Statically Determinate Structures

Mark A. Austin

University of Maryland

austin@umd.edu

ENCE 353, Fall Semester 2020

September 23, 2020

Overview

Part 3

Introduction

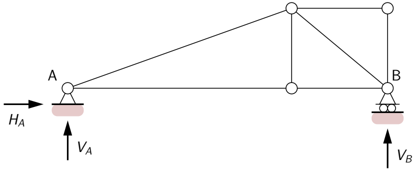
Need for Mathematical Test

Three cases to consider:

Test Structure A: Determinate.

Can compute:

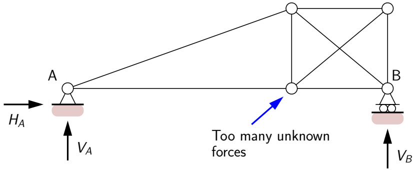
- Support reactions. ✓
- Member forces. ✓



Test Structure B: Indeterminate.

Can compute:

- Support reactions. ✓
- Member forces. ✗

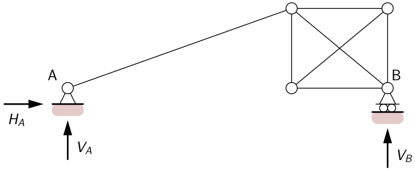


Need for Mathematical Test

Test Structure C: Unstable.

Can compute:

- Support reactions. ✗
- Member forces. ✗



Key Points:

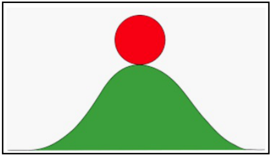
- Intuition on notions of determinacy will not scale. We need a mathematical test to classify structures.
- Initial inclination is to design for A and avoid B – it's complicated and probably won't work. Unless, there are benefits to B?

Stability

Stability

Stability of Dynamical Systems

Instability occurs when some of the system outputs (e.g., displacement) can increase without bounds. In structural analysis, equilibrium of displacements corresponds to a minimum energy state.

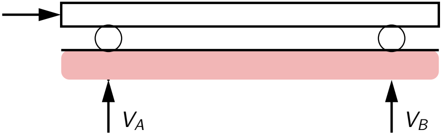


Instability of Structures:

- **External Instability.** When support reactions are either concurrent forces about a point or parallel.
- **Internal Instability.** When a mechanism exists.

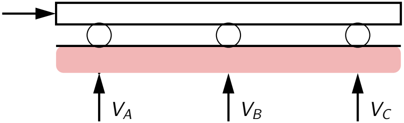
External Instability

Example 1: Reaction forces are parallel.



Three equations of equilibrium but only two reactions ($r < 3n$).

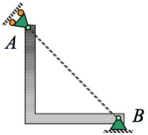
Example 2: Three parallel reaction forces

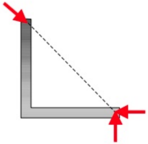


Three equations of equilibrium and three reactions ($r = 3n$). Still unstable.

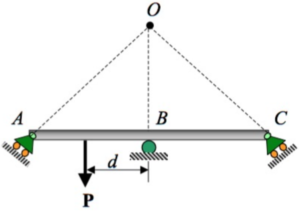
External Instability

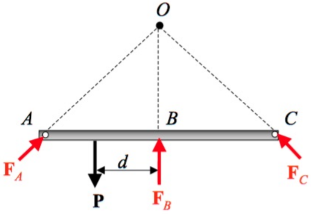
Example 3. Reaction forces are concurrent about point B.





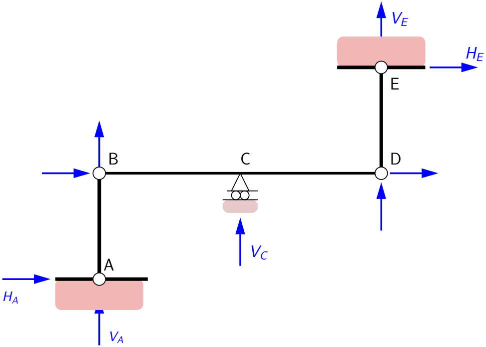
Example 4. Reaction forces are concurrent.





Internal Instability

Example 5. Structural configuration forms a pendulum mechanism.

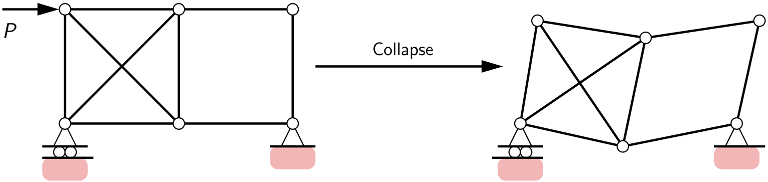


Three members: $n = 3$. No reactions r $= \{H_A, V_A, \cdots, H_E\} = 9$.

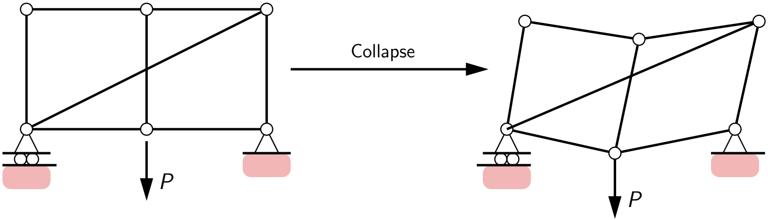
Test: r - 3n $= 0 \rightarrow$ statically determinate.

Internal Instability

Example 6: Internal Mechanism.



Example 7: Sometime internal mechanism are hard to identify.



Relating Stability to Linear Matrix Equations

Aside: If we compute the reactions and then systematically write the equations of equilibrium for each joint $\rightarrow 2j$ equations, which can be put in matrix form:

$$[A]\,[X] = [B]\,. \tag{2}$$

Here,

- X is a vector of truss element forces.
- A is a matrix of geometry and boundary conditions.
- B is a vector of applied loads.

When the system is statically determinate we can write:

$$[A]\,[X] = [B] \rightarrow [X] = \left[A^{-1}\right][B]\,. \tag{3}$$

Relating Stability to Linear Matrix Equations

Equations 3 only exist when $\left[A^{-1}\right]$ exists.

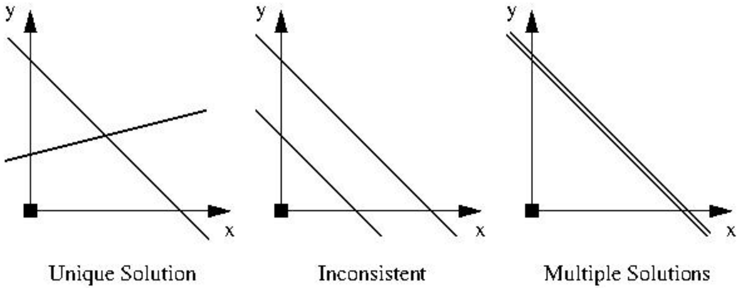
And this requires that the individual equations be linearly independent.

In Example 2, $\left[A^{-1}\right]$ does not exist because the reactions are co-linear, meaning that V_A can be written as a linear combination of V_B and V_C, i.e.,

$$\sum F_y = 0 \rightarrow V_A + V_B + V_C = 0. \tag{4}$$

Equations in Two Dimensions

Three Types of Solutions:



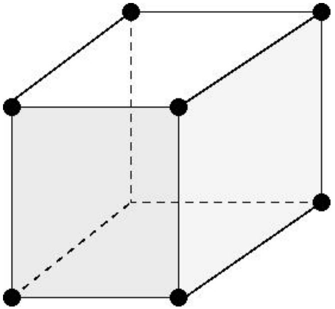
- Unique solution when two lines meet at a point.
- No solutions when two lines are parallel but not overlapping.
- Multiple solutions when two lines are parallel and overlap.

Equations in Three Dimensions

Also Three Types of Solutions:

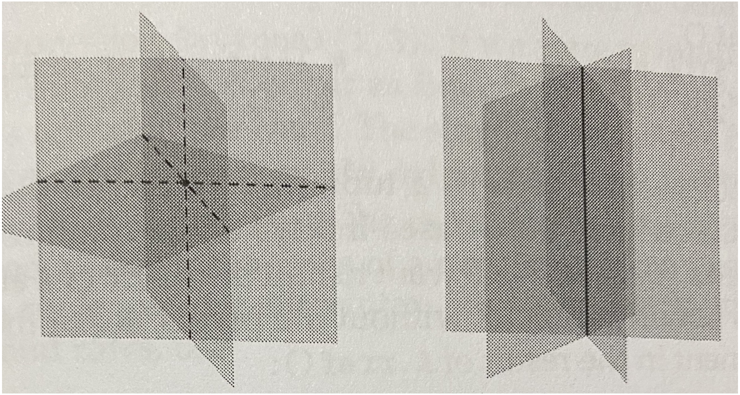
Each equation corresponds to a plane in three dimensions (e.g., think walls, floor and ceiling in a room).

- Unique solution when three planes intersect at a corner point.
- Multiple solutions where three planes overlap or meet along a common line.
- No solutions when three planes are parallel, but distinct, or pairs of planes that intersect along a line (or lines).



Equations in Three Dimensions

One Solution/Infinite Solutions:



Equations in Three Dimensions

No Solutions: