Department of Civil and Environmental Engineering, Spring Semester, 2025

<u>ENCE 353 Final Exam, Open Notes and Open Book</u>

Name : ________________________

Exam Format and Grading. This take home final exam is open notes and open book. You need to comply with the university regulations for academic integrity.

Answer Question 1. Then answer **three of the four** remaining questions. Partial credit will be given for partially correct answers, so please show all your working.

IMPORTANT: Only the **first four questions** that you answer will be graded, so please **cross out the question you do not want graded** in the table below. Also, before submitting your exam, check that **every page has been scanned correctly**.

Question	Points	Score
1	20	
2	10	
3	10	
4	10	
5	10	
Total	50	

Question 1: 20 points

COMPULSORY: Computing Displacements with the Method of Virtual Forces. Figure 1 is a front elevation view of a cantilevered beam carrying a single point load P. EI is constant along the beam structure A-B-C-D.

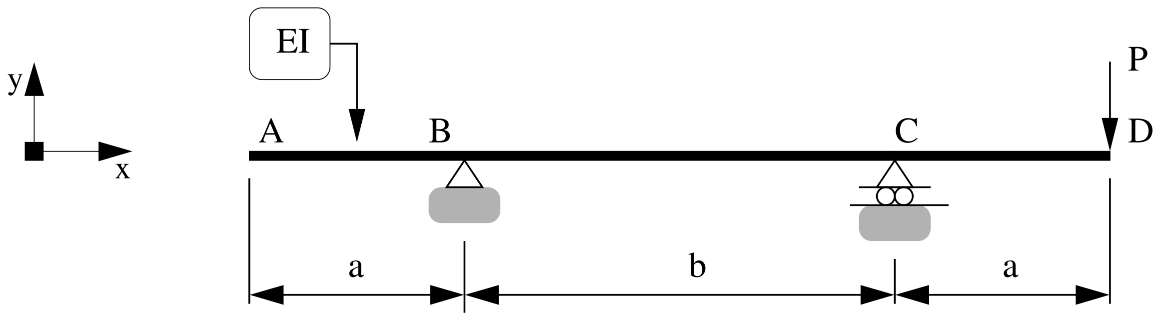

Figure 1: Cantilevered beam carrying a single applied load P.

[1a] (3 pts) Use the method of **virtual displacements** to compute the support reactions at B and C.

[1b] (2 pts) Draw and label the M/EI diagram.

[1c] (5 pts) Use the method of **virtual forces** to show that the virtical deflection (measured downwards) at D is:

$$y_D = \frac{Pa^2}{3EI}\,(a+b). \tag{1}$$

Show all of your working.

[1d] (5 pts) Use the method of **virtual forces** to show that the virtical deflection (measured downwards) at A is:

$$y_A = \frac{Pa^2 b}{6EI} \tag{2}$$

Show all of your working.

[1e] (5 pts) Use the method of **virtual forces** to show that rotation of the beam (measured clockwise) at D is:

$$\theta_D = \frac{Pa}{6EI}\left[2b + 3a\right] \tag{3}$$

Show all of your working.

Question 2: 10 points

OPTIONAL: Simple Three-Pinned Arch. Figure 2 is a front elevation view of a simple three-pinned arch that carries a total snow loading of 3WL uniformly distributed over its upper section.

