Introduction to Structural Analysis

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Overview

Part 2

Introduction

Definition of Structural Mechanics

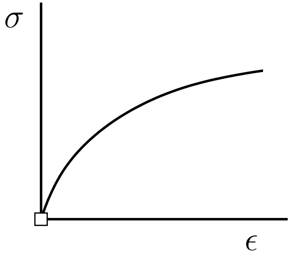
Mechanics. Branch of science that deals with response of matter to forces.

Civil Engineering:

- Structural mechanics (σ - ϵ): material displacement.
- Geomechanics (σ - ϵ): pressure, temperature, displacements.
- Fluid mechanics (σ - ϵ): pressure, velocities.

Other domains:

- Biomechanics (σ - ϵ): eye, heart, biological systems that grow!

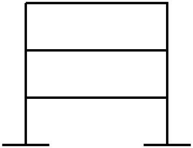


Connecting Mechanics to Analysis

Pathway from Mechanics to System-Level Behavior

From material-level mechanics to building-system response:

Material Response —integrate→ Section Response —integrate→ Beam Response —assemble→ Building System Response

Material Response	Section Response	Beam Response	Building System Response
Stress $\sigma(x, y)$ Strain $\epsilon(x, y)$	Curvature $\phi(x) = \left[\frac{M(x)}{EI}\right]$	Deflection $y(x)$ Slope dy/dx	

How will the integration work?

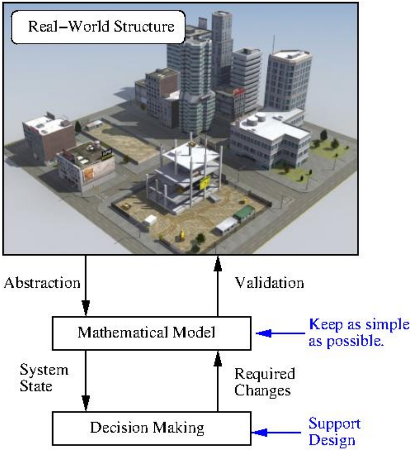
- Analytical Procedures: The math needs to be "nice" ...
- Numerical Proedures: Compute approximate solutions → linear algebra, numerical algorithms, structural analysis and finite elements.

Connecting Analysis to Design

Framework for Analysis and Design

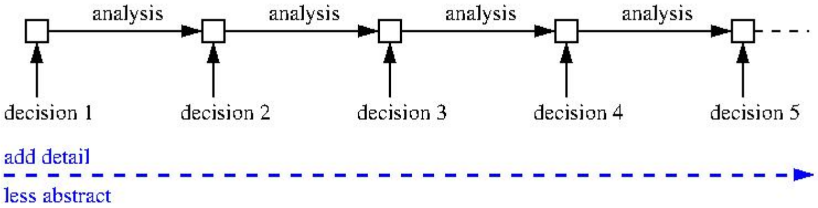
Creating an Analysis Model

- Abstract from consideration details not needed for decision making.
- Validate that model captures essential aspects of real-world behavior.
- Decision making needed for design.
- Perfect is the enemy of good. Mathematical model and decision making does not need to be perfect in order to be useful.



Connecting Analysis to Design

Structural Design. Sequence of analyses punctuated by decision making.

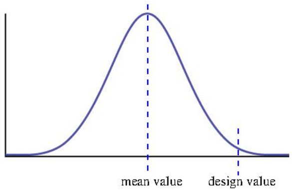


- Determine types and magnitudes of loads and forces acting on the structure.
- Determine context of project: geometric constraints, architectural constraints, geological conditions, urban regulations, cost, schedule, etc.

Connecting Analysis to Design

- Generate structural system alternatives.
- Analyze one or more of the alternatives.
- Select and perform detailed design.
- Implement/build.

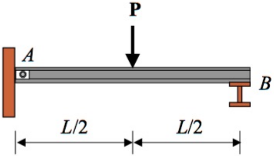
Analysis and decision making procedures complicated by uncertainties in loading, material properties, etc. State-of-the-art methods compensate for uncertainties with safety factors.



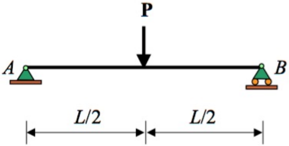
New structural systems may also require an experimental testing phase to verify behavior and achievable system performance.

Connecting Analysis to Design

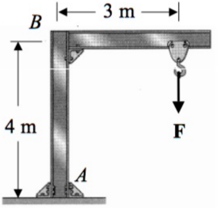
Real-World and Idealized Abstractions



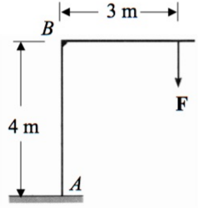
actual beam



idealized beam



actual structure



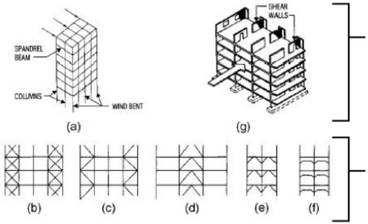
idealized structure

Connecting Analysis to Design

Formal Approaches to Behavior Modeling and Decision Making

Appropriate formalisms depend on the design domain of interest.

