Statically Determinate Structures

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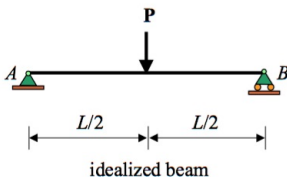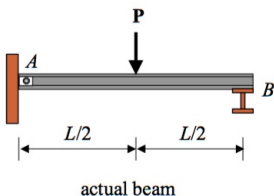## Overview

Part 1

Introduction

Quick Review

Real-World and Idealized Abstractions

Statically Determinate Structure

- Can use statics to determine reactions and distribution of element-level forces.

Statically Indeterminate Structure

- Statics alone are not enough to find reactions. Need to find additional information (e.g., material behavior).
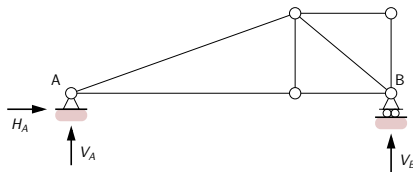
Need for Mathematical Test

Three cases to consider:
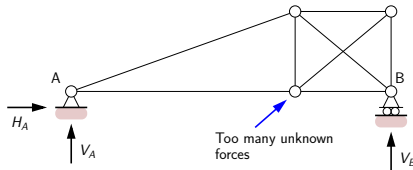
Test Structure A: Determinate.

Can compute:
- Support reactions. ✓
- Member forces. ✓

Test Structure B: Indeterminate.

Can compute:
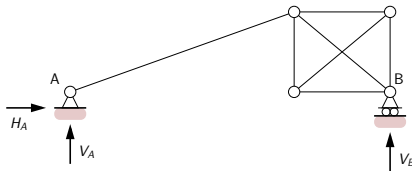- Support reactions. ✓
- Member forces. ✗

Need for Mathematical Test

Test Structure C: Unstable.

Can compute:

- Support reactions. ✗
- Member forces. ✗

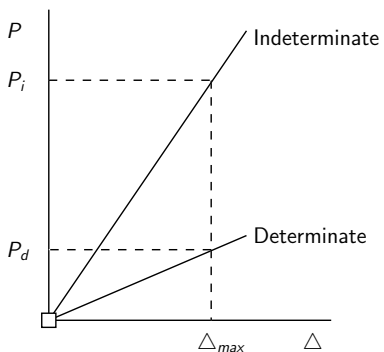Key Points:

- Intuition on notions of determinacy will not scale. We need a mathematical test to classify structures.
- Initial inclination is to design for A and avoid B – it's complicated and probably won't work. Unless, there are benefits to B?

Benefits of Indeterminacy

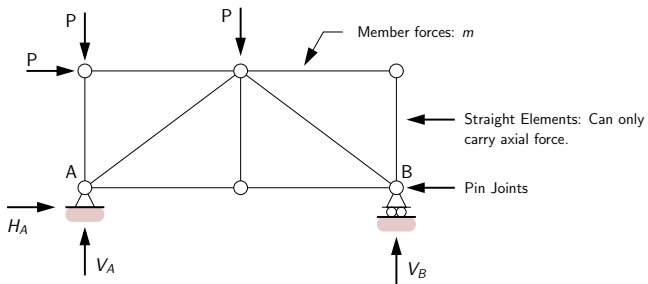Generally, indeterminate structures are stiffer than determinate structures.

Materials such as steel/concrete are displacement constrained.

For a maximum allowable displacement ($\triangle_{max}$), the load carrying capacity of indeterminate structures (P_i) is greater than determinate structures (P_d).

Statical Determinacy of Trusses

Trusses

Formulae: If the truss has j joints $\rightarrow$ 2j equations of equilibrium.

$$\sum F_x = 0, \quad \sum F_y = 0. \tag{1}$$

Unknowns: No of reactions r, and no of member forces m.

Determinacy of Trusses

Test covers three categories:

- Truss is statically determinate: $m + r = 2j$.
- If $m + r < 2j \leftarrow$ Truss is unstable.
- If $m + r > 2j \leftarrow$ Truss is statically indeterminate.

Note. Tests are necessary but not sufficient.

For our three test cases:

Test Structure A: $r = 3$, $m = 7$, and $j = 5$.

- $m + r - 2j = 0 \rightarrow$ statically determinate.

Test Structure B: $r = 3$, $m = 8$, and $j = 5$.

- $m + r - 2j > 0 \rightarrow$ statically indeterminate.

Introduction   Statical Determinacy of Trusses   Statical Determinacy of Planar Structures   Indeterminacy of Beams   Indeterminacy o

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Determinacy of Trusses

Test Structure C: $r = 3$, $m = 7$, and $j = 5$.

