

Solutions to Homework 1

Question 1: 10 points. Write a program that solves for all positive integer pairs, i.e., $a, b \geq 0$,

$$\sqrt{a} + \sqrt{b} = \sqrt{n} \quad (1)$$

where $n = 2025$.

Hint: 2025 happens to be a perfect square, with the prime factorization of $3 \cdot 3 \cdot 3 \cdot 3 \cdot 5 \cdot 5$. You can use this fact and a little bit of number theory to see that there will only be 46 solutions (i.e., (a,b) pairs).

Python Source Code:

```
# =====
# TestIntegerSolutions2025.py: Compute positive integer solutions to:
#
#         math.sqrt(a) + math.sqrt(b) = math.sqrt(2025).
#
# where a and b are integers greater than or equal to zero.
#
# Notice: 2025 is a perfect square (3*3*5)*(3*3*5).
#
# Written by: Mark Austin                                January, 2025
# =====

import math

# main method ...

def main():
    print("--- Enter TestIntegerSolutions2025.main()    ... ");
    print("--- ===== ... ");

    n = 2025

    # Part 1: Use nested for loop to exhaustively test all cases.
    #         : Naively test for equality ...

    print("--- ")
    print("--- Part 1: Nested loops, naive test for equality ...")

    i = 1
```

```

for a in range(0, n+1):
    for b in range(0, n+1):
        error = math.sqrt(a) + math.sqrt(b) - math.sqrt(n)
        if error == 0:
            print("--- soln {:2d}: a = {:6d}, b = {:6d}: solution !! ...".format(i,a,b) );
            i = i + 1

# Part 2: Use nested for loop to exhaustively test all cases.
#       : Account for imprecise number representation ...

print("--- ")
print("--- Part 2: Nested loops, account for inexact arithmetic ...")

i = 1
for a in range(0, n+1):
    for b in range(0, n+1):
        error = math.sqrt(a) + math.sqrt(b) - math.sqrt(n)
        if math.fabs(error) < 0.000000000000001:
            print("--- soln {:2d}: a = {:6d}, b = {:6d}: solution !! ...".format(i,a,b) );
            i = i + 1

# Part 3: Use number theory to reduce number of cases ...

print("--- ")
print("--- Part 3: Use number theory to reduce number of cases to evaluate ...")

mmax = 45
m = list( range(0, mmax + 1) );

# Traverse list and verify expression values ...

for i in m:
    j = mmax - i
    a = i*i
    b = j*j
    error = math.sqrt(a) + math.sqrt(b) - math.sqrt(n)
    if math.fabs(error) < 0.000000000000001:
        print("--- a = {:6d}, b = {:6d}: solution !! ...".format(a,b) );
    else:
        print("--- a = {:6d}, b = {:6d}: not a solution !! ...".format(a,b) );

print("--- ===== ... ");
print("--- Leave TestIntegerSolutions2025.main() ... ");

# call the main method ...

if __name__ == "__main__":
    main()

```

Program Output: The abbreviated textual output is:

```

--- Enter TestIntegerSolutions2025.main() ...
--- ===== ...

