Non linear

– Decay of Electromagnetic Waves in unmagnetized plasma

Light waves

$$\omega^2 = k^2c^2 + \omega_p^2$$

TRY

$$\omega_2^2 = k_2^2c^2 + \omega_p^2$$

$$\omega_1^2 = k_1^2c^2 + \omega_p^2 \qquad = (\omega_2+\omega_3)^2$$

$$\omega_3^2 = k_3^2c^2 + \omega_p^2$$

$$(\omega_2+\omega_3)^2 = |\underset{\sim}{k}_2 + \underset{\sim}{k}_3|^2 c^2 + \omega_p^2$$

$\cancel{2\omega_2\omega_3}$

$$\omega_2^2 + 2\omega_2\omega_3 + \omega_3^2 = \left[(k_2^2 + k_3^2) + 2\underset{\sim}{k}_2\cdot\underset{\sim}{k}_3\right]^{c} + \omega_p^2$$

Requires $\quad \omega_p^2 + 2\omega_2\omega_3 = 2\underset{\sim}{k}_2\cdot\underset{\sim}{k}_3\,c^2$

$$\cancel{\omega_p^2 + 2\sqrt{\omega_p^2+k_2^2c^2}\sqrt{\omega_p^2+k_3^2c^2}} = 2\underset{\sim}{k}_2\cdot\underset{\sim}{k}_3\,c^2$$

$$\frac{\omega_2}{|k_2|} > c \qquad \frac{\omega_3}{|k_3|} > c$$

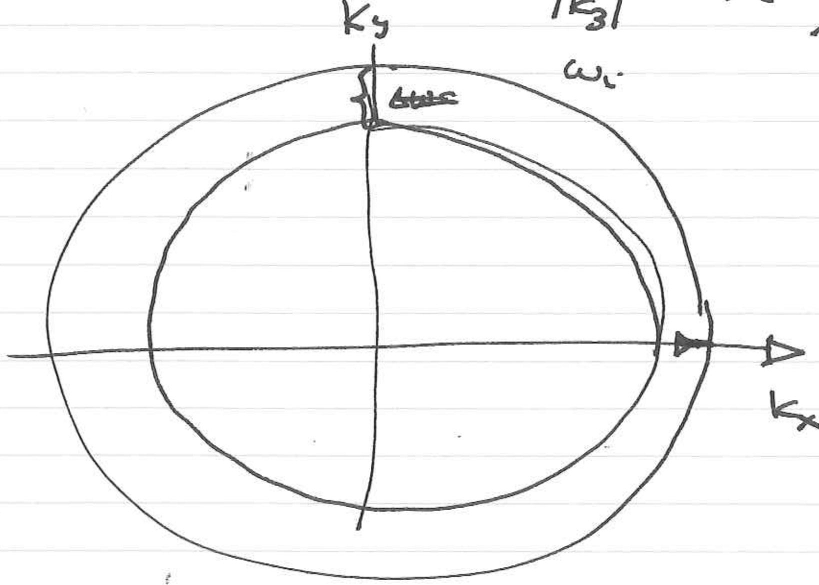
But $\quad \dfrac{\omega_2}{|k_2|} > c \quad \dfrac{\omega_3}{|k_3|} > 0 \quad$ can't be satisfied!

TWO LIGHT WAVES + 1 PLASMA wave

$$\omega_1^2 = k_1^2 c^2 + \omega_p^2$$

$$\omega_2^2 = k_2^2 c^2 + \omega_p^2$$

$\omega_3 = \omega_p$ ($\underset{\sim}{k}_{3}$ could be anything so long as $\frac{\omega}{|k_3|} > V_{the}$)



$\omega_2 = \omega_1 - \omega_p$ & $\underset{\sim}{k}_2$ could be in any direction

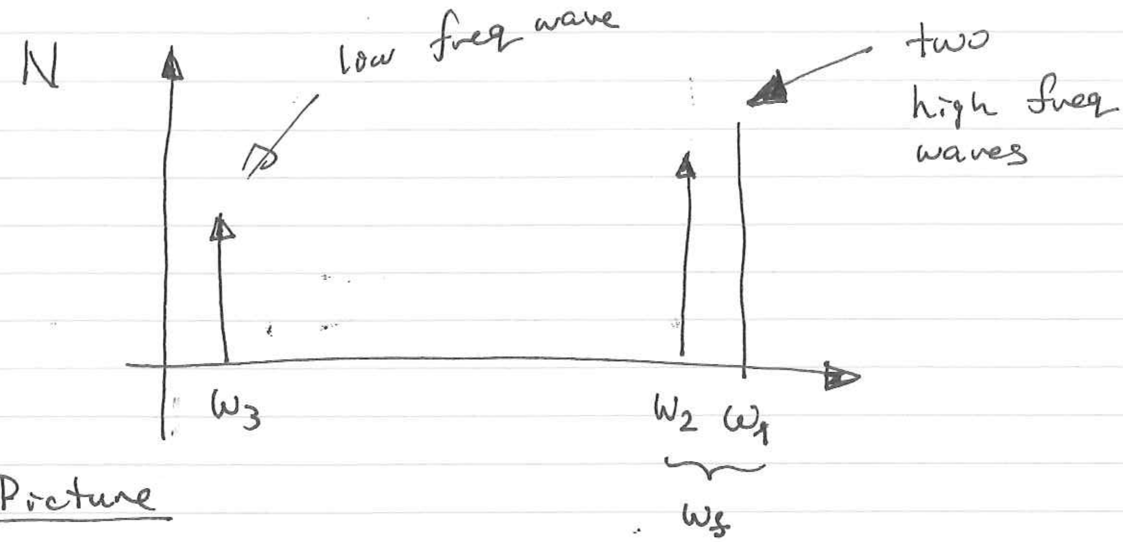
$\underset{\sim}{k}_3 = \underset{\sim}{k}_1 - \underset{\sim}{k}_2$

ANOTHER POSSIBILITY

$\omega_2 = \omega_3 = \omega_p$ — two plasma waves

$\omega_1 = 2\omega_p$ — Light wave

* Calculation of Coupling is easy to understand in the case of coupling of high and low frequency waves



Physical Picture

- Two high frequency waves beat together to make a low frequency force
  - * Ponderomotive force
  - * acts mainly on electrons

- Low frequency force makes a low frequency density modulation *(plasma wave, ion sound wave)

- density modulation modulates elements of susceptibility $\underset{\approx}{\chi} \propto n$
  - * couples together high frequency waves

Part # 1 Ponderomotive force

Cold fluid Equations

$$\frac{\partial \underset{\sim}{u}}{\partial t} + \underset{\sim}{u}\cdot\nabla\underset{\sim}{u} = \frac{q}{m}\left(\underset{\sim}{E} + \frac{\underset{\sim}{u}\times\underset{\sim}{B}}{c}\right)$$

$$\underset{\sim}{E} = \underset{\sim}{E}_f + \underset{\sim}{E}_s \qquad \underset{\sim}{E}_f = Re\left\{\hat{\underset{\sim}{E}}_f(\underset{\sim}{x},t)\,e^{-i\omega_f t}\right\}$$

$$\underset{\sim}{u} = \underset{\sim}{u}_f + \underset{\sim}{u}_s \qquad \underset{\sim}{u}_f = Re\left\{\hat{\underset{\sim}{u}}_f(\underset{\sim}{x},t)\,e^{-i\omega_f t}\right\}$$

fast ~~field~~ flow is the linear high frequency response

$$\frac{\partial \underset{\sim}{u}_f}{\partial t} = \frac{q}{m}\underset{\sim}{E}_f$$

Slow flow contains nonlinear terms

$$\frac{\partial u_s}{\partial t} = \frac{q}{m}\left(\underset{\sim}{E}_s\right) - \left\langle \underset{\sim}{u}_f\cdot\nabla\underset{\sim}{u}_f\right\rangle_{\omega_f} + \frac{q}{m}\left\langle\frac{\underset{\sim}{u}_f\times\underset{\sim}{B}_f}{c}\right\rangle_{\omega_f}$$

this means average over fast time $T_f = \frac{2\pi}{\omega_f}$

Note $\quad \underset{\sim}{u}_f \cdot \nabla \underset{\sim}{u}_f = \nabla \frac{1}{2} \underset{\sim}{u}_f \cdot \underset{\sim}{u}_f - \underset{\sim}{u}_f \times \nabla \times \underset{\sim}{u}_f$

$$\frac{\partial \underset{\sim}{u}_s}{\partial t} = \frac{q}{m} \underset{\sim}{E}_s - \nabla \tfrac{1}{2} \langle \underset{\sim}{u}_f \cdot \underset{\sim}{u}_f \rangle_{\omega_f}$$

$$+ \Big\langle \underset{\sim}{u}_f \times \Big[\frac{q \underset{\sim}{B}_f}{mc} + \nabla \times \underset{\sim}{u}_f \Big] \Big\rangle_{\omega_f}$$

This term aveages to zero

$$\frac{\partial \underset{\sim}{u}_f}{\partial t} = \frac{q}{m} \underset{\sim}{E}_f$$

$$-\frac{1}{c} \frac{\partial \underset{\sim}{B}_f}{\partial t} = \nabla \times \underset{\sim}{E}_f$$

$$\frac{\partial}{\partial t} \left[\frac{q \underset{\sim}{B}_f}{mc} + \nabla \times \underset{\sim}{u}_f \right] = \frac{q}{mc} \frac{\partial \underset{\sim}{B}_f}{\partial t} + \nabla \times \frac{\partial \underset{\sim}{u}_f}{\partial t}$$

$$= -\frac{q}{m} \nabla \times \underset{\sim}{E}_f + \nabla \times \frac{q}{m} \underset{\sim}{E}_f = 0$$

ponderomotive force

$$m \frac{\partial \underset{\sim}{u}_s}{\partial t} * = q \underset{\sim}{E}_s + \underset{\sim}{F}_p$$

ponderomotive potential

$$\underset{\sim}{F}_p = - \nabla V_p$$

$$\underset{\sim}{V}_p = \frac{1}{2} m \langle \underset{\sim}{u}_f \cdot \underset{-}{u}_f \rangle$$

$$\underset{\sim}{u}_f = \frac{1}{2}\left(\hat{\underset{-}{u}}_f e^{-i\omega_f t} + \hat{\underset{\sim}{u}}_f^* e^{i\omega_f t} \right) = \mathrm{Re}\{ \hat{\underset{\sim}{u}}_f e^{-i\omega_f t} \}$$

$$V_p = \frac{m}{\cancel{8}4} |\hat{\underset{\sim}{u}}_f|^2$$

$$\hat{\underset{-}{u}}_f = \frac{q}{m(-i\omega_f)} \hat{\underset{\sim}{E}}_f$$ FROM EQ. of motion

$$\boxed{\overline{V}_p = \frac{q^2 |\hat{\underset{\sim}{E}}_f|^2}{4\omega_f^2 m}} \boxed{= \frac{1}{4} \left| \frac{q\hat{\underset{\sim}{A}}}{mc^2} \right|^2}$$

* PONDEROMOTIVE FORCE same sign for e, i
* much bigger for electrons
* pushes particles out of high field region.