- $m + r - 2j = 0 \rightarrow$ statically determinate?

Bottom Line:

- Last test says statically determinate, but actually the test is faulty because structure is unstable.
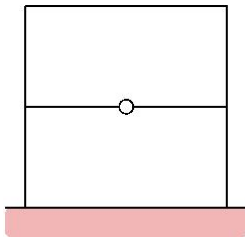
Statical Determinacy of Planar Structures

Planar Frame Structures

Three equations of equilibrium for each free body diagram:

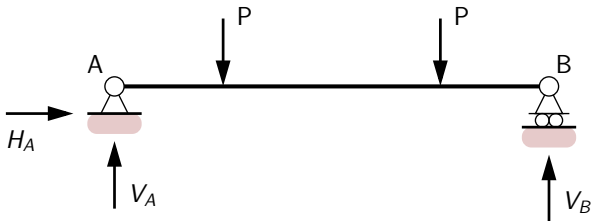If structure has n members and r
unknown reactions,

Test:

- If $r = 3n \rightarrow$ statically determinate.

- If $r > 3n \rightarrow$ statically
  indeterminate.

- If $r < 3n \rightarrow$ structure is unstable.
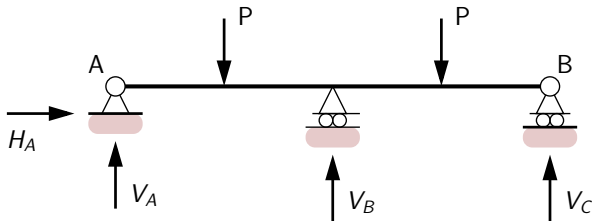
Planar Structures

Example 1.

$n = 1$. $r = \{H_A, V_A, V_B\} = 3$.

Test: $r - 3n = 0 \rightarrow$ statically determinate.
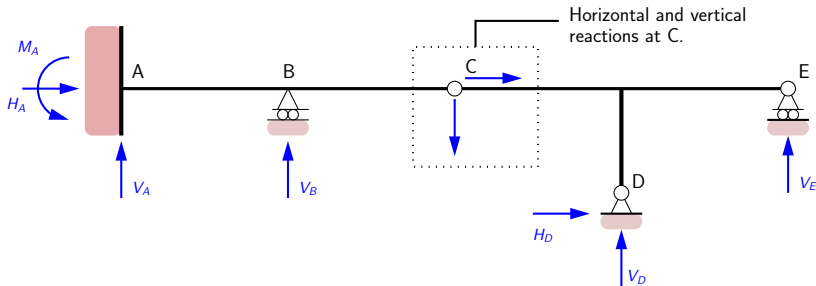
Planar Structures

Example 2.

$n = 1$. $r = \{H_A, V_A, V_B, V_C\} = 4$.

Test: r - 3n = 1 > 0 $\rightarrow$ statically indeterminate.

Planar Frame Structures

Example 3.

Horizontal and vertical reactions at C.

Two members: $n = 2$.

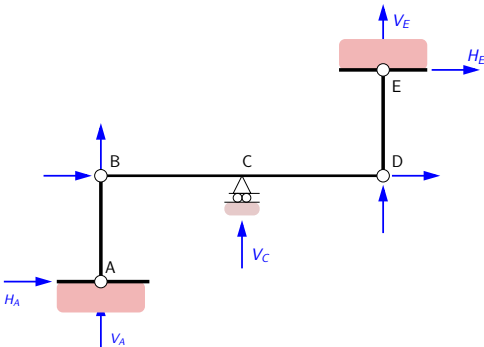No reactions $r = \{H_A, V_A, M_A, \cdots, V_E\} = 9$.

Test: r - 3n = 3 > 0 → statically indeterminate to degree 3.

Planar Frame Structures

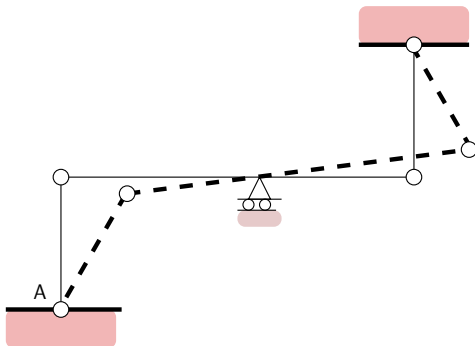**Counter Example 4.** Example demonstrates test is necessary but not sufficient.

Three members: $n = 3$. No reactions r $= \{H_A, V_A, \cdots, H_E\} = 9$.

Test: r - 3n $= 0 \rightarrow$ statically determinate.

Planar Frame Structures

But this configuration is also a mechanism, i.e.,

Conclusion: Test is necessary but not sufficient!