```

```

---
--- Part 1: Nested loops, naive test for equality ...
--- soln 1: a =      0, b = 2025: solution !! ...
--- soln 2: a =      1, b = 1936: solution !! ...
--- soln 3: a =      4, b = 1849: solution !! ...
--- soln 4: a =      9, b = 1764: solution !! ...
--- soln 5: a =     16, b = 1681: solution !! ...
--- soln 6: a =     25, b = 1600: solution !! ...
--- soln 7: a =     36, b = 1521: solution !! ...
--- soln 8: a =     49, b = 1444: solution !! ...
--- soln 9: a =     64, b = 1369: solution !! ...
--- soln 10: a =    81, b = 1296: solution !! ...
--- soln 11: a =   100, b = 1225: solution !! ...
--- soln 12: a =   121, b = 1156: solution !! ...
--- soln 13: a =   144, b = 1089: solution !! ...
--- soln 14: a =   169, b = 1024: solution !! ...
--- soln 15: a =   196, b =  961: solution !! ...
--- soln 16: a =   225, b =  900: solution !! ...
--- soln 17: a =   256, b =  841: solution !! ...
--- soln 18: a =   289, b =  784: solution !! ...
--- soln 19: a =   324, b =  729: solution !! ...
--- soln 20: a =   361, b =  676: solution !! ...
--- soln 21: a =   400, b =  625: solution !! ...
--- soln 22: a =   441, b =  576: solution !! ...
--- soln 23: a =   484, b =  529: solution !! ...
--- soln 24: a =   529, b =  484: solution !! ...
--- soln 25: a =   576, b =  441: solution !! ...
--- soln 26: a =   625, b =  400: solution !! ...
--- soln 27: a =   676, b =  361: solution !! ...
--- soln 28: a =   729, b =  324: solution !! ...
--- soln 29: a =   784, b =  289: solution !! ...
--- soln 30: a =   841, b =  256: solution !! ...
--- soln 31: a =   900, b =  225: solution !! ...
--- soln 32: a =   961, b =  196: solution !! ...
--- soln 33: a =  1024, b =  169: solution !! ...
--- soln 34: a =  1089, b =  144: solution !! ...
--- soln 35: a =  1156, b =  121: solution !! ...
--- soln 36: a =  1225, b =  100: solution !! ...
--- soln 37: a =  1296, b =   81: solution !! ...
--- soln 38: a =  1369, b =   64: solution !! ...
--- soln 39: a =  1444, b =   49: solution !! ...
--- soln 40: a =  1521, b =   36: solution !! ...
--- soln 41: a =  1600, b =   25: solution !! ...
--- soln 42: a =  1681, b =   16: solution !! ...
--- soln 43: a =  1764, b =    9: solution !! ...
--- soln 44: a =  1849, b =    4: solution !! ...
--- soln 45: a =  1936, b =    1: solution !! ...
--- soln 46: a = 2025, b =    0: solution !! ...
---
--- Part 2: Nested loops, account for inexact arithmetic ...
--- soln 1: a =      0, b = 2025: solution !! ...
--- soln 2: a =      1, b = 1936: solution !! ...

```

... lines of output removed ...

```

--- soln 44: a = 1849, b = 4: solution !! ...
--- soln 45: a = 1936, b = 1: solution !! ...
--- soln 46: a = 2025, b = 0: solution !! ...
---
--- Part 3: Use number theory to reduce number of cases to evaluate ...
--- a = 0, b = 2025: solution !! ...
--- a = 1, b = 1936: solution !! ...
--- a = 4, b = 1849: solution !! ...

... lines of output removed ...

--- a = 1849, b = 4: solution !! ...
--- a = 1936, b = 1: solution !! ...
--- a = 2025, b = 0: solution !! ...
--- ===== ...
--- Leave TestIntegerSolutions2025.main() ...

```

Question 2: 10 points. A square of sheet metal having side length $2L$ cm has four pieces cut out symmetrically from the corners as shown in Figure 1.

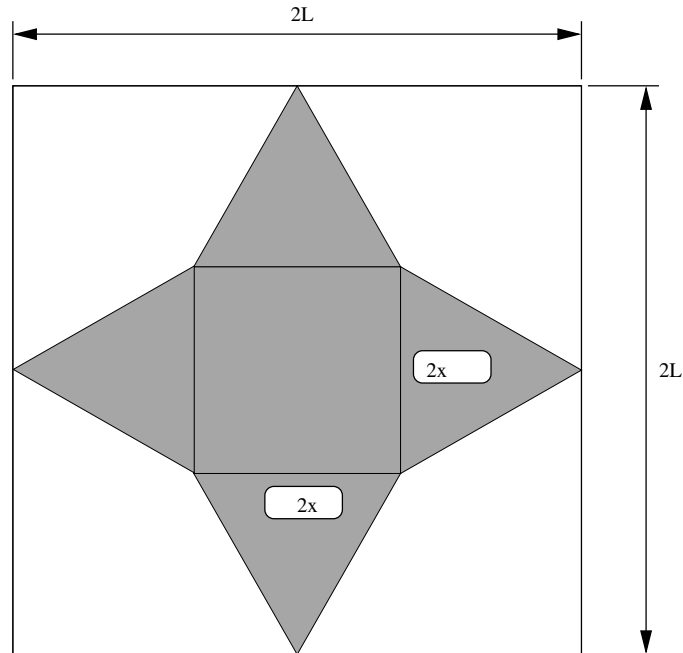


Figure 1: Sheetmetal schematic for a folded pyramid.

