Cable Problem Solutions

Department of Civil and Environmental Engineering, Fall Semester, 2018

ENCE 353 Final Exam, Open Notes and Open Book

Name: AUSTIN.

Exam Format and Grading. The exam will be 2 hrs plus five minutes to read the questions.

Answer question 1. Then answer **three of the five** remaining questions.

Only the first four questions that you answer will be graded, so please **cross out the two questions you do not want graded** in the table below. Partial credit will be given for partially correct answers, so please show all your working.

After you have finished working on the exam, look at the bonus problem for additional credit. No partial credit for this part of the exam.

Question	Points	Score
1	20	
2	10	
3	10	
4	10	
5	10	
6	10	
Bonus	5	
Total	50	

Question 6: 10 points

OPTIONAL: Compute Support Reactions in a Suspension Bridge. Figure 6 is a front elevation view of a suspension bridge that has three spans, two support towers, and anchor supports at the bridge endpoints.

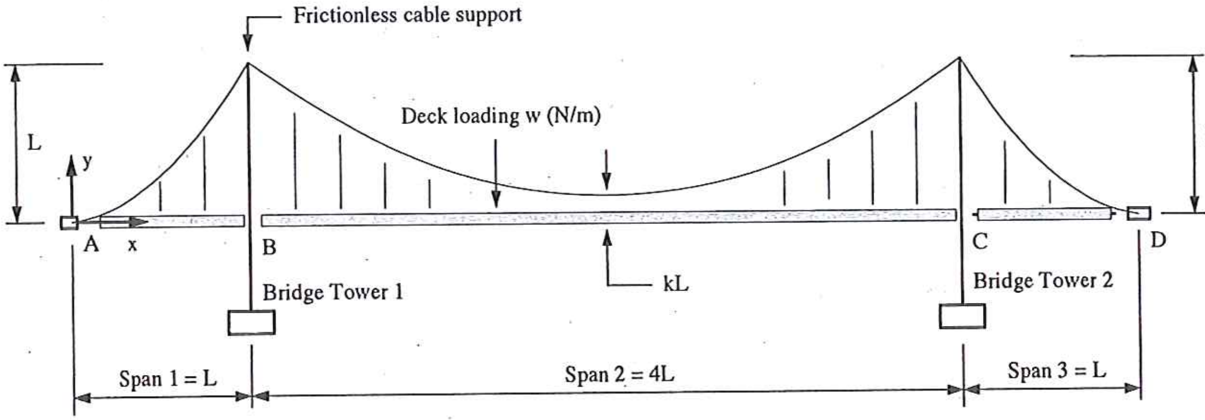

Figure 6: Elevation view of a three-span suspension bridge.

Spans 1, 2 and 3 have lengths L, $4L$ and L, respectively, and carry a uniform load w (N/m). The towers have height L above the bridge deck level. At the mid-point of Span 2, the lowest point of the cable profile is kL above the bridge deck.

The purpose of this question is to work step by step toward the computation of vertical support reactions at the cable anchor points (i.e., at points A and D), and at the towers (i.e., at points B and C). For the purposes of this analysis, assume that the bridge cable weight is bundled into the applied loads w, and can otherwise be ignored. Also, assume that the cable passes through the top of the towers on a frictionless support and, as a result, the horizontal component of cable force will be constant along the entire bridge.

[6a] (2 pts) By examining equilibrium of the cable profile in Span 2, show that the horizontal component of cable force is:

$$H = \left[\frac{2}{1-k}\right] wL. \quad (6)$$

$$\frac{d^2y}{dx^2} = \frac{w}{H}$$

$$y(x) = \frac{wx^2}{2H}$$

$$y(2L) = (1-k)L \rightarrow H = \left[\frac{2}{1-k}\right] wL.$$

$$= \frac{w\,44^2}{2H} \Rightarrow (1-k)L = \frac{2L^2 w}{H}$$

[6b] (3 pts) Show that the equation of the cable profile in Span 1 is:

$$y(x) = \left[\frac{1-k}{4L}\right]x^2 + \left[\frac{3+k}{4}\right]x \tag{7}$$

$$\frac{d^2y}{dx^2} = \frac{W}{H}$$

$$\frac{dy}{dx} = \frac{Wx}{H} + A$$

$$y(x) = \frac{Wx^2}{2H} + Ax + B$$

$$\left.\begin{array}{l} y(0) = 0 \\ y(L) = L \end{array}\right\} \quad \begin{array}{l} B = 0 \\ A = \left(1 - \frac{WL}{2H}\right) \end{array}$$

$$y(x) = \frac{Wx^2}{2H} + \left(1 - \frac{WL}{2H}\right)x$$

but $H = \left(\frac{2}{1-k}\right)WL$

$$\Rightarrow y(x) = \left(\frac{1-k}{4L}\right)x^2 + \left(\frac{3+k}{4}\right)x.$$

[6c] (3 pts) Use the results from parts [6a] and [6b] to show that the vertical reaction at the anchor support (i.e., at point A) is:

$$V_A = \left[\frac{3+k}{2-2k}\right]wL \tag{8}$$

acting downwards.

$$\left.\frac{dy}{dx}\right|_{x=0} = \left[\frac{Wx}{H} + 1 - \frac{WL}{2H}\right]_{x=0} = \frac{V}{H}$$

$$H = \left(\frac{2}{1-k}\right)WL \Rightarrow V_A = \left(\frac{3+k}{2-2k}\right)WL.$$

[6d] (2 pts) Hence, show that the vertical support reaction at Tower 1 is:

$$V_B = \left[\frac{9 - 5k}{2 - 2k}\right] wL. \tag{9}$$

acting upwards.

$$V_B = 3wL + V_A = \left(\frac{6 - 6k}{2 - 2k}\right) wL + \frac{(3 + k)}{(2 - 2k)} wL$$

$$= \left(\frac{9 - 5k}{2 - 2k}\right) wL.$$