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Energy Methods in Structural Analysis.

Goal: Determine Internal forces & displacements in statically determinate structures.

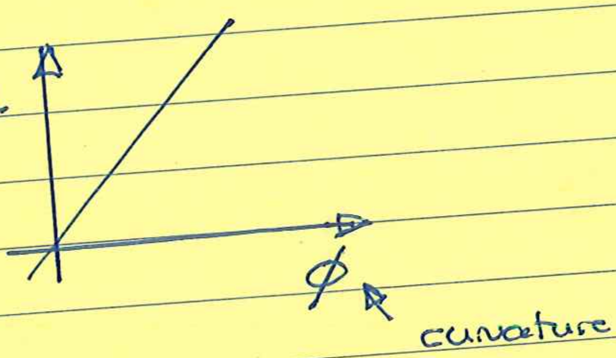
Guiding Principles

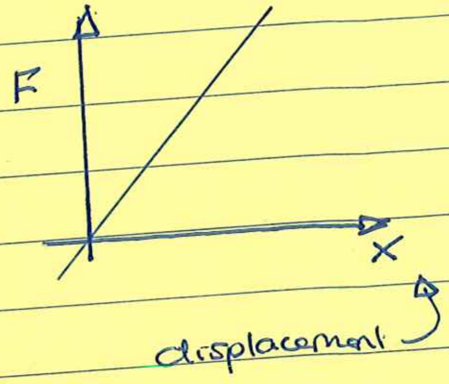
When a load is applied to a structure:

1. Applied loads & reactions must be in static equilibrium (no mechanisms).
2. Displacements need to be geometrically compatible (structures are not allowed to fall apart).

Energy in Structural Systems.

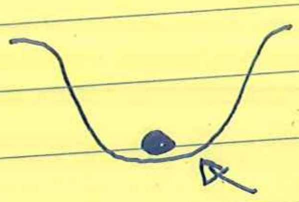
Sources:

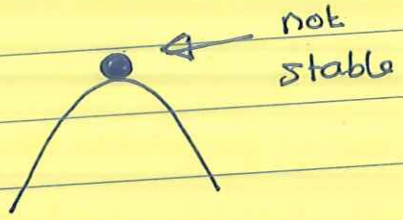




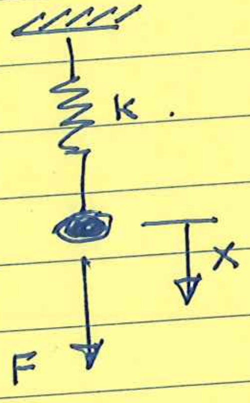
Equilibrium ⟺ minimum potential energy state.

(B)





<u>Simple Example</u>

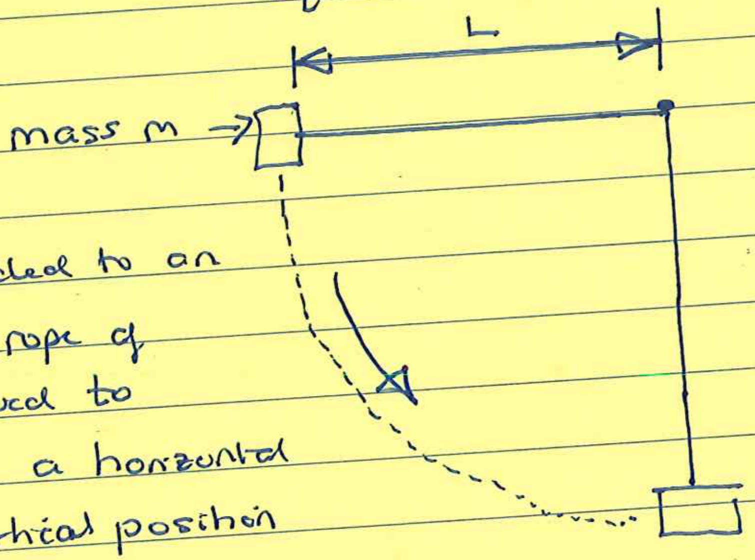


$$\text{Potential energy } E = \underbrace{\tfrac{1}{2}kx^2}_{\text{potential energy stored in spring}} - \underbrace{Fx}_{\text{Work done by applied load}}$$

$$\text{Equilibrium} \Rightarrow \left(\frac{dE}{dx}\right) = 0.$$

$$\frac{dE}{dx} = \underbrace{kx - F}_{kx = F \text{ is equation of equilibrium.}} = 0$$

<u>Home Exercise</u>:



A mass m, connected to an inextensible light rope of length L, is allowed to swing down from a horizontal position, to a vertical position as shown in adjacent diagram.

Ⓒ Questions:

1. Show that the maximum tension in the rope is 3mg. (hint: consider gravity + angular acceleration effects).

2. Explain why length of the rope does not affect the maximum tension?

3. Now suppose that the rope is replaced by a piece of extensible rubber having spring constant k, and of the same length L.