Assuming that L is a constant and $L > 2x$, then the remaining metal can be folded into a pyramid having volume:

$$\text{Volume}(x) = \frac{4x^2}{3} \sqrt{L^2 - 2Lx} \quad \text{cm}^3. \quad (2)$$

The maximum volume occurs when $x = 2L/5$ cm.

Write a Python program that sets a value for length $L = 10$ cm, and then systematically computes and print volumes for appropriate values of x . Organize your output into a tidy table, e.g., something like:

```

--- Pyramid: L = 10.0 cm
+-----+-----+
| X (cm) | Volume (cm^3) |
+-----+-----+
| 0.00 | 0.00 |
| 0.50 | 3.16 |
| 1.00 | 11.93 |
| 1.50 | 25.10 |
| 2.00 | 41.31 |
| 2.50 | 58.93 |

```

	3.00		75.89	
	3.50		89.46	
	4.00		95.41	
	4.50		85.38	
	5.00		0.00	
+-----+-----+				

Python has a package called `prettytable` (i.e., `pip3 install prettytable`) which you might find useful. A small test program for pretty tables can be found in: `python-code.d/basics/TestPrettyTable01.py`.

Python Source Code:

```
# =====
# TestPyramidVolume02.py: Compute pyramid volume. Display results
#                          in a prettytable.
#
# Written by: Mark Austin                      January, 2025
# =====

import math
import numpy as np

from prettytable import PrettyTable

# Compute pyramid volume (for x <= 0.5L).

def PyramidVolume(L, x):
    return (4.0/3.0)*x*x*math.sqrt(L*L - 2*L*x);

# Main function ...

def main():
    print("--- Enter TestPyramidVolume.main()          ... ");
    print("--- ===== ... ");
    print("");

    # Create table of coordinates and pyramid volumes ...

    L = 10 # <--- L = 10 cm ...

    # Create array of x coordinates ...

    coords = np.linspace(0,5.0,11)

    # Compute pyramid volumes ...

    volume = []
    for x in coords:
        vol = PyramidVolume(L,x);
        volume.append( vol );

    print("--- ");
```

```

print("--- Create table of pyramid volumes ...");
print("--- ");

table01 = PrettyTable()
table01.add_column( "X (cm)", coords )
table01.add_column( "Volume (cm^3)", volume )

table01.float_format = ".2"          # print floating point numbers to 2 decimal places ...
table01.align["Volume (cm^3)"] = "r" # right alignment of volume column ...

print("--- Pyramid: L = {:.1f} cm".format(L));
print(table01)

print("--- ");
print("--- ===== ... ");
print("--- Leave TestPyramidVolume02.main()      ... ");

# call the main method ...

main()

```

Program Output:

```

--- Pyramid: L = 10.0 cm
+-----+-----+
| X (cm) | Volume (cm^3) |
+-----+-----+
| 0.00 | 0.00 |
| 0.50 | 3.16 |
| 1.00 | 11.93 |
| 1.50 | 25.10 |
| 2.00 | 41.31 |
| 2.50 | 58.93 |
| 3.00 | 75.89 |
| 3.50 | 89.46 |
| 4.00 | 95.41 |
| 4.50 | 85.38 |
| 5.00 | 0.00 |
+-----+-----+

```

Question 3: 10 points. Figure 2 shows a two-dimensional grid of masses.