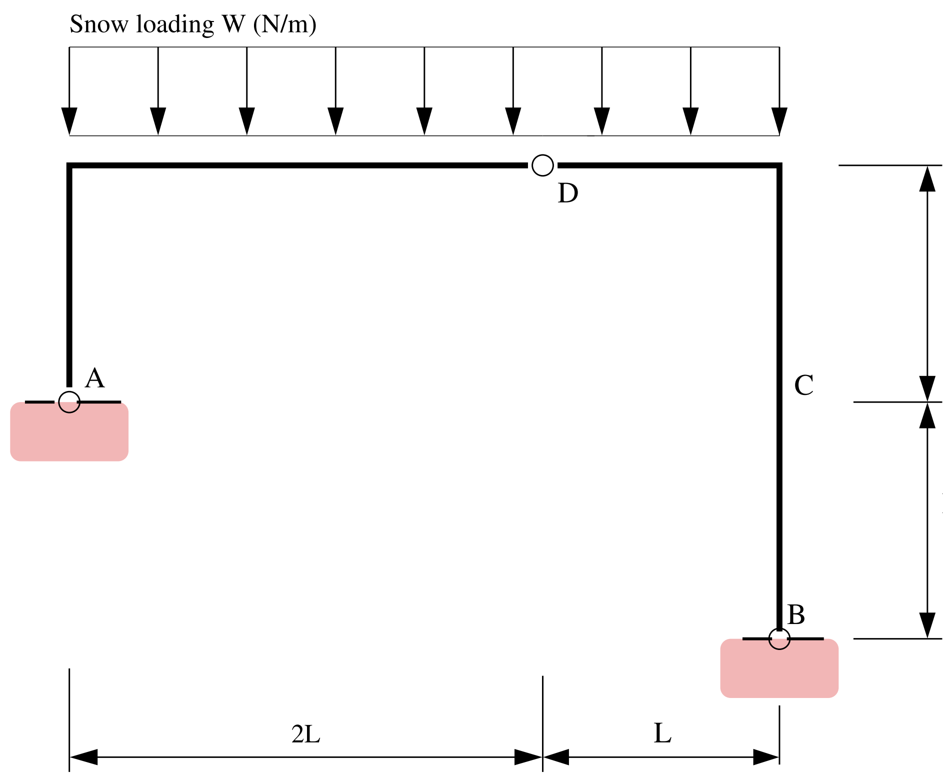

Figure 2: Front elevation view of a three-pinned arch that supports a snow loading.

[2a] (6 pts) Compute the vertical and horizontal components of reaction force at supports A and B as a function of W and L.

Queation 2a continued:

[2b] (4 pts) Draw and label the bending moment diagram.

Question 3: 10 points

OPTIONAL: Elastic Curve for Beam Deflections. Figure 3 is a front elevation view of a simply supported beam that carries a triangular load.

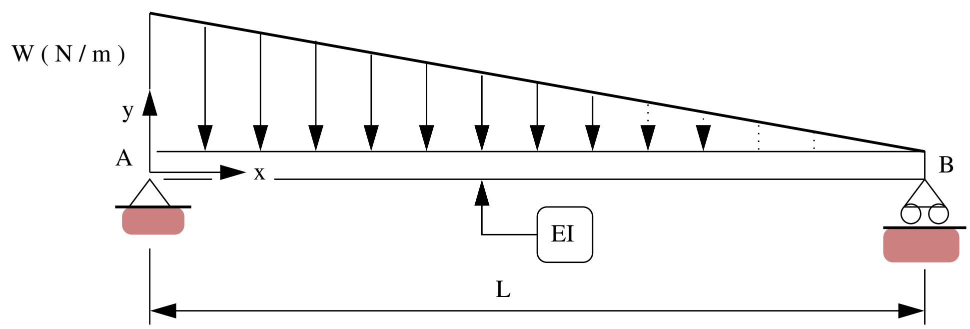

Figure 3: Simply supported beam carrying a triangular load.

The load decreases from W (N/m) at point A to zero at point B. Thus, the total beam loading is WL/2.

[3a] (4 pts). Starting from first principles of engineering, show that the bending moment at point x is:

$$M(x) = \left[\frac{W}{6L} \right] x \left(L - x \right) \left(2L - x \right). \tag{4}$$

[3b] (4 pts). Show that the elastic curve for beam deflection is given by (notice that in Figure 4, the y axis is pointing upwards):

$$y(x) = \left[\frac{WL^4}{360EI}\right]\left(3\left[\frac{x}{L}\right]^5 - 15\left[\frac{x}{L}\right]^4 + 20\left[\frac{x}{L}\right]^3 - 8\left[\frac{x}{L}\right]\right) \tag{5}$$

[3c] (2 pts). Show that the maximum beam curvature occurs at x = $\left[1 - \frac{1}{\sqrt{3}}\right] L$.

Question 4: 10 points

OPTIONAL: Analysis of Forces in a Cable Structure. Figure 4 is a front elevation view of a cable structure and bridge deck that carries a snow loading.

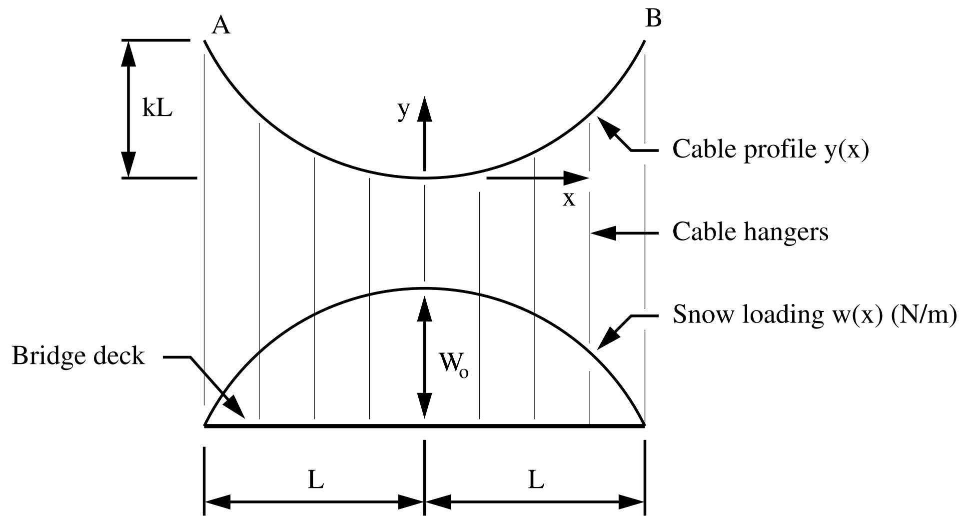

Figure 4: Cable structure carrying a snow loading.

The bridge deck and cable system have total span 2L. The cable sag is kL, where $0 < k < 1$. The distribution of snow loading follows a parabolic shape:

$$w(x) = \frac{W_o}{L^2}\left[L^2 - x^2\right] \quad (N/m). \tag{6}$$

[4a] (4 pts) Starting from first principles of engineering (i.e., the differential equation for cable behavior) show that the horizontal component of cable force is:

$$H = \frac{5W_oL}{12k}. \tag{7}$$

[4b] (3 pts) Determine the vertical and horizontal components of cable force at the supports A and B.

[4c] (3 pts) Determine whether or not the cable system forces and snow loads are in equilibrium?

Question 5: 10 points

OPTIONAL: Use Superposition and Principle of Virtual Work to Compute Displacements. Consider the articulated cantilever beam structure shown in Figure 5.

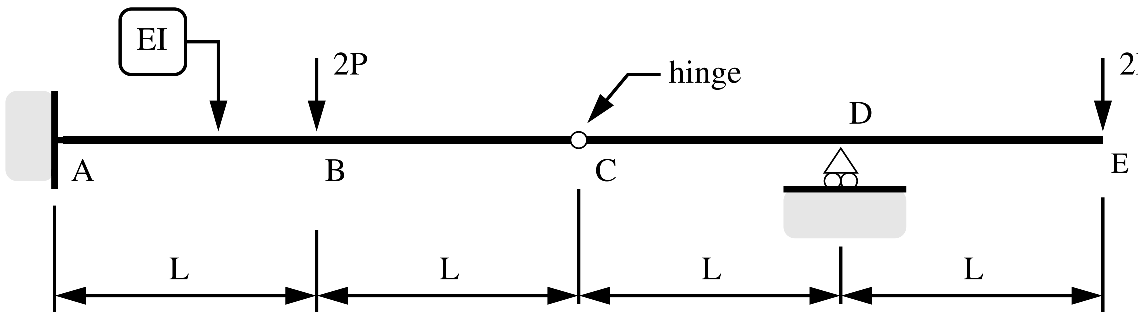

Figure 5: Elevation view of articulated cantilever beam structure.

At Point A, the cantilever is fully fixed (no movement) to a wall. Point C is a hinge. Both members have cross section properties EI. Point loads **2P** (N) are applied at nodes B and E as shown in the figure.

[5a] (3 pts). Draw and label a diagram indicating how the **principle of superposition** can be used to simplify the analysis of the multi-span beam structure.

[5b] (2 pts). Qualitatively sketch the deflected shape. Indicate regions of tension/compression, and any points where slope of the beam is discontinuous.

[5c] (5 pts). Use the method of **virtual forces** to compute the **vertical displacement** at E.

Show all of your working.

Question [5c] continued: