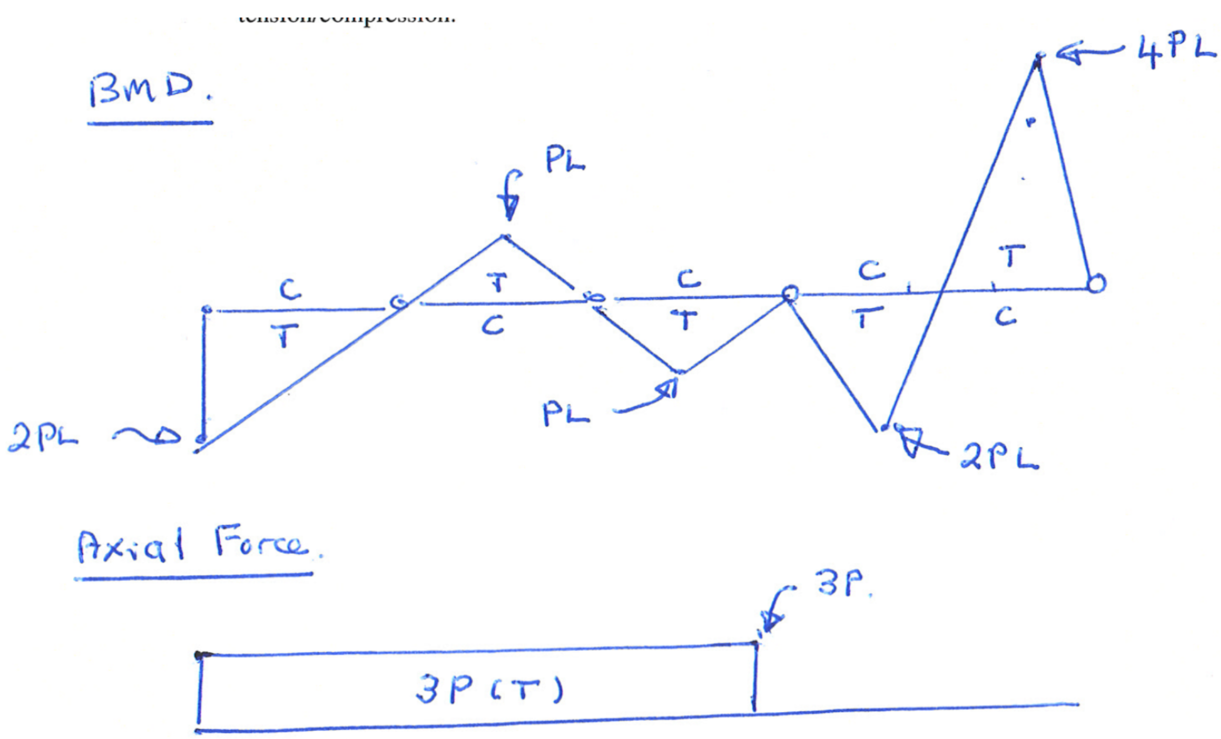
$$\sum M_E = 0, \qquad FL + G(2L) = (4P)(3L) \quad \longrightarrow \quad F + 2G = 12P. \tag{1}$$

$$\sum V = 0, \qquad F + G + 2P = 4P \quad \longrightarrow \quad F + 2G = 12P. \tag{2}$$

From equations 1 and 2, G = 10P and F = -8P.

Part [1e]. Draw and label diagrams showing how the **bending moment** and **axial force** vary along the beam, nodes A through H. Clearly indicate on your bending moment diagram, regions that are in tension/compression.

Sol'n: The beam is in tension for nodes A – E.



Question 2: 10 points.

Classifiy each of the structures in Figure 3 as statically determinate, statically indeterminate, stable or unstable. For those structures that are indeterminate, specify the degree of indeterminancy.

Sol'n: ...

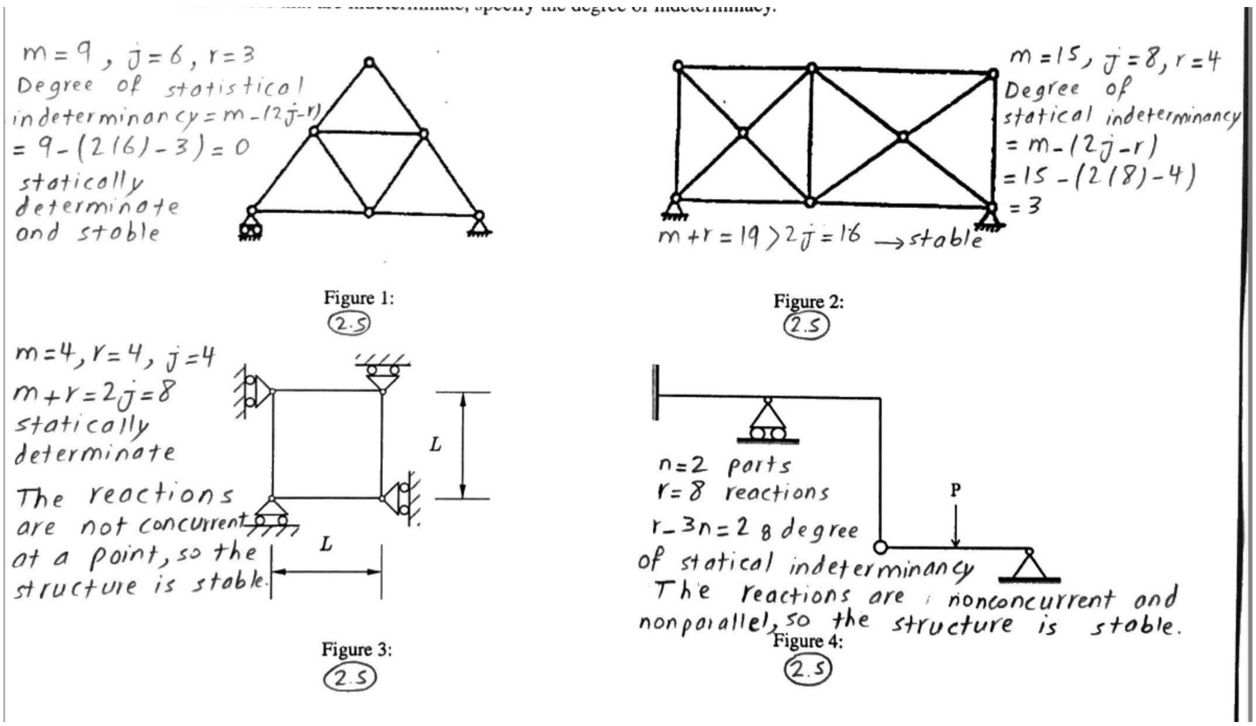


Figure 3: Assortment of statically determinate and indeterminate frame structures.

Question 3: 10 points.

Consider the crane tower structure shown in Figure 4.

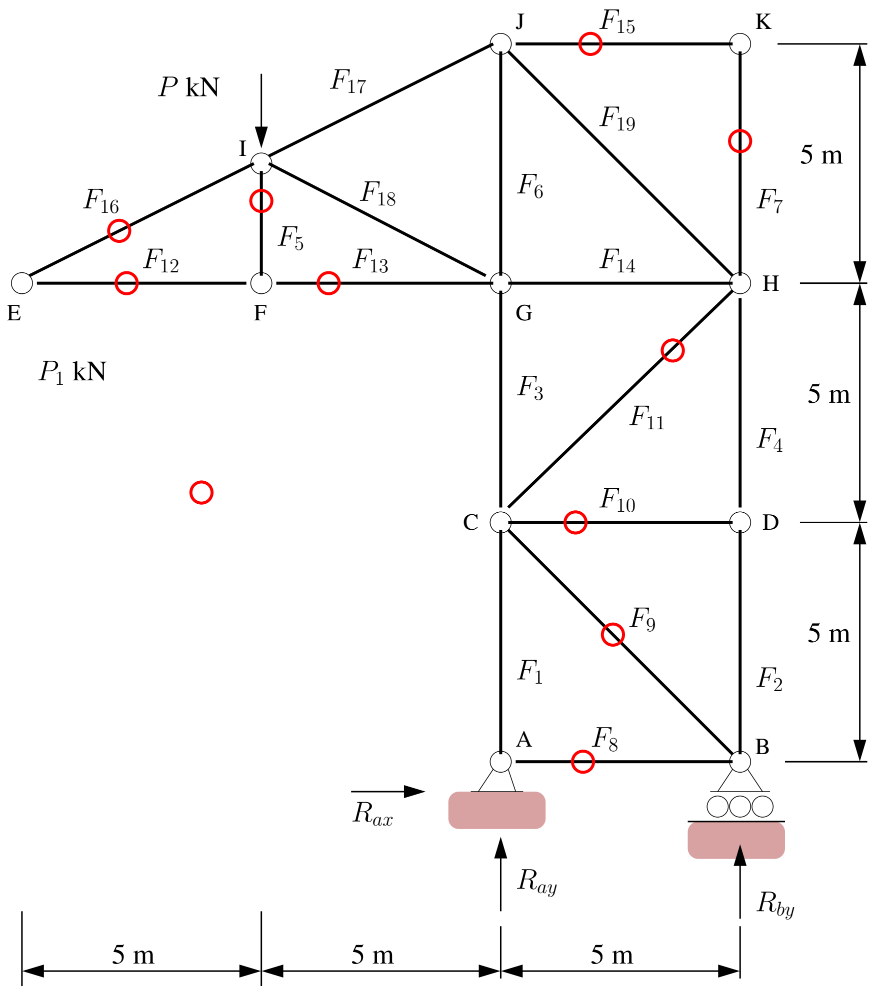


Figure 4: Elevation view of a simple crane tower.

A single point load **P** (kN) is applied at node I as shown in the figure.

Part [3a]. Compute the support reactions at A and B.

Sol'n: First, take moments about A (for the whole structure):

$$\sum M_A = 0 \longrightarrow R_{by} = -P\ kN. \tag{3}$$

Look at equilibrium in vertical direction (for the whole structure):

$$\sum V = 0 \longrightarrow R_{ay} + R_{by} = P\ kN \longrightarrow R_{ay} = 2P\ kN. \tag{4}$$

Finally, consider equilibrium in horizontal direction:

$$\sum H = 0 \longrightarrow R_{ax} = 0\ kN. \tag{5}$$

Part [3b]. Identify all of the zero-force members. If you wish, you can simply copy and annotate Figure 4.

Sol'n: Ten zero-force members (see red circles on Figure 4).

Part [3c]. Using the method of joints (or otherwise) compute the distribution of tension and compression forces throughout the crane structure. Draw and label a diagram showing the distribution of forces in the simplified crane tower structure.

Sol'n: There are 11 joints, therefore 22 equations of equilibrium to consider. Note, however, that joints E, F, and K, only connect to zero-force elements. So, we will only look at equilibrium for the eight remaining joints:

At Joint A:

$$\sum H = 0 \quad R_{ax} + F_8 = 0 \longrightarrow F_8 = 0\ kN. \tag{6}$$

$$\sum V = 0, \quad R_{ay} + F_1 = 0 \longrightarrow F_1 = -2P\ kN(C). \tag{7}$$

At Joint B:

$$\sum H = 0, \quad F_8 + \frac{1}{\sqrt{2}} F_9 = 0 \longrightarrow F_9 = 0\ kN. \tag{8}$$

$$\sum V = 0, \quad F_2 + \frac{1}{\sqrt{2}} F_9 + R_{by} = 0 \longrightarrow F_2 = P\ kN(T). \tag{9}$$

At Joint C:

$$\sum H = 0, \quad \frac{1}{\sqrt{2}} F_9 + F_{10} + \frac{1}{\sqrt{2}} F_{11} = 0 \longrightarrow F_{11} = 0\ kN. \tag{10}$$

$$\sum V = 0, \quad F_1 - F_3 + \frac{1}{\sqrt{2}}F_9 - \frac{1}{\sqrt{2}}F_{11} = 0. \longrightarrow F_3 = -2P\ kN(C). \tag{11}$$

At Joint D:

$$\sum H = 0, \quad F_{10} = 0\ kN. \tag{12}$$

$$\sum V = 0, \quad F_2 - F_4 = 0 \longrightarrow F_4 = P\ kN(T). \tag{13}$$

At Joint G:

$$\sum H = 0, \quad F_{13} - F_{14} + \frac{2}{\sqrt{5}}F_{18} = 0. \longrightarrow F_{18} = -\frac{\sqrt{5}}{2}P\ kN(C). \tag{14}$$

$$\sum V = 0, \quad -F_3 + F_6 + \frac{1}{\sqrt{5}}F_{18} = 0. \longrightarrow F_6 = -\frac{3}{2}P\ kN(C). \tag{15}$$

At Joint H:

$$\sum H = 0, \quad \frac{1}{\sqrt{2}}F_{11} + F_{14} + \frac{1}{\sqrt{2}}F_{19} = 0. \longrightarrow F_{14} = -P\ kN(C). \tag{16}$$

$$\sum V = 0, \quad F_4 - F_7 + \frac{1}{\sqrt{2}}F_{11} - \frac{1}{\sqrt{2}}F_{19} = 0 \longrightarrow F_{19} = \sqrt{2}P\ kN(T). \tag{17}$$

At Joint I:

$$\sum H = 0, \quad \frac{2}{\sqrt{5}}F_{16} - \frac{2}{\sqrt{5}}F_{17} - \frac{2}{\sqrt{5}}F_{18} = 0. \longrightarrow F_{17} = \frac{\sqrt{5}}{2}P\ kN(T). \tag{18}$$

$$\sum V = 0, \quad P + F_5 + \frac{1}{\sqrt{5}}F_{16} - \frac{1}{\sqrt{5}}F_{17} + \frac{1}{\sqrt{5}}F_{18} = 0. \quad \text{(check equilibrium)} \tag{19}$$

At Joint J:

$$\sum H = 0, \quad F_{15} - \frac{2}{\sqrt{5}}F_{17} + \frac{1}{\sqrt{2}}F_{19} = 0. \quad \text{(check equilibrium)} \tag{20}$$

$$\sum V = 0, \quad F_6 + \frac{1}{\sqrt{5}} F_{17} + \frac{1}{\sqrt{2}} F_{19} = 0 \quad \text{(check equilibrium)} \tag{21}$$

Part [3d]. If the maximum force any member can support is 10 kN in tension and 7 kN in compression, determine the maximum value of P that the crane tower can safely carry.

Sol'n: Find limiting cases:

Maximum tensile force is $\sqrt{2}$ P (kN).

Maximum compressive force is -2P (kN).

Limiting constraint is: -2P = 7 kN, therefore P_{max} = 3.5 kN.

Question 4: 20 points. Consider the leaning tower structure shown in Figure 5.

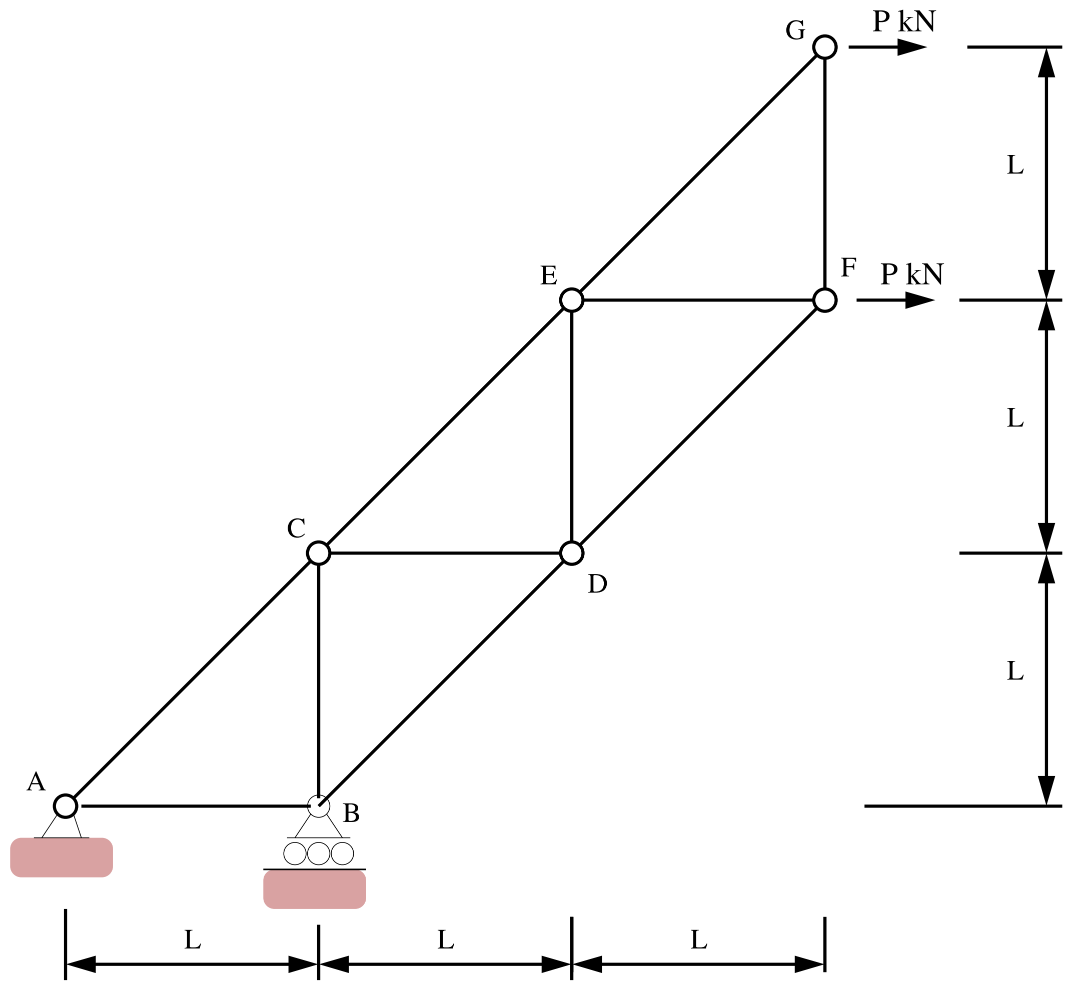


Figure 5: Elevation view of a leaning tower structure.

Horizontal loads **P** (kN) are applied at nodes F and G as shown in the figure.

Note: No joints j = 7, no members m = 11, and no reactions r = 3. Hence $m + r = 2j \longrightarrow$ statically determinate.

Part [4a]. Compute the **total support reactions** at A and B.

Sol'n: First, take moments about A (for the whole structure):

$$\sum M_A = 0 \longrightarrow R_{by}L = 2PL + 3PL = 5PL \longrightarrow R_{by} = 5P\ kN. \quad (22)$$

Next, look at equilibrium in vertical direction (for the whole structure):

$$\sum V = 0 \longrightarrow R_{ay} + R_{by} = 0\, kN \longrightarrow R_{ay} = -5P\, kN. \tag{23}$$

Summing forces in the horizontal direction, R_{ax} = - 2P kN. Hence, the total reaction force at A is: $\left[2^2 + 5^2\right]^{1/2}$ = $\sqrt{29}$ kN.

Part [4b]. Using the method of joints (or otherwise) compute the distribution of tension and compression forces throughout the structure. Show all of your working.

Sol'n: Systematically look at equilibrium at nodes A, B, C, D, G and F.

At Joint A:

$$\sum V = 0, \quad \frac{F_{ac}}{\sqrt{2}} = 5P \longrightarrow F_{ac} = 2\sqrt{2}P\, kN(T). \tag{24}$$

$$\sum H = 0 \quad \frac{F_{ac}}{\sqrt{2}} + F_{ab} = 2 \longrightarrow F_{ab} = -3P\, kN(C). \tag{25}$$

At Joint B:

$$\sum H = 0, \quad \frac{F_{bd}}{\sqrt{2}} + 3P = 0 \longrightarrow F_{bd} = -3\sqrt{2}P\, kN(C). \tag{26}$$

$$\sum V = 0, \quad 5P + F_{bc} - \frac{3\sqrt{2}}{\sqrt{2}} = 0 \longrightarrow F_{bc} = -2P\, kN(C). \tag{27}$$

At Joint C:

$$\sum V = 0, \quad \frac{F_{ce}}{\sqrt{2}} + 2P - 5P = 0 \longrightarrow F_{ce} = 3\sqrt{2}P\, kN(T). \tag{28}$$

$$\sum H = 0, \quad \frac{F_{ce}}{\sqrt{2}} + F_{cd} = 5P \longrightarrow F_{cd} = 2P\, kN(T). \tag{29}$$

At Joint D:

$$\sum H = 0, \quad \frac{F_{df}}{\sqrt{2}} = 2P - 3P \longrightarrow F_{df} = -\sqrt{2}P\ kN(C). \tag{30}$$

$$\sum V = 0, \quad F_{de} + \frac{F_{df}}{\sqrt{2}} + 3P = 0 \longrightarrow F_{de} = -2P\ kN(C). \tag{31}$$

At Joint G:

$$\sum H = 0, \quad \frac{F_{eg}}{\sqrt{2}} = P \longrightarrow F_{eg} = \sqrt{2}P\ kN(T). \tag{32}$$

$$\sum V = 0, \quad \frac{F_{eg}}{\sqrt{2}} + F_{fg} = 0 \longrightarrow F_{fg} = -P\ kN(C). \tag{33}$$

At Joint F:

$$\sum V = 0, \quad P + \frac{F_{df}}{\sqrt{2}} = 0 \longrightarrow F_{df} = -\sqrt{2}P\ kN(C). \tag{34}$$

$$\sum H = 0, \quad F_{ef} + \frac{F_{df}}{\sqrt{2}} = P \longrightarrow F_{ef} = 2P\ kN(T). \tag{35}$$

At Joint E: Can validate equilibrium by checking $\sum V = \sum H = 0$.

Part [4c]. Now suppose that the maximum tensile force any member can support is 10 kN, and that the maximum allowable compressive force is:

$$P_{ci} = 8\left(\frac{L}{L_i}\right)^2 kN, \tag{36}$$

where L_i is the length of the i-th element, and P_{ci} is the maximum allowable compressive force of the i-th element before buckling.

Determine the maximum value of P (kN) that the leaning tower can safely carry.

Sol'n: From the analysis:

Maximum tensile force = $5\sqrt{2}P$ (T).

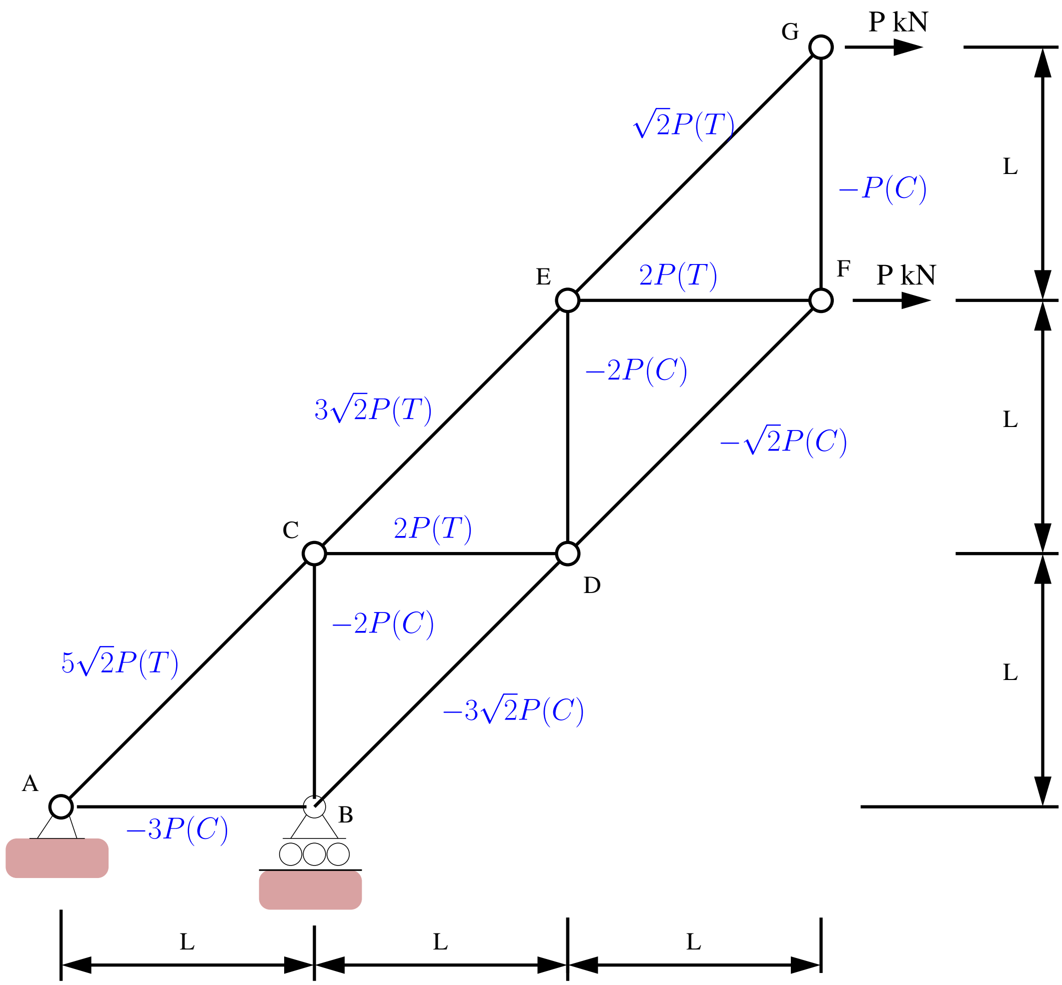


Figure 6: Elevation view of a leaning tower structure.

Maximum compressive force = $-3\sqrt{2}P$ (C) in element BD.

Limiting constraint in tension:

$$5\sqrt{2}P \leq 10kN \longrightarrow P \leq \sqrt{2}P. \tag{37}$$

Limiting constraint in compression:

$$3\sqrt{2}P \leq \frac{8}{2} = 4kN \longrightarrow P \leq \frac{4}{3\sqrt{2}}P. \tag{38}$$

Compression case limits allowable load.