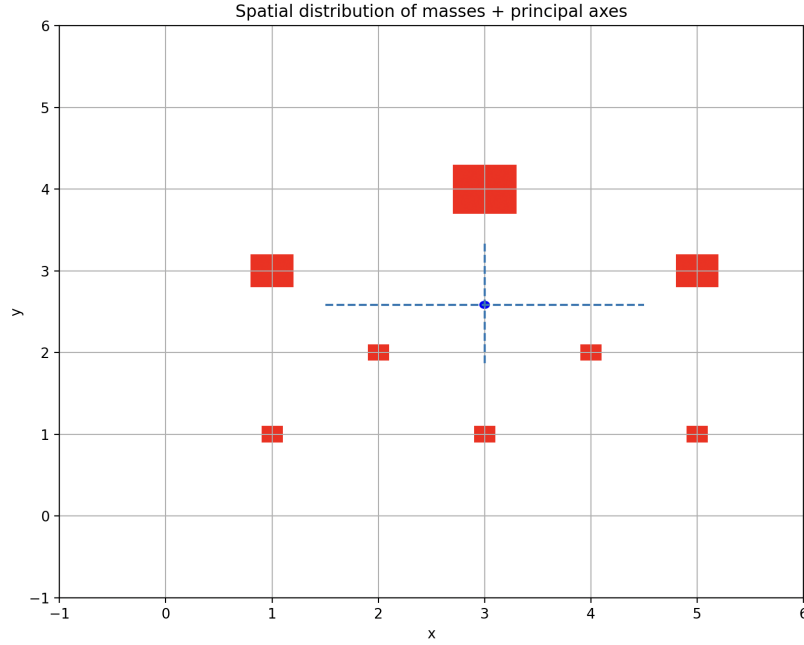


Figure 2: Two-dimensional grid of masses + principal axes.

If the total number of point masses is denoted by N , then the total mass of the grid, M , is given by

$$M = \sum_{i=1}^N m_i \quad (3)$$

The coordinates of the grid centroid, $(\bar{x}, \bar{y})$, are defined by:

$$M\bar{x} = \sum_{i=1}^N x_i \cdot m_i \quad \text{and} \quad M\bar{y} = \sum_{i=1}^N y_i \cdot m_i \quad (4)$$

The moments of inertia about the x- and y-axes are given by:

$$I_{xx} = \sum_{i=1}^N y_i^2 \cdot m_i \quad \text{and} \quad I_{yy} = \sum_{i=1}^N x_i^2 \cdot m_i \quad (5)$$

respectively. Similarly the cross moment of inertia is given by

$$I_{xy} = \sum_{i=1}^N x_i \cdot y_i \cdot m_i \quad (6)$$

With solutions to equations 4 - 6 in hand, the corresponding moments of inertia about the centroid are given by the parallel axes theorem (Google: parallel axis theorem moments of inertia). Finally, the orientation of the principle axes are given by

$$\tan(2\theta) = \left[\frac{2I_{xy}}{I_{xx} - I_{yy}} \right] \quad (7)$$

Now suppose that the (x,y) coordinates and masses are stored in two arrays;

```
mass = np.array( [ 1.0, 1.0, 1.0, 1.0, 1.0, 2.0, 3.0, 2.0 ] );

coord = np.array( [ ( 1.0, 1.0 ),
                    ( 2.0, 2.0 ),
                    ( 3.0, 1.0 ),
                    ( 4.0, 2.0 ),
                    ( 5.0, 1.0 ),
                    ( 5.0, 3.0 ),
                    ( 3.0, 4.0 ),
                    ( 1.0, 3.0 ) ] );
```

Write a Python program to evaluate equations 3 – 7, and create a plot in Python similar to Figure 2. Add the centroid and principal axes (drawn with the appropriate orientation) to your plot.

