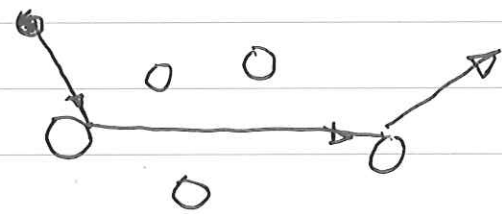
Collisions in Plasmas

General Comments

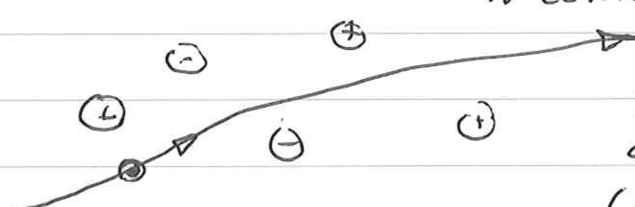
1 Collisions are not like colliding Billiard Balls

- Billiard Balls — collisions occur one ball at a time



- Large angle deflection

- Plasma Particles — a given particle is colliding with all the other particles within a Debye sphere (Coulomb Force is long range)



- Many small angle deflection
Diffusion and slowing down in velocity space

2. Due to disparity in mass $m_i >> m_e$ collision occur at different rates for ions and electrons, different collisional processes occur at different rates

$$\overset{1}{Z\nu_e} > \overset{2}{\nu_e} \gg \underset{(m_i/m_e)^{1/2}}{\overset{3}{\nu_i}} \gg \underset{(m_i/m_e)^{1/2}}{\overset{4}{\nu_{eq}}}$$

average ionization stage

1. Electrons isotropize $f_e \to f_e(|v|)$

2. Electrons thermalize with themselves $f_e(|v|) \to \frac{n}{(2\pi T_e/m_e)^{3/2}} \exp(-\frac{1}{2} m_e v^2/T_e)$

3. Ions thermalize $f_i(v) \to \frac{n}{(2\pi T_i/m_i)^{3/2}} \exp(-\frac{1}{2} m_i v^2/T_i)$

4 electrons and ions equilibrate $T_e \leftrightarrow T_i$

$$\nu_e \sim \frac{D_{vv}}{v^2} \quad \sim \frac{4\pi n e^4}{m^2 v^3} \Lambda$$

diffusion coefficient

"Coulomb" logarithm

$\frac{1}{v^3}$

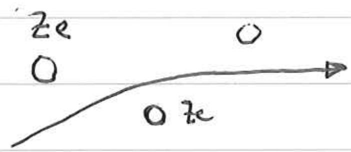
$$\nu \sim \frac{\omega_p^4}{n v^3} \Lambda$$

$\lambda_d = V_{te}/\omega_{pe}$ Debye

$$\nu \sim \left(\frac{V_{te}}{v}\right) \left[\frac{\omega_p}{n \lambda_d^3}\right]$$

$n\lambda_d^3$ = # particles in Debye cube $>> 1$

Where does the Z come from?



for equal total ion charge

$Z e n_i = e n_e$

Putting more charge in each scatterer event though there are fewer scatterers results in higher diffusion. Think in terms of random walk in angle

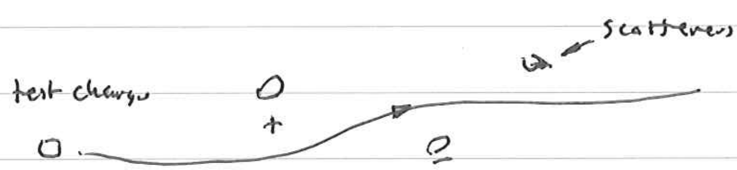
step size $\nabla\theta \sim Z$

time between collisions $\tau \sim Z$

$D \sim \frac{\Delta\theta^2}{\tau} \sim Z$ collisions are elastic

---

Where does the factor $(m_i/m_e)^{1/2}$ come from?



Suppose for the moment the scatterers are fixed.

$$\frac{d}{dt}\underline{v}_e = \frac{q}{m_e}\underline{E}(\underline{x}(t))$$

$$\frac{d\underline{x}}{dt} = \underline{v}_e$$

let $\underline{u} = \lambda \underline{v}$ ← scaling factor

$\tau = t/\lambda$

$$\boxed{\frac{1}{m_i} = \frac{\lambda^2}{m_e}}$$

$$\lambda = \sqrt{\frac{m_e}{m_i}}$$

$$\frac{dx}{dt} = \underline{v}_e \qquad \frac{dx}{d\tau} = \lambda \frac{dx}{dt} = \lambda \underline{v} = u$$

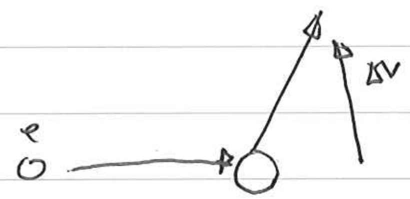
$$\frac{dx}{d\tau} = u$$

$$\frac{du}{d\tau} = \lambda^2 \frac{dv}{dt} = \frac{\lambda^2 q}{m_e}\underline{E}(\underline{x}(t))$$

~~14~~) equilibration   ions are bowling balls

electrons " ping-pong balls

to lowest order there is no energy exchange in a collision



From momentum conservation

$$m_e \Delta \underline{V}_e = -m_i \Delta V_i$$

thus $\frac{\langle \Delta V_i \rangle^2}{\tau} = \frac{m_e^2}{m_i^2} \frac{\langle \Delta V_e^2 \rangle}{\tau}$ $\frac{m_i \langle \Delta V_i \rangle^2}{\tau} \sim \frac{m_e}{m_i} \frac{m_e \langle \Delta V_e^2 \rangle}{\tau}$

~~so~~ rate of ion diffusion in energy $\sim \frac{m_e}{m_i}$ rate of electron diffusion

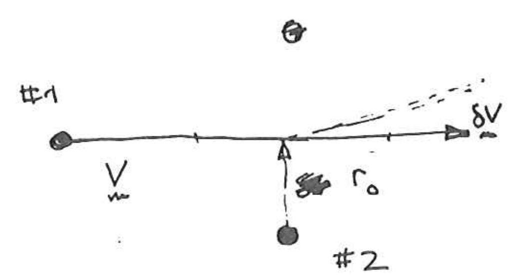
Collisions ~~Using~~ ~~Using Vlasov Eqn~~ In deriving Liouville from Vlasov

$$F_N(\underset{\sim}{x}_1, \underset{\sim}{v}_1; \underset{\sim}{x}_2, \underset{\sim}{v}_2 \cdots\cdots, t) \cong F_1(\underset{\sim}{x}_1, \underset{\sim}{v}_1, t) F_1(\underset{\sim}{x}_2, \underset{\sim}{v}_2, t) \cdots$$

Assumption of iid eliminates detailed effects of individual particles on each other. i.e. it eliminates collisions.

Collisions may still be important over long time scales.

To estimate effect of collisions consider effect of one stationary charge on another moving charge



#1 is deflected by #2

$\delta V \simeq \quad \delta t \cdot \underset{\sim}{E} \dfrac{e}{m}$ $\qquad \delta t =$ time during which Field is felt

$$\delta t \sim \frac{r_0}{V} \qquad \underset{\sim}{E} \sim \frac{e}{r_0^2}$$

$$\delta V = \frac{e^2}{r_0 V m}$$

LETS CONSTRUCT A DIFFUSION COEFFICIENT IN VELOCITY SPACE USING RANDOM WALK ARGUMENTS

$$\frac{\partial}{\partial t} f = D \nabla_v^2 f$$

D = velocity space diffusion coefficient

$$D = \frac{(\delta v)^2}{\Delta T}$$

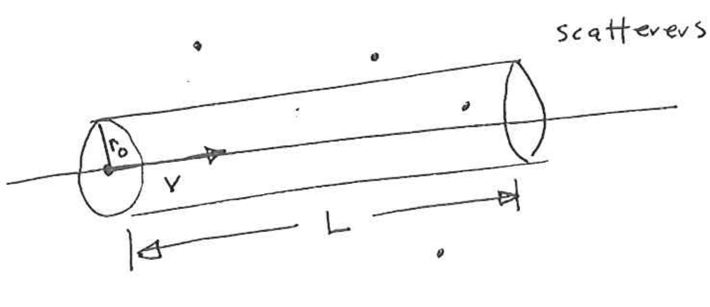
δv = deflection in a collision

ΔT = time between collisions

$$\delta v = \frac{e^2}{m v r_0}$$

collision is when electron passes within r_0 of scatterer

How long between "collisions" depends on density



$$L = v \Delta T$$

we can expect another collision when $\text{Volume} = \pi r_0^2 L = \frac{1}{n}$

~~ΔT~~ $L = 1/\pi n r_0^2$

$\Delta T = 1/v\pi n r_0^2$

$$\frac{(\delta v)^2}{\Delta T} = \frac{\pi e^4 n r_0^2 v}{m^2 v^2 r_0^2} = \frac{\pi e^4 n}{v m^2} = D$$

notice r_0 dissapears

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial \underset{\sim}{v}} \cdot D \frac{\partial f}{\partial \underset{\sim}{v}}$$