- Physical aspects of behavior are often characterized by differential equations.
- Logical aspects of system design can be captured by binary and multi-valued logic variables and boolean equations.



Structural behavior corresponds to solution of differential equations.

Selection of the structural system can be written mathematically as a problem in multi–valued logic.

Connecting Analysis to Design

Structural Behavior

Time-dependent behavior corresponds to solutions of:

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$$[M]\frac{d^2x}{d^2t} + [C]\frac{dx}{dt} + [K]\,x = P(t). \tag{1}$$

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Here,

- M, C, and K are (n × n) matrices,
- x is a (n × 1) vector of displacements,
- P(t) is a vector of external loads applied to the structural degrees of freedom.

Design Parameters

- Selection of the best structural system (e.g., braced system) from a list of options.
- Size of the beams, columns, and bracing (if required).

Theory of Structures

Statically Determinate Structures

Definition. Can use statics to determine reactions and distribution of element-level forces. Determinacy is not affected by details of loading.

Two-Dimensional Problems

$$\sum F_x = 0, \ \sum F_y = 0, \ \sum M_z = 0. \tag{1}$$

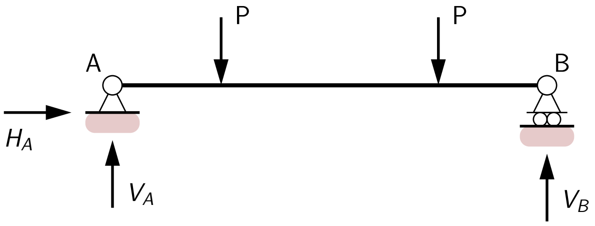
Three-Dimensional Problems

$$\sum F_x = 0, \ \sum F_y = 0, \ \sum F_z = 0. \tag{2}$$

$$\sum M_x = 0, \ \sum M_y = 0, \ \sum M_z = 0. \tag{3}$$

Statically Determinate Structures

Example 1. Simply supported beam:

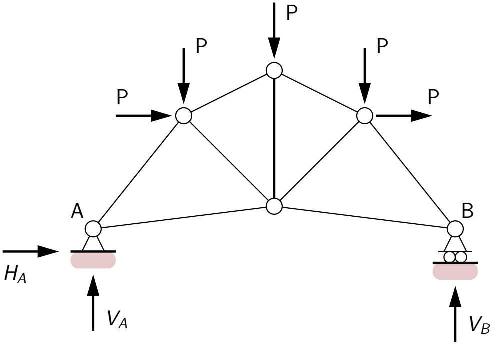


Three equations of equilibrium: $\sum F_x = 0, \ \sum F_y = 0, \ \sum F_z = 0.$

Three unknowns: V_A, H_A and V_B $\rightarrow$ Can use statics to solve.

Statically Determinate Structures

Example 2. Small truss structure:

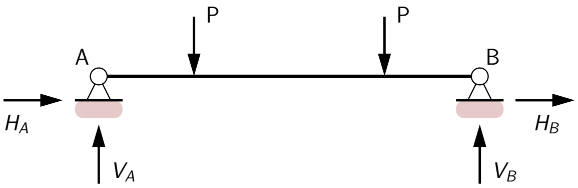


- Use statics to find support reactions V_A, H_A and V_B.
- Compute member forces by considering equilibrium of individual joints.

Statically Indeterminate Structures

Definition. Statics alone are not enough to find reactions. Need to find additional information (e.g., material behavior).

Example 1. Simply supported beam:

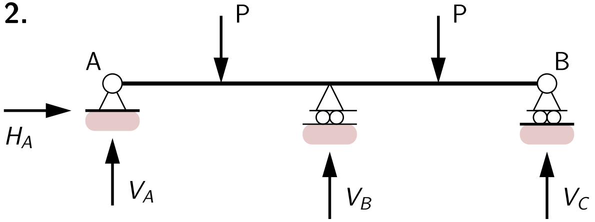


Three equations of equilibrium: $\sum F_x = 0, \ \sum F_y = 0, \ \sum F_z = 0.$

Four unknowns: V_A, H_A, V_B and H_B $\rightarrow 4 > 3 \rightarrow$ statically indeterminate to degree 1.

Statically Indeterminate Structures

Example 2.



Three equations of equilibrium. Four unknowns: V_A, H_A, V_B and V_C $\rightarrow 4 > 3 \rightarrow$ statically indeterminate to degree 1.

Example 3. Multi-material Truss Element.

Material behavior defined by $\sigma - \epsilon$ characteristics. Need to maintain geometric compatibility.