Python Source Code:

```
# =====
# TestMomentsInertia02.py: Compute moments of inertia about the
# origin, centroid (x,y), and orientation of principal axes.
#
# Written by: Mark Austin January, 2025
# =====

import math
import numpy as np
import matplotlib.pyplot as plt

from matplotlib.patches import Rectangle
from matplotlib.patches import Circle
from matplotlib.lines import Line2D

# Main function ...
```

```

def main():
    print("--- Enter TestMomentsInertia02.main()          ... ");
    print("--- ===== ... ");
    print("");

    # Homework 01: Mass and coordinate arrays

    mass = np.array( [1.0, 1.0, 1.0, 1.0, 1.0, 2.0, 3.0, 2.0])
    coord = np.array( [ ( 1.0, 1.0 ), ( 2.0, 2.0 ), ( 3.0, 1.0 ), ( 4.0, 2.0 ),
                        ( 5.0, 1.0 ), ( 5.0, 3.0 ), ( 3.0, 4.0 ), ( 1.0, 3.0 ) ] );

    # Computing inertias about the axes and origin.

    Ixx = 0.0; Iyy = 0.0; Ixy = 0.0;
    for i in range(len(mass)):
        Ixx = Ixx + mass[i]*coord[i][1]*coord[i][1]
        Iyy = Iyy + mass[i]*coord[i][0]*coord[i][0]
        Ixy = Ixy + mass[i]*coord[i][0]*coord[i][1]

    # Print results ...

    print("--- Ixx (about axis)   = {:.2f}   ...".format(Ixx) );
    print("--- Iyy (about axis)   = {:.2f}   ...".format(Iyy) );
    print("--- Ixy (about origin) = {:.2f}   ...".format(Ixy) );

    # Compute centroid of masses (x,y) ...

    M = sum (mass);
    print("--- Total mass = {:.2f}   ...".format(M) );

    fmMassX = 0.0; fmMassY = 0.0;
    for i in range(len(mass)):
        fmMassX = fmMassX + mass[i]*coord[i][0];
        fmMassY = fmMassY + mass[i]*coord[i][1];

    centroidX = fmMassX/M;
    centroidY = fmMassY/M;

    print("--- Centroid (x,y) = ({:.2f}, {:.2f})   ...".format(centroidX, centroidY) );

    # Use parallel axis theorem to compute interias about centroid ...

    IxxCentroid = Ixx - M*centroidY*centroidY;
    IyyCentroid = Iyy - M*centroidX*centroidX;
    IxyCentroid = Ixy - M*centroidX*centroidY;

    print("--- Ixx (about centroid) = {:.2f}   ...".format(IxxCentroid) );
    print("--- Iyy (about centroid) = {:.2f}   ...".format(IyyCentroid) );
    print("--- Ixy (about centroid) = {:.2f}   ...".format(IxyCentroid) );

    # Compute orientation of principal axes ...

    angle = 0.0;
    if(Ixx != Iyy):
        angle = (1.0/2.0)*math.atan(IxyCentroid/(IxxCentroid - IyyCentroid));

```

```

print("--- Mohr's circle: angle = {:.3f} radians ...".format(angle) );

# Plot masses and coordinates ...
# Define Matplotlib figure and axis

fig, ax = plt.subplots()

# Draw masses as small/medium-sized rectangles ...

for i in range(len(mass)):

    xcoord = coord[i][0];
    ycoord = coord[i][1];

    dm = mass[i];
    if dm == 1:
        width = 0.2;
    elif dm == 2:
        width = 0.4;
    else:
        width = 0.6;

    ax.add_patch(Rectangle( (xcoord - width/2, ycoord-width/2), width, width, facecolor='red'))

# Draw centroid ...

radius = 0.05;
ax.add_patch( Circle( (centroidX, centroidY), radius, facecolor='blue') )

# Plot major principal axis ...

axislength = 1.5
x1 = centroidX - axislength*math.cos(angle);
y1 = centroidY - axislength*math.sin(angle);
x2 = centroidX + axislength*math.cos(angle);
y2 = centroidY + axislength*math.sin(angle);

xcoords = [ x1, x2 ]
ycoords = [ y1, y2 ]
ax.add_line( Line2D(xcoords, ycoords, linestyle='--') )

# Plot minor principal axis ...

x1 = centroidX - axislength/2.0*math.sin(angle);
y1 = centroidY + axislength/2.0*math.cos(angle);
x2 = centroidX + axislength/2.0*math.sin(angle);
y2 = centroidY - axislength/2.0*math.cos(angle);

xcoords = [ x1, x2 ]
ycoords = [ y1, y2 ]
ax.add_line( Line2D(xcoords, ycoords, linestyle='--') )

plt.title('Spatial distribution of masses + principal axes')
plt.ylabel('y')

```

```

plt.xlabel('x')
plt.ylim( -1, 6)
plt.xlim( -1, 6)
plt.grid(True)
plt.show()

print("--- ===== ... ");
print("--- Leave TestMomentsInertia02.main()      ... ");

# call the main method ...

if __name__ == "__main__":
    main()

```

Program Output: The abbreviated textual output is as shown in the question description.

```

--- Enter TestMomentsInertia02.main()      ...
--- ===== ...

--- Ixx (about axis)    = 95.00  ...
--- Iyy (about axis)    = 134.00  ...
--- Ixy (about origin) = 93.00  ...
--- Total mass = 12.00  ...
--- Centroid (x,y) = (3.00, 2.58)  ...
--- Ixx (about centroid) = 14.92  ...
--- Iyy (about centroid) = 26.00  ...
--- Ixy (about centroid) = 0.00  ...
--- Mohr's circle: angle = -0.000 radians ...
--- ===== ...
--- Leave TestMomentsInertia02.main()      ...

```

The graphical output is shown in Figure 2.

Question 4: 10 points. Figure 3 shows the cross-section of a T-shaped beam (also called T-beam). Reinforced concrete T-beams are commonly found in buildings and highway bridges.

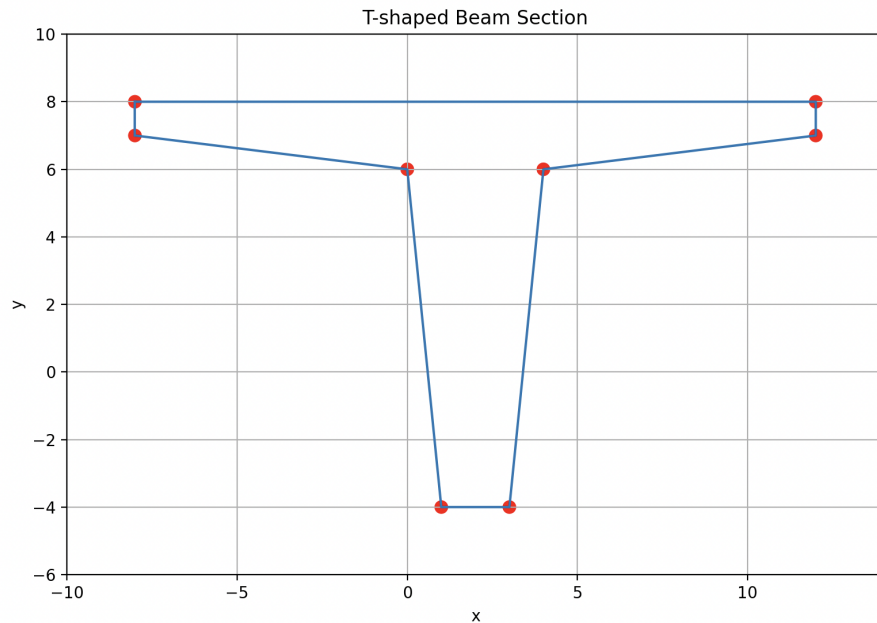


Figure 3: T-shaped beam cross section.

Under service load conditions, T-beams are expected to behave elastically, with very small displacements and no long-term damage. From a mechanics standpoint, the associated elastic analysis procedures require a knowledge of the section area and centroid, and moments of inertia. The purpose of this question is to take a first step toward the development of python code that will compute these section properties automatically. Later on (i.e., homeworks 2 and 3) we will step things up a bit by adding holes to the cross section, and modeling the whole cross section as an object.

Getting Started. The T-beam shown in Figure 3 has (x, y) coordinates stored as two columns of a numpy array:

```
coord = np.array( [ ( -8.0,  8.0 ),
                    ( 12.0,  8.0 ),
                    ( 12.0,  7.0 ),
                    (  4.0,  6.0 ),
                    (  3.0, -4.0 ),
                    (  1.0, -4.0 ),
                    (  0.0,  6.0 ),
                    ( -8.0,  7.0 ) ] );
```

Write a Python program that will:

1. Compute and print the minimum and maximum polygon coordinates in both the x and y directions.
2. Compute and print the minimum and maximum distance of the polygon vertices from the coordinate system origin.
3. Create a plot of the T-beam similar to Figure 3.
4. Write functions `perimeter()` and `area()` to compute the perimeter and area of the T-beam, respectively.

