FIRE: THE FIRST-YEAR INNOVATION & RESEARCH EXPERIENCE

QUANTUM MACHINE LEARNING

Powering Quantum Algorithms with Quantum Fourier Transform

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Introduction

Quantum computers exponentially speed up calculations using the properties unique to qubits (quantum bits), superposition, and entanglement. These properties are used to create gates which are then chained together to form algorithms. One such algorithm is the fourier transform which is a mathematical technique that decomposes functions, or vectors, into spatial or temporal frequency using sine and cosine basis vectors.

$$g(\alpha) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(x)\, e^{-i\alpha x}\, dx$$

Eqn. 1: The general equation of Fourier Transform.

Fourier transform has many practical applications in classical computing such as signal processing, differential equations, and image processing. Quantum fourier transform (QFT) applies the same concept to quantum state vectors.

Quantum Gates in QFT

First, we present the general equation of QFT:

$$y_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-2} x_n \omega_N^{nk}, \text{k} = 0,1,\ldots,\text{N-1},$$

Eqn. 2: The general equation of Quantum Fourier Transform.

In QFT, we apply three quantum gates. The primary gate for the QFT is the Hadamard gate (H gate). The H gate is what puts a qubit into a superposition of states. Next, the controlled phase rotations (CP) gates rotate the phase of a target qubit depending on if each control qubit is in the state |1> or not. For a target qubit q_n you perform n rotations for n control qubits in state |1>. There are n rotations for each q_n qubit starting at a rotation of $\pi/2^n$ and decreasing n until n=0. Finally we use the swap gate to preserve the correct ordering of the qubits. See **Fig. 1.** for bloch spheres illustrating how the qubit changes after applying the H gate.

Acknowledgements

We would like to thank our faculty advisor, Dr. Shabnam Jabeen for continuous support and guidance. We also thank IBM Quantum Lab for providing the resources to make this project possible.

References

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2. *Oliveira, Ivan S., et al. "Fundamentals of Quantum Computation and Quantum Information." NMR Quantum Information Processing, Elsevier Science B.V., 28 Sept. 2007,* *https://www.sciencedirect.com/science/article/pii/B9780444527820500051.*

Results

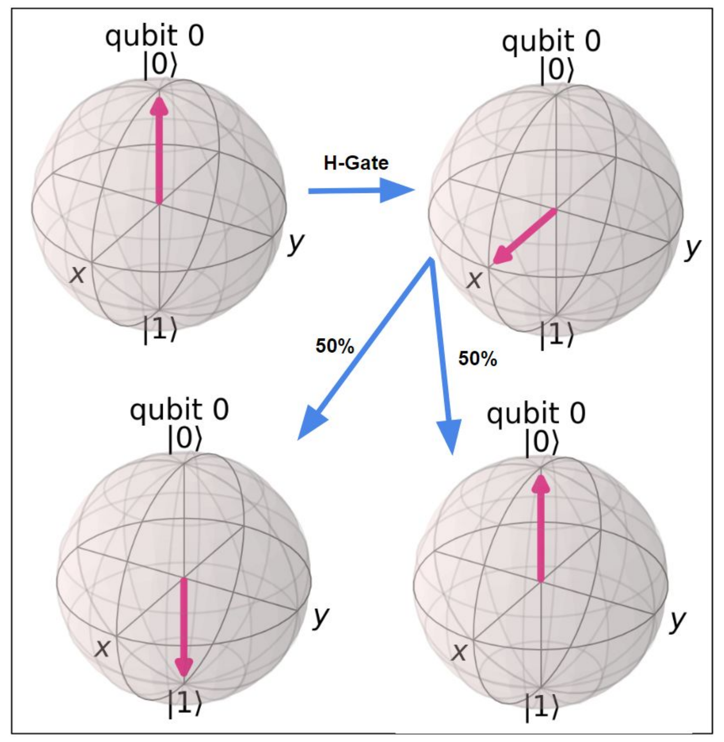

Fig. 1: Block sphere demonstrating the H-Gate. The qubit has 50% chance of |0> state and 50% of |1> state.

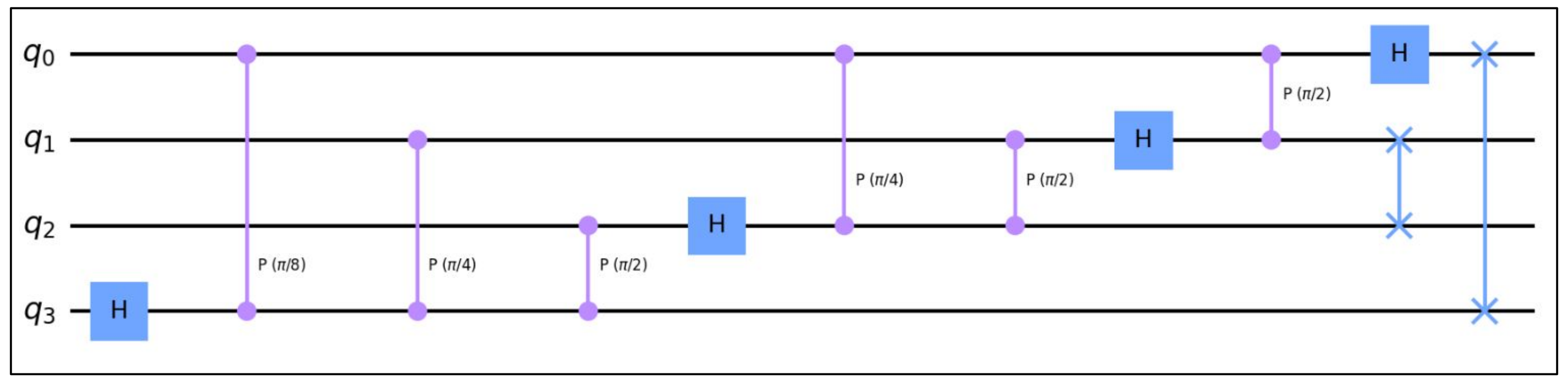

Fig. 2: QFT Circuit Diagram

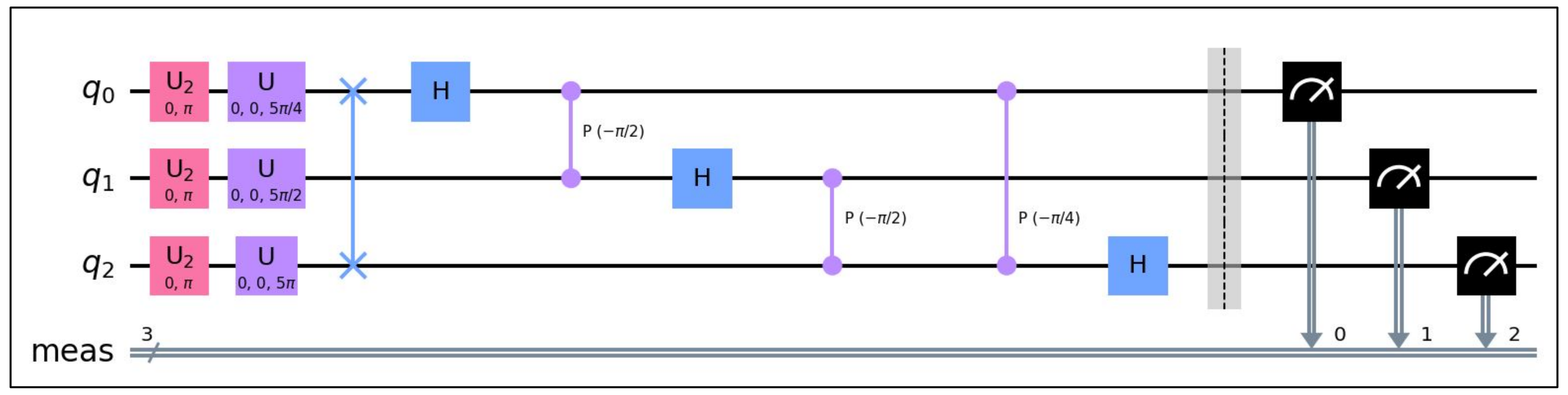

Fig. 3: QFT Inverse Circuit Diagram

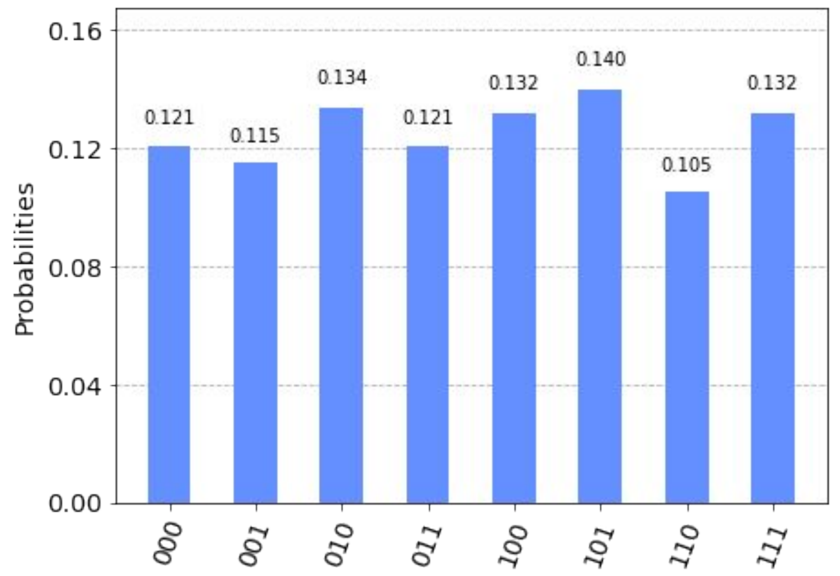

Fig. 4: Quantum computer results after applying QFT to the integer 5.

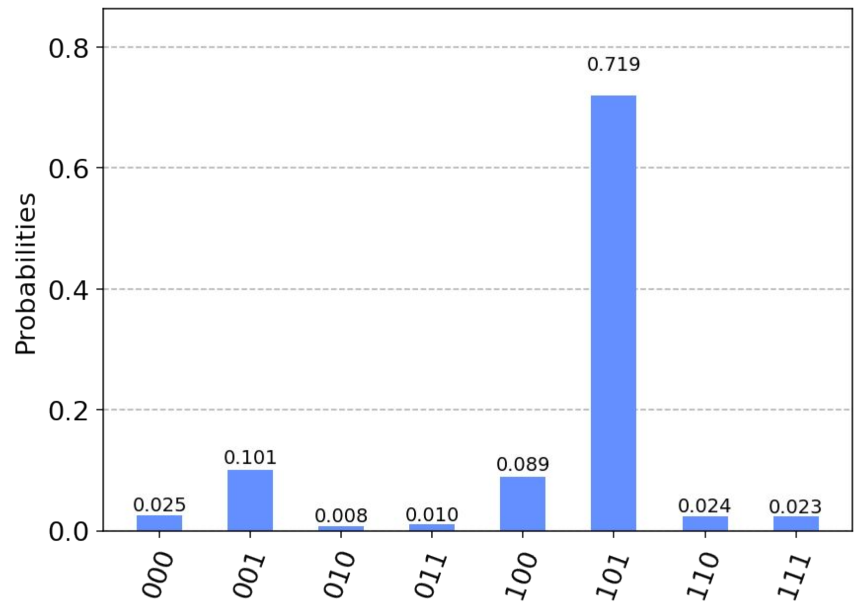

Fig. 5: Quantum computer results after applying inverse QFT to the integer 5.

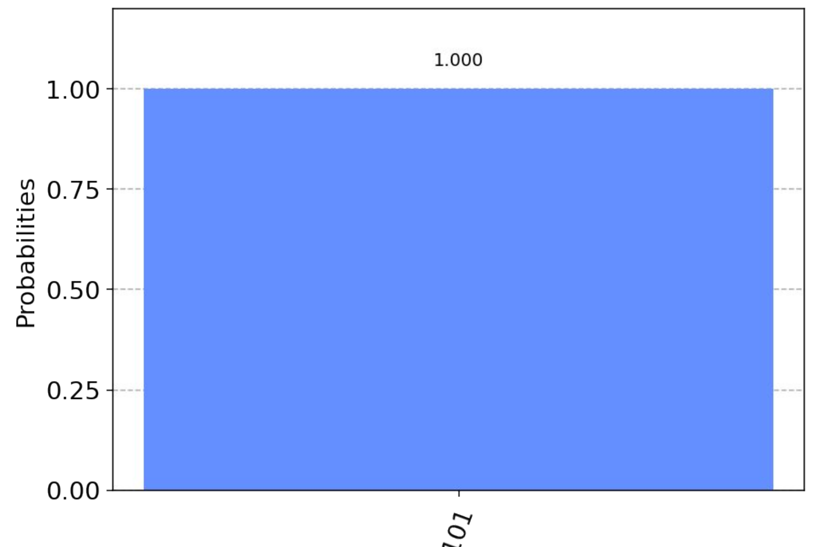

Fig. 6: Applying inverse QFT to the integer 5 using the AER simulator.

Discussion & Results

In **Fig. 2.** we present the QFT circuit. Starting with the last qubit, we apply the H gate and a CP gate for each of the preceding qubits. This process is repeated for the remaining three qubits. The final step in the circuit is to apply the swap gate, represented by the blue vertical lines. In **Fig.3.**, we present the inverse QFT circuit. The inverse QFT algorithm starts by creating a circuit and applies QFT to it. Then we use the inverse method from Qiskit to invert this circuit. Lastly, we add this circuit back to the original and decompose it. The inverse circuit validates our QFT circuit. In **Fig. 4**, we present the results of quantum fourier transform on the integer 5 using the IBM quantum computer (Oslo, Sweden backend). In **Fig. 5.**, we see the inverse QFT applied onto the integer 5 in fourier basis. We observe the largest probability is binary 101 (5 in decimal). In **Fig. 6**, we see the inverse QFT in the AER simulator shows a 100% probability of 5. In the future, QFT can be implemented to massively increase efficiency in the way of running several tasks simultaneously.

Future Work

We have presented here an example study of Quantum Fourier Transform and results have been verified on a quantum simulator as well as one of the real IBM quantum computers. Some of us are continuing to use these and other quantum algorithms to solve complex problems, especially in the field of physics and astronomy.