"collison frequency"

$$(D/v^2) = \frac{\pi e^4 n}{m^2 v^3}$$

$\nu \; \cancel{\Delta T} \propto n \qquad \overset{\nu}{\cancel{\Delta T}} \propto v^{-3}$

LET TRY TO BE more careful

LETS SUM UP THE EFFECTS OF COLLISIONS OCCURING with different r_0

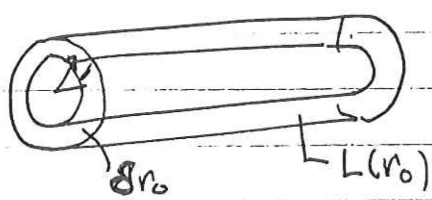
$$D = \sum_{r_0} \frac{e^4}{m^2 v^2 r_0^2 \Delta T(r_0)} \qquad D = \sum_{r_0} \frac{\delta v^2(r_0)}{\Delta T(r_0)}$$

$T(r_0) =$ Time between collisions with ~~differ~~ a ~~pattern~~ particular r_0

$$\pi r_0 \delta r_0 L = \frac{1}{n}$$

$$L = v \Delta T$$

$$\Delta T = \frac{1}{\pi r_0 \delta r_0 v n}$$



$$D = \sum_{r_0} \pi r_0 \delta r_0 \, \delta v^2(r_0) \, v n$$

$$\text{Volume} = \pi r_0 dr_0 L(r_0) = \frac{1}{n} \qquad L(r_0) = \Delta T(r_0) v$$

$$\boxed{\delta v(r_0) = \frac{e^2}{m v r_0}}$$

$$\frac{1}{\Delta T(r_0)} = \frac{v}{L(r_0)} = \pi r_0 dr_0 n v$$

$$D = \int \frac{e^4}{m^2 v^2 r_0^2} \cdot \pi r_0 dr_0 n v = \frac{\pi n e^4}{m^2 v} \int_{r_{min}}^{r_{max}} \frac{dr_0}{r_0}$$

Integral does not converge

(consequence of inverse sq. law

Estimate of D_{vv}

$$D = \sum_{r_0} \frac{1}{\Delta T} (\delta v)^2$$

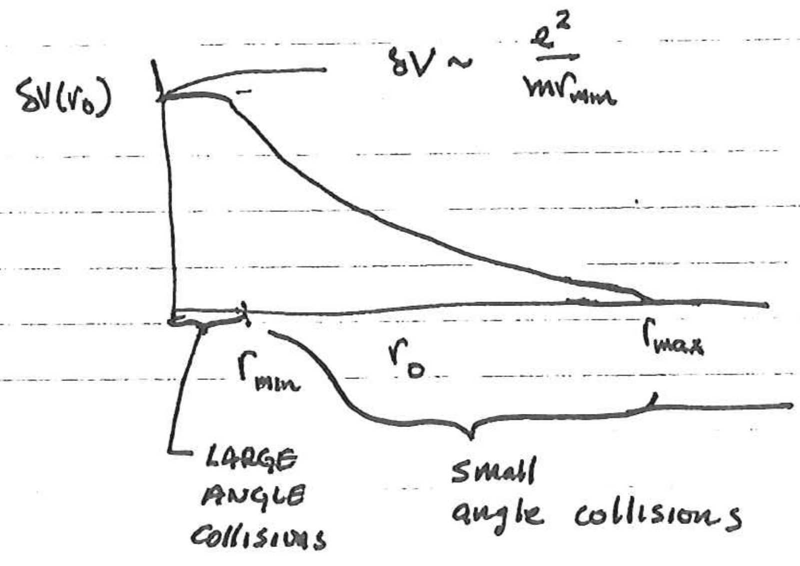
$$\cancel{D_{vv} = \int_0^\infty \frac{dr_0}{r_0} \frac{\pi n e^4}{m^2 v}}$$

$$\frac{1}{\Delta T} = \pi r_0 dr_0 n v$$

$$D = \int \pi r_0 dr_0 \, n v \left(\delta v(r_0) \right)^2$$

$$\delta v(r_0) = \frac{e^2}{m v_0 r_0}$$

estimate



$$D_{LA} = \int_0^{r_{min}}$$

$$D_{LA} = \int \pi r_{min}^2 n v \frac{e^2}{m^2 v^2 r_{min}^2} = \frac{n \pi e^2}{m^2 v} \, 1$$

$$D_{SA} = \int_{r_{min}}^{r_{max}} \pi r_0 dr_0 \, n v \frac{e^2}{m^2 v^2 r_0^2} = \frac{\pi n e^2}{m^2 v} \ln\left(\frac{r_{max}}{r_{min}}\right)$$

EXTRA PHYSICS REQUIRED

$$D = \frac{\pi n e^4}{m^2 V} \ln \left| \frac{r_{o\,max}}{r_{o\,min}} \right|$$

$r_{o\,min}$ is easy ~~suppose we were to~~

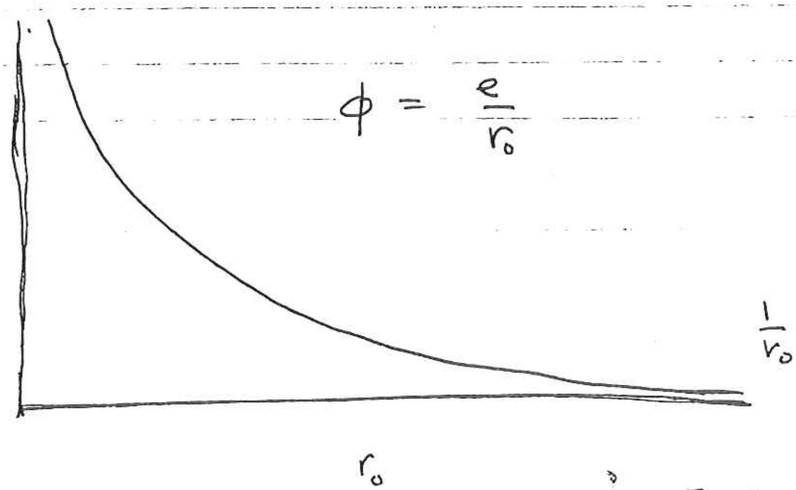
We assumed small deflections ~~For mula~~ Formula's For small deflections require $\delta V \ll V$

$$\delta V = \frac{e^2}{r_o V m} \simeq V \qquad r_{o\,min} = \frac{mV^2}{e^2} \qquad \boxed{\frac{e^2}{r_{o\,min}} = mV^2}$$

closest approach

$r_{o\,max}$

$\phi(r_o)$

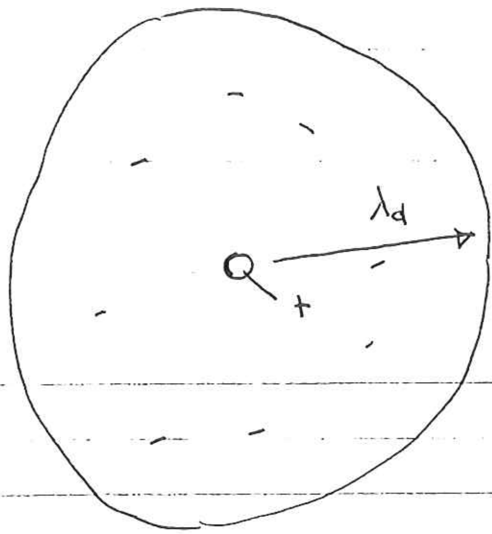


Force gets weak very slowly as $r \to \infty$

$$F \sim \frac{1}{r_o^2}$$

other particles will shield

$$\lambda_d^2 = \frac{T}{4\pi n e^2}$$



$r > \lambda_d$

Field goes to zero

$$r_{o\,max} = \lambda_d$$

$$r_{o\,min} = \frac{\frac{1}{2} e^2}{\frac{1}{2} m v^2}$$

$$\cancel{\ln \left| \frac{r_{o\,max}}{r_{o\,min}} \right| = \frac{1}{2} \ln \left| \frac{\lambda_d^2}{r_{o\,min}^2} \right| = \frac{1}{2} \ln \left| \frac{T}{4\pi n e^2} \frac{m^2 v^4}{e^4} \right|}$$

$$\frac{r_{o\,min}}{\lambda_d} = \frac{4\pi e^2 n}{T \, 4\pi n} = \frac{1}{\frac{4\pi}{3} n \lambda_d^3}$$

$$= \frac{1}{\text{\# particles in debye sphere}}$$

SIZE OF $\frac{r_{max}}{r_{min}} = \frac{\lambda_d}{e^2/mv^2} \simeq \frac{\lambda_d T}{e^2} \times \frac{4\pi n}{4\pi n}$

T

$\simeq 4\pi n \lambda_d^3$

$=$ 3x # of particles in Debye sphere $>> 1$

If # particles is LARGE small angle collisions dominate