Python Source Code:

```
# =====
# TestPolygonTshapedBeam01.py: Compute centroid and moments of
# inertia for a T-shaped beam cross section.
#
# Written by: Mark Austin                                January 2025
# =====

import math
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.patches import Circle
from matplotlib.lines import Line2D

# Function to print two-dimensional matrices ...

def PrintMatrix(name, a):
    print("Matrix: {:s} ".format(name) );
    for row in a:
        for col in row:
            print("{:8.4f}".format(col), end=" ")
        print("")

# Compute polygon perimeter ...

def perimeter( coord ):
    norows = coord.shape[0];

    dperimeter = 0.0;
    for i in range(1,norows):
        dx = coord[i][0] - coord[i-1][0];
        dy = coord[i][1] - coord[i-1][1];
        dperimeter = dperimeter + math.sqrt( dx*dx + dy*dy );

    dx = coord[norows-1][0] - coord[0][0];
    dy = coord[norows-1][1] - coord[0][1];
    dperimeter = dperimeter + math.sqrt( dx*dx + dy*dy );

    return dperimeter;

# Compute polygon area ...

def area( coord ):
```

```

norows = coord.shape[0];

darea = 0.0;
for i in range(1,norows):
    dx = coord[i][0] - coord[i-1][0];
    dy = coord[i][1] + coord[i-1][1];
    darea = darea + dx*dy/2.0;

dx = coord[0][0] - coord[norows-1][0];
dy = coord[0][1] + coord[norows-1][1];
darea = darea + dx*dy/2.0;

return darea;

# =====
# main method ...
# =====

def main():
    print("--- Enter TestPolygonTshapedBeam01.main() ... ");
    print("--- ===== ... ");

    print("--- Part 1: Initialize t-beam coordinates ... ");

    coord = np.array( [ ( -8.0,  8.0 ),
                        ( 12.0,  8.0 ),
                        ( 12.0,  7.0 ),
                        (  4.0,  6.0 ),
                        (  3.0, -4.0 ),
                        (  1.0, -4.0 ),
                        (  0.0,  6.0 ),
                        ( -8.0,  7.0 ) ] );

    PrintMatrix("T-beam Coordinates", coord);

    print("--- ");
    print("--- Part 2: Max/min coordinate positions ... ");
    print("--- ");

    print("--- Min x = {:8.3f} ...".format( min ( coord[:,0] ) ) );
    print("--- Max x = {:8.3f} ...".format( max ( coord[:,0] ) ) );
    print("--- Min y = {:8.3f} ...".format( min ( coord[:,1] ) ) );
    print("--- Max y = {:8.3f} ...".format( max ( coord[:,1] ) ) );

    print("--- ");
    print("--- Part 3: Compute and print distance of coords from origin ... ");
    print("--- ");

    distance = np.zeros( coord.shape[0] )
    for i in range( coord.shape[0] ):
        x = coord[i][0];
        y = coord[i][1];
        distance[i] = math.sqrt(x**2 + y**2)

    print("--- Min distance from origin = {:8.3f} ...".format( min ( distance ) ) );

```

```

print("--- Max distance from origin = {:.3f} ...".format( max ( distance ) ) );

print("--- ");
print("--- Part 4: Compute and print perimeter and area of polygon ... ");
print("--- ");

print("--- Section perimeter = {:.3f} ...".format( perimeter(coord) ) );
print("--- Section area          = {:.3f} ...".format( area(coord) ) );

print("--- ");
print("--- Part 5: Plot section verticies and edges ...");
print("--- ");

# Define Matplotlib figure and axis

fig, ax = plt.subplots()

# Draw vertices as small circles ...

width = 0.2;
for i in range(len(coord)):
    xcoord = coord[i][0]; ycoord = coord[i][1];
    ax.add_patch( Circle( (xcoord, ycoord), width, facecolor='red' ) )

# Draw section edges ...

x1 = coord[:,0];
y1 = coord[:,1];
x2 = [ coord[0][0], coord[ len(coord)-1][0] ];
y2 = [ coord[0][1], coord[ len(coord)-1][1] ];

ax.add_line( Line2D(x1, y1) )
ax.add_line( Line2D(x2, y2) )

plt.title('T-shaped Beam Section')
plt.ylabel('y')
plt.xlabel('x')
plt.ylim( -6, 10)
plt.xlim( -10, 14)
plt.grid(True)
plt.show()

print("--- ===== ... ");
print("--- Enter TestPolygonTshapedSection01.main() ... ");

# call the main method ...

if __name__ == "__main__":
    main()