* The ponderomotive force is not really a new kind of force. It is just a manifestation of the E & B Lorenz force.

<u>Interesting puzzles</u>

suppose $\hat{\underset{\sim}{E}}_f = \underset{\sim}{a}_x \hat{E}_x(z)$

$\hat{E}_f$ has an x-component that depends on z

$\underset{\sim}{F}_p - \nabla V_p$ is in what direction?

$\underset{\sim}{F}_p$ is in z-direction even though

$\underset{\sim}{E}$ is in x direction

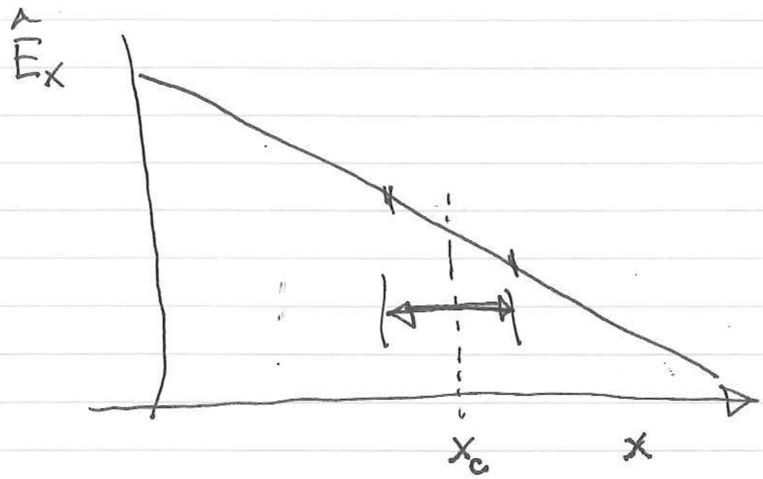
How can this be? $\hat{E}_x \rightarrow u_x$

$\dfrac{\partial \hat{E}_x}{\partial z} \rightarrow \hat{B}_y$

$F_z = \dfrac{q}{c}(u_x B_y)$ Ponderomotive force is due to $\underset{\sim}{v} \times \underset{\sim}{B}$ Lorenz force.

Suppose $\hat{\underline{E}}_f = \underset{\sim}{a}_x \hat{E}_x(x)$

$\nabla \times \hat{\underset{\sim}{E}}_f = 0 \qquad \underset{\sim}{B}_f = 0 \qquad$ no $\underset{\sim}{V} \times \underset{\sim}{B}$



Particle oscillates back and forth in non uniform field

Results in net force out of strong field

Analysis of Raman Instability

$\underset{\sim}{E} = \underset{\sim}{E}_f + \underset{\sim}{E}_s$ decompose $\underset{\sim}{E}$ into fast and slow

FOR FAST ~~E~~ FIELDS • COMBINE in Maxwell's Eqn

$$\nabla \times \underset{\sim}{B}_f = \frac{4\pi}{c}\underset{\sim}{J}_f + \frac{1}{c}\frac{\partial E_f}{\partial t} \qquad -\frac{1}{c}\frac{\partial \underline{B}_f}{\partial t} = \nabla \times \underline{E}_f$$

$$\nabla \times \frac{1}{c}\frac{\partial B_f}{\partial t} = -\nabla\times\nabla\times \underline{E}_f = \frac{4\pi}{c^2}\frac{\partial \underline{J}_f}{\partial t} + \frac{1}{c^2}\frac{\partial E_f}{\partial t}$$

for E_f

$\nabla \cdot \underset{\sim}{E}_f,\ n_f \simeq 0$

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\underset{\sim}{E}_f - \nabla^2 \underset{\sim}{E}_f = -\frac{4\pi}{c^2} q_e \frac{\partial}{\partial t}(n\underset{\sim}{u}_f) = -\frac{4\pi q_e^2}{m_e^2 c^2} n \underset{\sim}{E}_f$$

FLUID EQUATIONS FOR FAST FLOW

$$\frac{\partial}{\partial t} n \underset{\sim}{u}_f = \frac{q}{m} n \underset{\sim}{E}_f$$

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)\underset{\sim}{E}_f = \frac{\omega_{po}^2}{c^2}\left(\frac{n}{n_0}\right)\underset{\sim}{E}_f$$

this will have a slow component

$$n = n_0 + n_s$$

<u>For slow fields</u> (assume plasma wave is linear)

$$\frac{\partial}{\partial t} n_0 u_s = \frac{n_0 q}{m}\left[\underset{\sim}{E}_s\right] + \frac{n_0}{m}\underset{\sim}{F}_p$$ ponderomotive force

$$\frac{\partial}{\partial t} n_s + \nabla \cdot n_0 \underset{\sim}{u}_s = 0$$ continuity

$$\nabla \cdot \underset{\sim}{E}_s = 4\pi q\, n_s$$

$$\frac{\partial^2}{\partial t^2} n_s + \nabla \cdot n_0 \frac{\partial u_s}{\partial t} = 0$$

$$\frac{\partial^2}{\partial t^2} n_s + \nabla \cdot \left[\frac{n_0 q}{m}\underset{\sim}{E}_s + \frac{n_0}{m}\underset{\sim}{F}_p\right] = 0$$

$$\boxed{\left(\frac{\partial^2}{\partial t^2} + \omega_{po}^2\right) n_s = -\nabla \cdot \frac{n_0}{m}\underset{\sim}{F}_p}$$

$$\underset{\sim}{F}_p = -\nabla \frac{m}{4}\left\langle\left[\hat{\underset{\sim}{u}}_f^* \cdot \hat{\underset{\sim}{u}}_f\right]\right\rangle_s$$

$$= -\nabla \frac{m}{4}\left\langle \frac{q\hat{\underset{\sim}{E}}_f^*}{m\omega_2} \cdot \frac{q\hat{\underset{\sim}{E}}_f}{m\omega_2} \right\rangle$$

Three wave Interactions

$$\underset{\sim}{E}_f = \mathrm{Re}\Big\{ \hat{\underset{\sim}{E}}_{f1}\, e^{i\underline{k}_1\cdot\underline{x} - i\omega_1 t} + \hat{\underset{\sim}{E}}_{f2}\, e^{i\underline{k}_2\cdot\underline{x} - i\omega_2 t} \Big\}$$

$$n_s = \mathrm{Re}\Big\{ \hat{n}_s\, e^{i\underline{k}_3\cdot\underline{x} - i\omega_3 t} \Big\}$$

$$\cancel{\langle \hat{\underset{\sim}{E}}_f^{*}\cdot\hat{\underset{\sim}{E}}_f \rangle_s = \hat{\underset{\sim}{E}}_{f1}^{*}\cdot\hat{\underset{\sim}{E}}_{f1} + |\hat{\underset{\sim}{E}}_{f2}|^2 + \hat{\underset{\sim}{E}}_{f1}^{*}\cdot\hat{\underset{\sim}{E}}_{f2}\, e^{i(\underline{k}_2-\underline{k}_1)\cdot x - i(\omega_2-\omega_1)t} + \hat{\underset{\sim}{E}}_{f2}^{*}\cdot\hat{\underset{\sim}{E}}_{f1}\, e^{i(\underline{k}_1-\underline{k}_2)\cdot x - i(\omega_1-\omega_2)t}}$$

$$(n_0 + n_s)\underset{\sim}{E}_f$$

$$= n_0 \underset{\sim}{E}_f + \frac{1}{2}\left(\hat{n}_s\, e^{i\underline{k}_3\cdot\underline{x} - i\omega_3 t} + \hat{n}_s^{*}\, e^{-i\underline{k}_3\cdot\underline{x} + i\omega_3 t}\right)$$

$$\frac{1}{2}\Big[\hat{\underset{\sim}{E}}_{f1}\, e^{i\underline{k}_1\cdot\underline{x} - i\omega_1 t} + \hat{\underset{\sim}{E}}_{f2}\, e^{i\underline{k}_2\cdot\underline{x} - i\omega_2 t} + c.c. \Big]$$

TERM RESONANT with wave # 1 $\underset{\sim}{k}_1 = \underset{\sim}{k}_2 + \underset{\sim}{k}_3$

$$\frac{1}{2}\left(n_0 \hat{\underset{\sim}{E}}_{f1} + \hat{n}_s \frac{1}{2} \hat{\underset{\sim}{E}}_{f2}\right)$$

TERM RESONANT with wave #2 $\underset{\sim}{k}_2 = \underset{\sim}{k}_1 - \underset{\sim}{k}_3$

$$\frac{1}{2}\left(n_0 \hat{\underset{\sim}{E}}_{f2} + \hat{n}_s^* \frac{1}{2} \hat{\underset{\sim}{E}}_{f1}\right)$$

<u>Wave Equation</u>

$$\frac{\partial}{\partial t} \rightarrow \left(-i\omega_i + \frac{\partial}{\partial t}\right)$$

$$\frac{\partial^2}{\partial t^2} \cong -\omega_i^2 - 2i\omega_i \frac{\partial}{\partial t}$$

~~Pon~~

Slow Equations

$$\left(\frac{\partial^2}{\partial t^2} + \omega_p^2\right)\frac{n_s}{n_0} = + \nabla \cdot \frac{\cancel{q}}{m} \nabla \cdot \frac{m}{4} \langle \hat{u}_f^* \cdot \hat{u}_f \rangle$$

$$\hat{u}_f = \frac{q\hat{E}_{f1}}{-im\omega_1} + \frac{q}{-im\omega_2} \hat{E}_{f2}\, e^{-i((\omega_2-\omega_1)t) + i(k_2-k_1)\cdot x}$$

$$\cancel{\left[\frac{\partial^2}{\partial t^2} + \omega_p^2\right] \hat{n}_s}$$

- component at $k_3 = (k_1 - k_2)$ $\quad \omega_3 = \omega_1 - \omega_2$

$$\left[\left(\frac{\partial}{\partial t} - i\omega_3\right)^2 + \omega_p^2\right]\frac{\hat{n}_s}{n_0} = -k_3^2 \frac{1}{4}\frac{q^2}{m^2}\frac{(\hat{E}_{f1}\cdot\hat{E}_{f2}^*)}{\omega_1\omega_2}$$

$$-\frac{1}{c^2}\left(\omega_1^2+2i\omega_1\frac{\partial}{\partial t}\right)\hat{\underset{\sim}{E}}_{f1}+k_1^2\hat{\underset{\sim}{E}}_{f1}=-\frac{\omega_p^2}{c^2}\left(\hat{\underset{\sim}{E}}_{f1}+\frac{1}{2}\frac{\hat{n}_s}{n_0}\hat{\underset{\sim}{E}}_{f2}\right)$$

$$-\frac{1}{c^2}\left(\omega_2^2+2i\omega_2\frac{\partial}{\partial t}\right)\hat{\underset{\sim}{E}}_{f2}+k_2^2\hat{\underset{\sim}{E}}_{f2}=-\frac{\omega_p^2}{c^2}\left(\hat{\underset{\sim}{E}}_{f2}+\frac{1}{2}\frac{\hat{n}_s^*}{n_0}\hat{\underset{\sim}{E}}_{f1}\right)$$

these terms cancel by virtue of DR

$$\boxed{\begin{aligned}-\frac{2i\omega_1}{c^2}\frac{\partial\hat{\underset{\sim}{E}}_{f1}}{\partial t}&=-\frac{1}{2}\frac{\omega_p^2}{c^2}\frac{\delta\hat{n}_s}{n_0}\hat{\underset{\sim}{E}}_{f2}\\-\frac{2i\omega_2}{c^2}\frac{\partial}{\partial t}\hat{\underset{\sim}{E}}_{f2}&=-\frac{1}{2}\frac{\omega_p^2}{c^2}\frac{\delta\hat{n}_s^*}{n_0}\underset{\sim}{E}_{f1}\end{aligned}}$$

$$\left(\frac{\partial^2}{\partial t^2}+\omega_p^2\right)\delta\hat{n}_s=-i\underset{\sim}{k}_3\cdot\frac{m}{4}i\underset{\sim}{k}_3\frac{q\hat{\underset{\sim}{E}}_{f2}^*}{m\omega_2}\cdot\frac{q\hat{\underset{\sim}{E}}_{f1}}{m\omega_1}$$

$$\left(\left(\frac{\partial}{\partial t}-i\omega_3\right)^2+\omega_p^2\right)\delta n_s=k_3^2\frac{q^2}{m\omega_1\omega_2}\underset{\sim}{E}_{f1}\cdot\underset{\sim}{E}_{f2}^*$$

$$\frac{2i\omega_2}{c^2}\frac{\partial}{\partial t}\hat{\underset{\sim}{E}}_{f2}^*=-\frac{1}{2}\frac{\omega_p^2}{c^2}\frac{\delta n_s}{n_0}\hat{\underset{\sim}{E}}_{f1}$$

Parametric

$$-2i\frac{\omega_1}{c^2}\frac{\partial}{\partial t}\hat{\underset{\sim}{E}}_{f1} = -\frac{1}{2}\frac{\omega_p^2}{c^2}\frac{\hat{n}_s}{n_0}E_{f2}$$

$$-2i\frac{\omega_2}{c^2}\frac{\partial}{\partial t}\hat{E}_{f2} = -\frac{1}{2}\frac{\omega_p^2}{c^2}\frac{\hat{n}_s^*}{n_0}E_{f1}$$

$$\left[\left(\frac{\partial}{\partial t}-i\omega_3\right)^2+\omega_p^2\right]\left(\frac{\hat{n}_s}{n_0}\right) = -k_3^2\,\frac{1}{4}\,\frac{q^2}{m^2\omega_1\omega_2}\hat{E}_{f1}\cdot\hat{\underset{\sim}{E}}_{f2}^*$$

Parametric decay of wave #1

$$|\hat{E}_{f1}| >> E_{f2}, n_s$$

conj.

$$2i\frac{\omega_2}{c^2}\frac{\partial}{\partial t}\hat{\underset{\sim}{E}}_{f2}^* = -\frac{1}{2}\frac{\omega_p^2}{c^2}\frac{\hat{n}_s}{n_0}\hat{\underset{\sim}{E}}_{f1}$$

$$\left[\left(\frac{\partial}{\partial t}-i\omega_3\right)^2+\omega_p^2\right]\left(\frac{\hat{n}_s}{n_0}\right) = -k_3^2\,\frac{1}{4}\,\frac{q^2}{m^2}\frac{\hat{E}_{f1}\cdot\hat{E}_{f2}^*}{\omega_1\omega}$$

look for solutions $e^{-i\delta\omega\,\cancel{\gamma}t}$

$$2i\frac{\omega_2}{c^2}\,\delta\omega\,\hat{E}_{f2}^* = -\frac{1}{2}\frac{\omega_p^2}{c^2}\frac{\hat{n}_s}{n_0}\hat{E}_{f1}$$

$$\left[(\gamma-i\omega_3)^2+\omega_p^2\right]\left(\frac{\hat{n}_s}{n_0}\right) = -k_3^2\,\frac{1}{4}\,\frac{q^2}{m^2\omega_1\omega_2}\hat{\underset{\sim}{E}}_{f1}\cdot\hat{E}_{f2}^*$$

$$-2i\frac{\omega_1}{c^2}\frac{\partial}{\partial t}\hat{\underset{\sim}{E}}_{f1} = -\frac{1}{2}\frac{\omega_p^2}{c^2}\frac{\hat{n}_s}{n_0}\hat{\underset{\sim}{E}}_{f2}$$

$$-2i\frac{\omega_2}{c^2}\frac{\partial}{\partial t}\hat{\underset{\sim}{E}}_{f2} = -\frac{1}{2}\frac{\omega_p^2}{c^2}\frac{\hat{n}_s^*}{n_0}\underset{\sim}{E}_{f1}.$$

$$\left[\left(\frac{\partial}{\partial t}-i\omega_3\right)^2+\omega_p^2\right]\left(\frac{\hat{n}_s}{n_0}\right) = -k_3^2\,\frac{1}{4}\frac{q^2}{m^2}\frac{\hat{\underset{\sim}{E}}_{f1}\cdot\hat{\underset{\sim}{E}}_{f2}^*}{\omega_1\omega_2}$$

Parametric Decay $\quad \hat{\underset{\sim}{E}}_{f1}$ is large (and constant)

$\hat{\underset{\sim}{E}}_{f2}$ and $\frac{\hat{n}_s}{n_0}$ are small and grow

$$2i\frac{\omega_2}{c^2}\frac{\partial}{\partial t}\hat{\underset{\sim}{E}}_{f2}^* = -\frac{1}{2}\frac{\omega_p^2}{c^2}\frac{\hat{n}_s}{n_0}\hat{\underset{\sim}{E}}_{f1}^*$$

$$\left[\left(\frac{\partial}{\partial t}-i\omega_3\right)^2+\omega_p^2\right]\frac{\partial}{\partial t}\left(\frac{\hat{n}_s}{n_0}\right) = +k_3^2\,\frac{1}{4}\frac{q^2}{m^2}\frac{\hat{\underset{\sim}{E}}_{f1}}{\omega_1\omega_2}\cdot$$

$$\frac{1}{2}\frac{\omega_p^2\,\hat{\underset{\sim}{E}}_{f1}^*}{2i\omega_2}\frac{\hat{n}_s}{n_0}$$

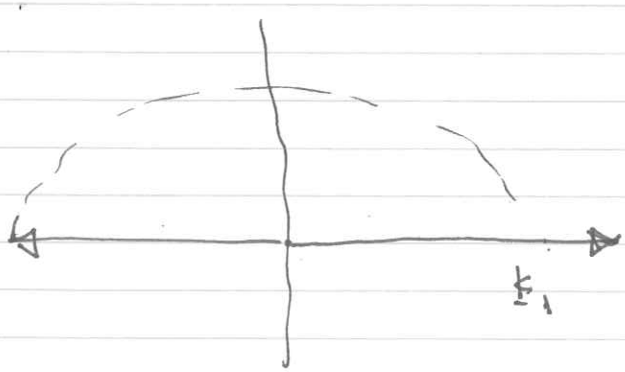
note $\omega_2 \simeq \omega_{-1}$

$$\delta\omega = \pm i \sqrt{\frac{\omega_p}{\omega_{-1}} k_3^2 |\hat{u}_{-1}|^2}$$

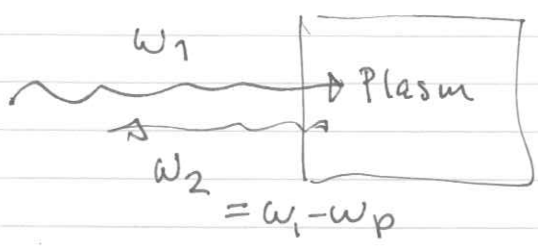
BIGGEST GROWTH rate occurs for biggest

$\underline{k}_3 = \underline{k}_1 - \underline{k}_2$

Backscatter !



Assumes long pulse



anomalous reflectivity

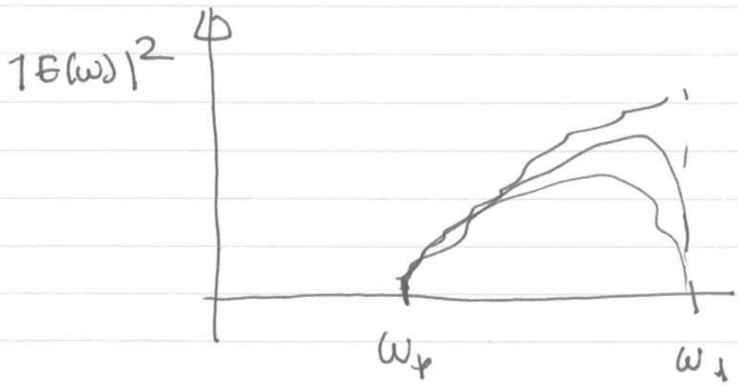
What limits growth of scattered light

Limits: * Pump depletion

* Particle trapping

Pump depletion Energy in freq 1 depleted

$\omega_1 \to \omega_2 + \cdot \quad \omega_2 \to \omega_2 - \omega_p ,$



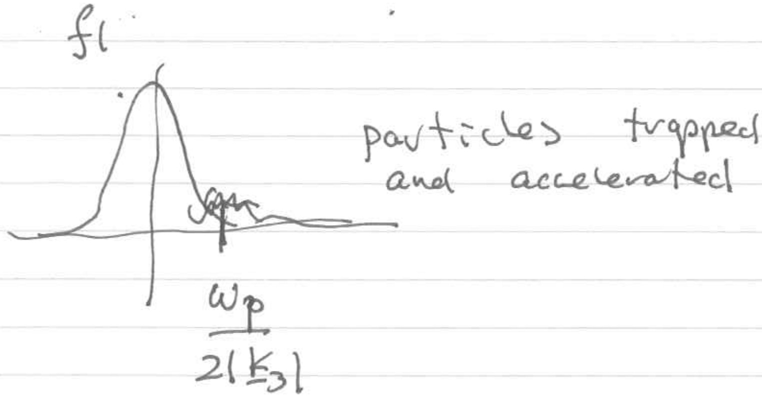
Particle trappin

plasma wave has freq ω_p

and wave vector $|\underset{\sim}{k}_3| \simeq 2|\underset{\sim}{k}_1| \sim \frac{2\omega_1}{c}$

$\frac{\omega_p}{|\underset{\sim}{k}_3|}$ = phase velocity of beat wave

$$= \left(\frac{\omega_p}{2\omega_1}\right) c \ll c$$



~~if grow~~

Dependence on matching criteria

suppose $\omega_3 \neq \omega_p$ but they are close

$$\delta\omega\left[\omega_p^2 - (\omega_3+\delta\omega)^2\right] = 2\omega_p \gamma_0^2$$

$$\cancel{\omega_p}\ \omega_3 = \omega_p + (\omega_3-\omega_p)$$

$$-\delta\omega\, 2\omega_p\left[\delta\omega + (\omega_3-\omega_p)\right] = 2\omega_p \gamma_0^2$$

$$-\delta\omega^2 - \delta\omega(\omega_3-\omega_p) = \gamma_0^2$$

$$\delta\omega = \frac{(\omega_3-\omega_p) \pm \sqrt{(\omega_3-\omega_p)^2 - 4\gamma_0^2}}{-2}$$

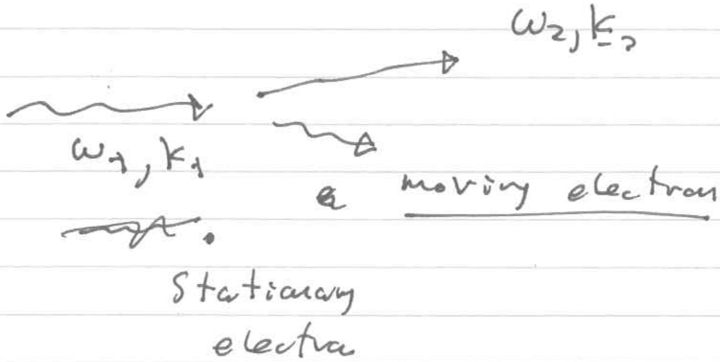
unstable if $(\omega_3-\omega_p)^2 < 4\gamma_0^2$

Exact freq match not required

Compton regime $\delta\omega >> \omega_p, \omega_3$

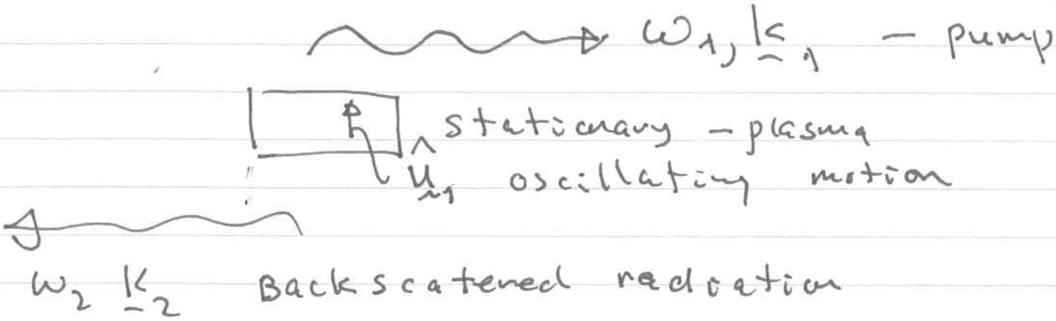
$$-\delta\omega^3 = \frac{1}{8}\frac{\omega_1\omega_p^2}{\omega_2}k_3|\hat{u}_1|^2$$

$$\delta\omega = e^{+i\frac{\pi}{3}}\left[\frac{1}{8}\frac{\omega_1\omega_p^2}{\omega_2}k_3|\hat{u}_1|^2\right]$$



All of the above can be applied to free electron lasers

Raman Back-scatering



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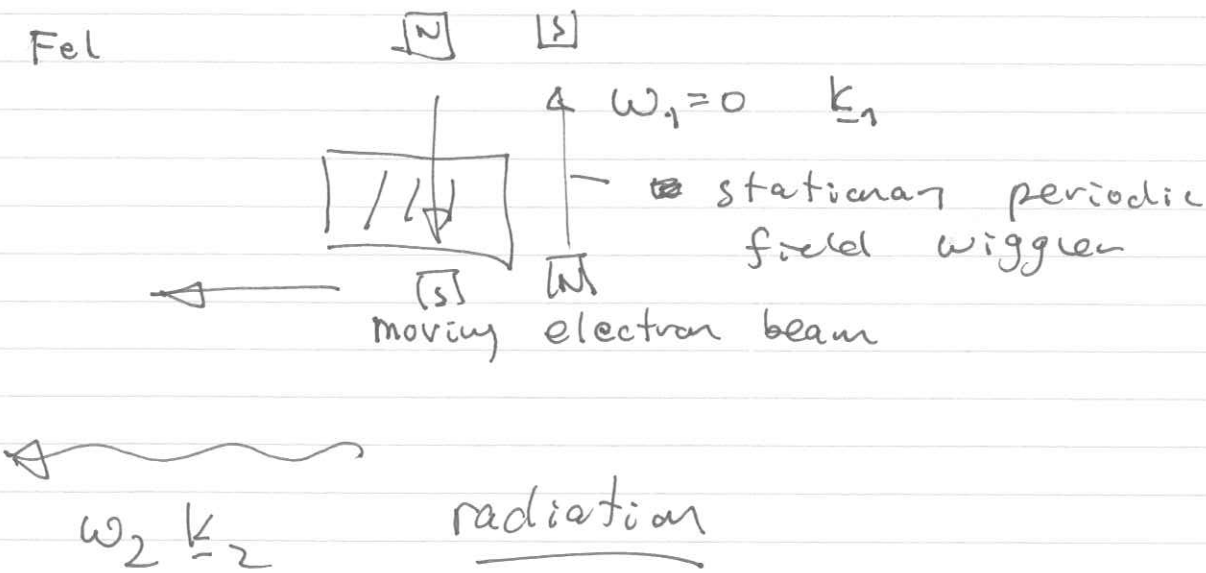
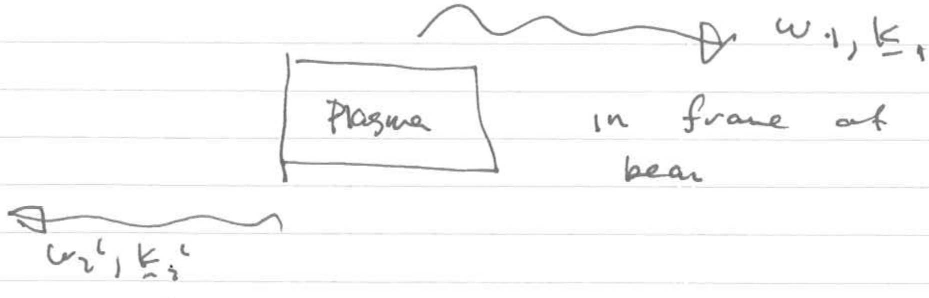


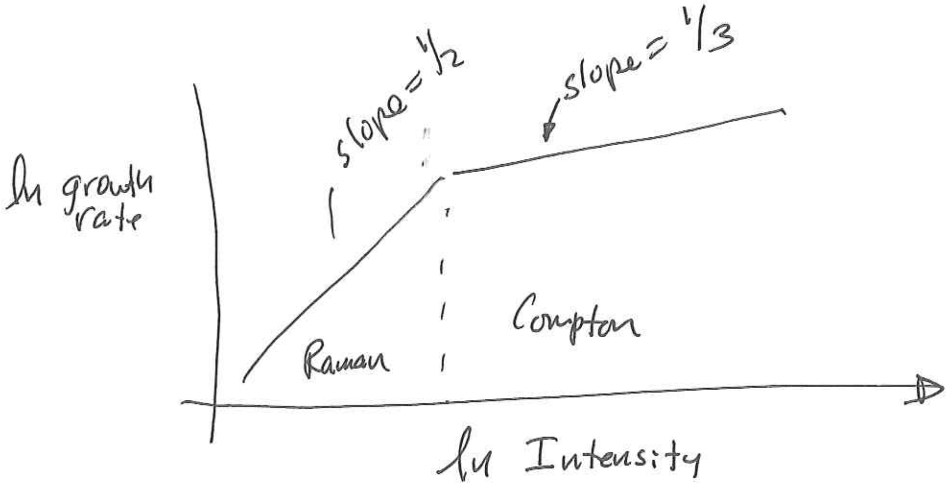


Compton Scattering

Strong Pump limit $\quad \delta\omega > \omega_p, \omega_3$

$$-(\delta\omega)^3 = \frac{1}{16}\frac{\omega_p^2}{\omega_f} k_3^2 |\hat{u}_f|^2$$

$$\delta\omega = e^{i\pi/3}\left(\frac{1}{16}\frac{\omega_p^2}{\omega_f} k_3 |u_f|^2\right)^{1/3}$$



$$\frac{2\omega_2}{c^2}\,\delta\omega\,\hat{\underset{\sim}{E}}_{f2}^{*} = -\frac{1}{2}\frac{\omega_p^2}{c^2}\hat{\underset{\sim}{E}}_{f1}^{*}\left(\frac{\hat{n}_s}{n_0}\right)$$

$$\left[\omega_p^2-(\delta\omega+\omega_3)^2\right]\left(\frac{\hat{n}_s}{n_0}\right) = -k_3^2\,\frac{1}{4}\,\frac{q^2}{m^2\omega_1\omega_2}\,\hat{\underset{\sim}{E}}_{f1}\cdot\hat{\underset{\sim}{E}}_{f2}^{*}$$

combine

$$\delta\omega\left[\omega_p^2-(\delta\omega+\omega_3)^2\right] = k_3^2\,\frac{1}{4}\,\frac{q^2}{m^2\omega_1\omega_2}\,\frac{\omega_p^2}{4\omega_2}\,|\underset{\sim}{E}_{f1}|^2$$

$\omega_1 \simeq \omega_2 = \omega_f$

$$= \frac{1}{16}\frac{\omega_p^2}{\omega_f}\,k_3^2\,|\underset{\sim}{u}_f|^2$$

oscillating velocity

Cubic DR

Three wave resonant case $\omega_3 = \omega_p$ (Raman)

$$\omega_p^2-(\delta\omega+\omega_3)^2 \simeq -2\,\delta\omega\,\omega_p$$

$$-\delta\omega^2 = \frac{1}{32}\frac{\omega_p}{\omega_f}k_3^2|u_f|^2 \qquad \text{unstable}$$

$$\delta\omega = \pm i\left[\frac{1}{32}\frac{\omega_p}{\omega_f}k_3^2|u_f|^2\right] \qquad \text{breaks down when}$$

$\delta\omega > \omega_p$