Department of Civil and Environmental Engineering, Fall Semester, 2025

ENCE 201 Midterm 2, Open Notes and Open Book

Name : ________________________

E-mail (print neatly!): ________________________

Exam Format and Grading. This take home midterm exam is open notes and open book. You need to comply with the university regulations for academic integrity.

There are three questions. Partial credit will be given for partially correct answers, so please show all your working.

Note: Please see the **class web page for instructions on how to submit your exam paper**.

Question	Points	Score
1	20	
2	10	
3	10	
Total	40	

Question 1: (20 points)

Recall from our class lectures that if $[A]$ is an $(n \times n)$ matrix then, in general, it can be factored into a product of lower and upper triangular matrices, i.e., $[A] = [L]\,[U]$ where $[L]$ and $[U]$ are also $(n \times n)$ matrices. Our case study programs with Python and sympy assume that the lower diagonal elements are unity (i.e., $L_{ii} = 1$), but this is only one way of enabling the factorization. A second possibility is to set the upper diagonal elements to unity (i.e., $U_{ii} = 1$). The key point here is that any set of constraints that reduces the total number of unknowns from $\left(n^2 + n\right)$ to n^2 might work.

[1a] (10 pts). Calculate the LU decomposition for the matrix

$$[A] = \begin{bmatrix} 1 & 1 & 0 \\ a & 8 & 4 \\ 0 & 1 & 2 \end{bmatrix}. \tag{1}$$

by assuming that $U_{ii} = 1$. This is a hand calculation – please show all of your working ...

[1a] continued ...

[1b] (3 pts). Hence, write down the det[A]?. Note: Do not calculate the determinate by the method of cofactors - there is a much faster way that is a one line calculation!

[1c] (4 pts). Use forward and backward substitution to show that the general solution to $[L]\,[U]\,[x] = [1, 2, b]^T$. can be written:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} (-4-2b)/(a-6) \\ (a+2b-2)/(a-6) \\ (ab-a-8b+2)/(2a-12) \end{bmatrix}. \tag{2}$$

This is a hand calculation, so show all of your working.

[1c] Continued ...

[1d] (3 pts). For what values (a,b) will the system have: (a) a unique solution, (b) zero solutions, (c) infinite solutions?

Question 2 (10 points)

This question covers numerical computation of roots to the test equation:

$$f(x) = x^4 - 12x^3 + 47x^2 - 66x + 18 = 0. \tag{3}$$

Preliminary work indicates that $f(x)$ has multiple roots at a and, thus, can be factored:

$$f(x) = (x - a)^m h(x). \tag{4}$$

Here $m \geq 2$.

[2a] (4 pts). Show that the Newton-Raphson update formula for equation 4 can be written:

$$x_{n+1} = x_n - \left[\frac{(x_n - a)h(x_n)}{mh(x_n) + (x_n - a)h'(x_n)} \right]. \tag{5}$$

[2b] (3 pts). Determine appropriate values of a, m and $h(x)$ for equation 3:

[2c] (3 pts). Starting from an initial value, $x_o = 0.0$, compute no more than three iterations of approximation to the lowest root of equation 3. I suggest your organize your computations into a table that shows iteration no, x, $(x-a)^m$, $h(x)$ and $f(x)$.

Question 3 (10 points)

This question covers function interpolation with the methods of divided differences and Lagrange Interpolation for the small dataset:

x	0.0	2.0	3.0	5.0
f(x)	0.0	8.0	27.0	125.0

[3a] (5 pts). Use the method of **divided differences** to find a polynomial of lowest order that will fit the dataset.

Be sure to show all of your working.

[3b] (5 pts). Check your answer in Part 3a by computing the functional form via the method of **Lagrange Interpolation**.

Be sure to show all of your working.