```

Program Output: The abbreviated textual output is:

```
--- Enter TestPolygonTshapedBeam01.main() ...
--- ===== ...
--- Part 1: Initialize t-beam coordinates ...

Matrix: T-beam Coordinates
-8.0000  8.0000
12.0000  8.0000
12.0000  7.0000
 4.0000  6.0000
 3.0000 -4.0000
 1.0000 -4.0000
 0.0000  6.0000
-8.0000  7.0000

---
--- Part 2: Max/min coordinate positions ...
---
--- Min x =   -8.000 ...
--- Max x =   12.000 ...
--- Min y =   -4.000 ...
--- Max y =    8.000 ...
---
--- Part 3: Compute and print distance of coords from origin ...
---
--- Min distance from origin =    4.123 ...
--- Max distance from origin =   14.422 ...
---
--- Part 4: Compute and print perimeter and area of polygon ...
---
--- Section perimeter =    60.224 ...
--- Section area      =    62.000 ...
---
--- Part 5: Plot section verticies and edges ...
---
--- ===== ...
--- Enter TestPolygonTshapedSection01.main() ...
```

Graphical output: see Figure 3.

Question 5: 10 points. Write a Python program that will compute and print a list of (x, y) pairs for:

$$y(x) = \left[\frac{(x^3 - 16x)}{(x - 4)(x + 5) \sin(x)} \right] \quad (8)$$

over the range $-10 \leq x \leq 10$ and in intervals of 0.25. You should find that $y(0)$ and $y(4)$ evaluate to not-a-number (NaN), and that $y(-5)$ evaluates to positive infinity.

Python 3 provides remarkably good builtin support for handling of run-time errors. Create a plot of $y(x)$ vs x – you should find that errors will be automatically handled within the matplotlib.pyplot environment.

Python Source Code:

```
# =====
# TestDifficultFunction04.py: Compute difficult function and see
# how Python handles divide by zero and NaN at runtime.
#
# Written by: Mark Austin January 2025
# =====

import math
import numpy as np
import matplotlib.pyplot as plt

# Implement test function, ignore difficulties ...

def testFunction(x):
    result = (x*x*x - 16*x)/(x-4)/(x+5)/math.sin(x)
    return result

# main method ...

def main():
    print("--- Enter TestDifficultFunction04.main() ... ");
    print("--- ===== ... ");
    print("");

    print("=====");
    print("          Coord          Value");
    print("          (x)          (y)");
    print("=====");

    # Define problem parameters.

    xcoord = np.linspace(-10,10,81)

    # Create list for y coordinates ...

    ycoord = [];
```

```

# Traverse xcoord array and compute y values ...

for x in xcoord:
    result = testFunction(x)
    print("          {:7.2f}    {:12.3f}".format(x,result) );
    ycoord.append(result)

print("=====");
print("");

# Plot y vs x for test function ...

plt.plot(xcoord,ycoord)
plt.title('plot y vs x for difficult function')
plt.ylabel('y')
plt.xlabel('x')
plt.xlim( -10, 10)
plt.grid(True)
plt.show()

print("--- ===== ... ");
print("--- Leave TestDifficultFunction04.main()      ... ");

# call the main method ...

if __name__ == "__main__":
    main()

```

Program Output: The abbreviated textual output is:

```

--- Enter TestDifficultFunction04.main()      ...
--- ===== ...

=====
      Coord      Value
      (x)      (y)
=====
      -10.00      -22.058
       -9.75      -36.939
       -9.50     -154.503

... lines of output removed ..

      -5.50      -23.386
      -5.25      -30.561
      -5.00         inf <--- divide by zero ...
      -4.75      14.260
      -4.50       4.603

... lines of output removed ..

      -0.50       0.811
      -0.25       0.798

```

```

0.00          nan <-- Not a number ...
0.25          0.818
0.50          0.853

... lines of output removed ..

3.50          -8.804
3.75          -5.811
4.00          nan <-- Not a number ...
4.25          -4.235
4.50          -4.119

... lines of output removed ..

9.50          -117.694
9.75          -28.446
10.00         -17.156
=====

--- ===== ...
--- Leave TestDifficultFunction04.main() ...

```

The graphical output is:

