Analysis of Beam Structures

Mark A. Austin

University of Maryland

austin@umd.edu
ENCE 353, Fall Semester 2020

September 3, 2020

Overview

1. Types of Beam Structure
2. Connection to Mechanics

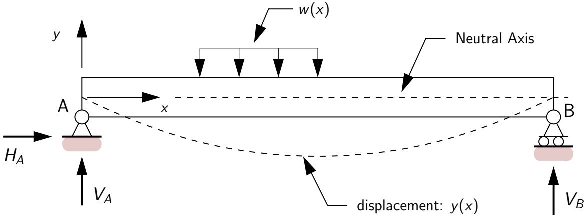
Part 1

3. Relationship between Shear Force and Bending Moment
   - Mathematical Preliminaries
   - Derivation of Equations
4. Examples

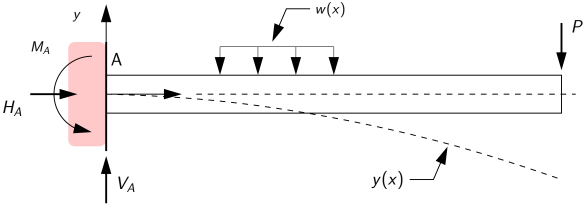
Types of Beam Structure

Types of Beam Structures

Simply Supported Beam:

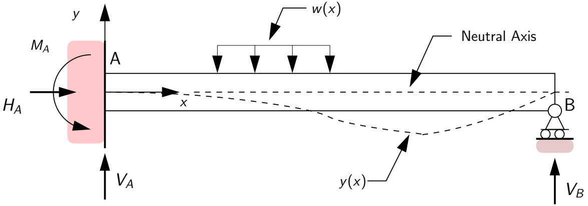


Cantilever:

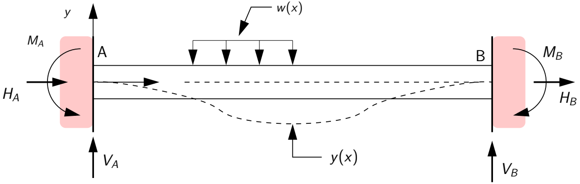


Types of Beam Structures

Supported Cantilever:



Fixed-Fixed Beam Structure:



Types of Beam Structures

Boundary Conditions

Simply Supported Beam

- $y(0) = y(L) = 0.$

Cantilever Beam

- $y(0) = 0, \ \frac{dy}{dx}|_{x=0} = 0$

Supported Cantilever Beam

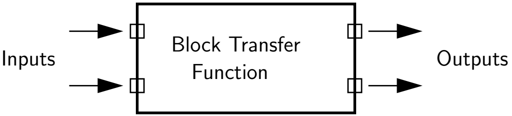
- $y(0) = y(L) = 0, \ \frac{dy}{dx}|_{x=0} = 0$

Fixed-Fixed Beam

- $y(0) = y(L) = 0, \ \frac{dy}{dx}|_{x=0} = \frac{dy}{dx}|_{x=L} = 0$

Basic Questions

Q1. What is the relationship between inputs and outputs?

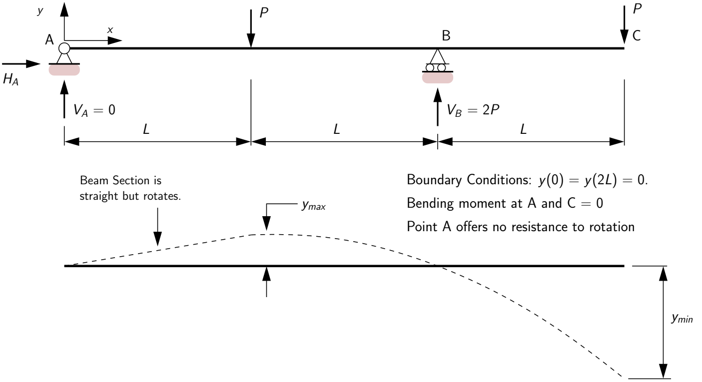


Inputs		Outputs
Applied loads (P and w)		Shear force – $V(x)$
Boundary conditions	mechanics →	Bending moment – $M(x)$
Beam geometry (L and I)		Axial force – $N(x)$
Material Properties (E)		Displacement – $y(x)$
		Rotation – $\theta(x) = \left[\frac{dy}{dx}\right]$

Decisions will be based on estimates of outputs.