   Show that the maximum tension reached in the rubber when the mass is allowed to swing down is given by.

$$T = 3mg\left[\frac{L+x}{L+2x}\right]$$

   where L is the unstretched length of the rubber & x is the maximum extension of the rubber.

4. Explain why, despite the mass falling a greater distance, the maximum tension in the rubber is less than the max tension in the rope.

Hint: For parts 3 & 4, think about the various forms of potential, kinetic & stored energy.

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Two Energy Theorems.

1. **Principle of Virtual Displacements.** (use equilibrium to compute forces and reactions).

   If a set of virtual displacements is imposed on a structure in equilibrium, then the change in external work done is balanced by the change of internal work.

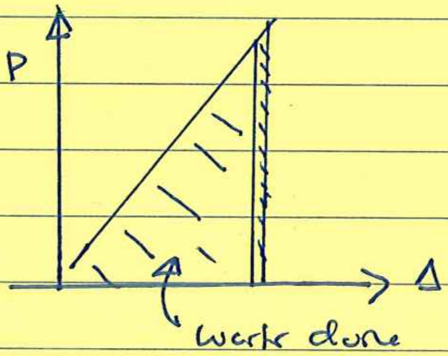
2. **Principle of Virtual Forces.** (use compatibility conditions to compute displacements).

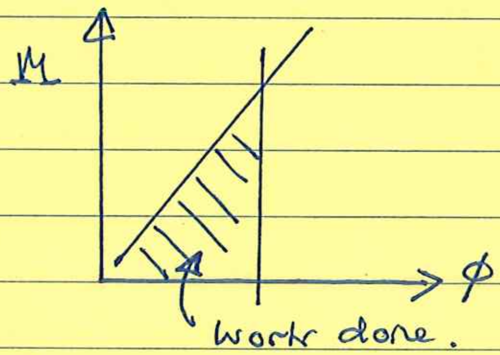
   If a set of virtual forces is imposed on a structure that satisfies conditions of geometric compatibility, then the change in external work done = change in internal work done.

Notation

Indicate : (a) Real system by superscript *
(b) Virtual system by superscript **

Recall.





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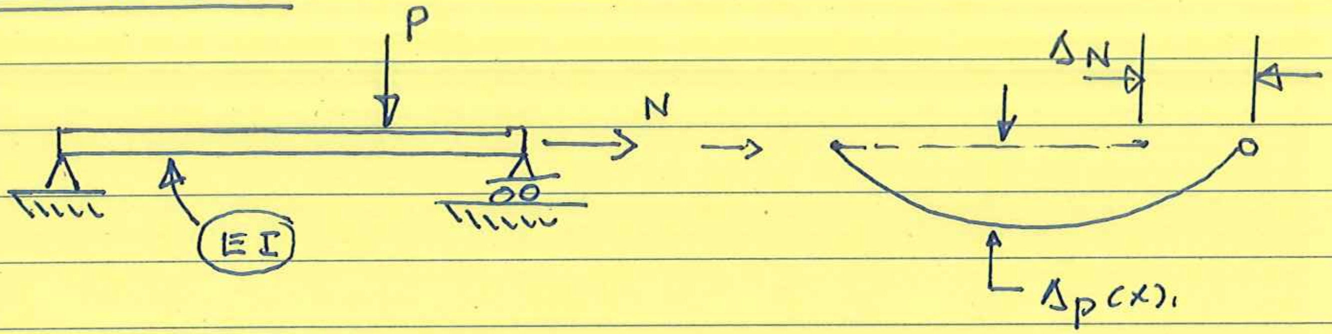
Test Problem.



Δ_N = axial extension due to applied load N.

$\Delta_p(x)$ = flexural displacement due to applied load P.

First Theorem (Virtual Displacements)

$$\underbrace{P^{*}\Delta_p^{**} + N^{*}\Delta_N^{**}}_{\text{External Energy}} = \underbrace{\int_0^L M^{*}\underset{\substack{\uparrow\\ \text{virtual}\\ \text{curvature}}}{\phi^{**}}\,dx + \int_0^L N^{*}\underset{\substack{\uparrow\\ \text{virtual}\\ \text{axial strain}}}{\epsilon^{**}}\,dx.}_{\text{internal energy.}}$$

Second Theorem (Virtual Forces).

$$P^{**}\Delta_p^{*} + N^{**}\Delta_N^{*} = \int_0^L M^{**}\underset{\uparrow}{\phi^{*}}\,dx + \int_0^L N^{**}\underset{\nearrow}{\epsilon^{*}}\,dx.$$

From mechanics $\phi^{*} = \left(\frac{M^{*}}{EI}\right)$ $\epsilon^{*} = \left(\frac{N^{*}}{AE}\right)$