Basic Questions

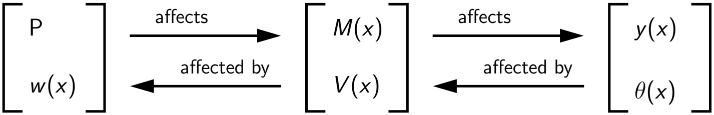
Typical problem: Given input parameters, compute $y(x)$, find location and magnitude of y_{min} and y_{max}.



For simple problems, can rely on intuition. Otherwise, need math and mechanics.

Basic Questions

Q2. What is the relationship among the outputs? Are they dependent?



Applied loads + geometry.

Bending moments + shear forces.

Displacements + rotations.

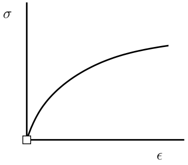
We will need to work with a chain of dependencies.

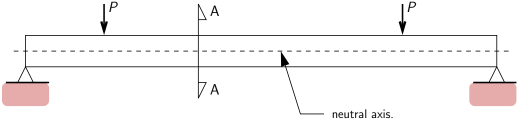
Q3. What is the relationship between $V(x)$ and $M(x)$? Are they independent? No!
We will see: $V(x) = \frac{dM(x)}{dx}$, but not always true!

Connection to Mechanics

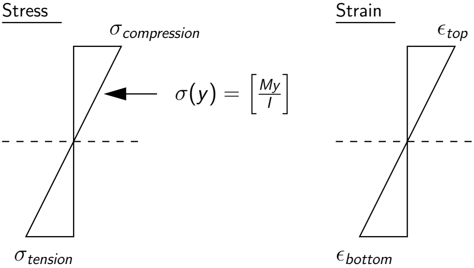
Connection to Mechanics

Problem Setup





Stress-Strain Relationships



$$\sigma(y) = \left[\frac{My}{I}\right]$$

Connection to Mechanics

For design purposes we need to make sure:

$$\sigma_{tension} < \sigma_{\text{max tension}} \tag{1}$$

and

$$\sigma_{compression} < \sigma_{\text{max compression}} \tag{2}$$

Also,

$$\epsilon_{\text{max compression}} \leq \epsilon(y) \leq \epsilon_{\text{max tension}} \tag{3}$$

These constraints limit the amount of load that a beam can carry.

Connection to Mechanics

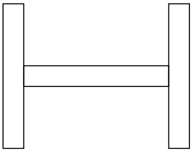
Section-Level Behavior

From a design standpoint we can reduce $\sigma(y)$ and $\epsilon(y)$ by increasing the moment of interia in

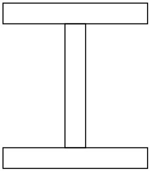
$$\sigma(y) = \left[\frac{My}{I} \right]. \tag{4}$$

To maximise I, maximize distance of material from neutral axis.

Poor Choice of Inertia



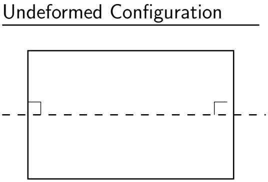
Good Choice of Inertia

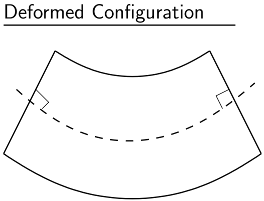


Assumptions on Beam Displacements

Assumptions. We will assume beam length / depth $\gg 10$.

Therefore, displacements will be dominated by flexural bending.





Sections remain perpendicular to the deformed neutral axis.

This is not the case for shear deformations.