Ionization what makes a plasma

As we supply energy to a neutral gas what determines the degree of ionization?

Two important processes

Electron impact ionization

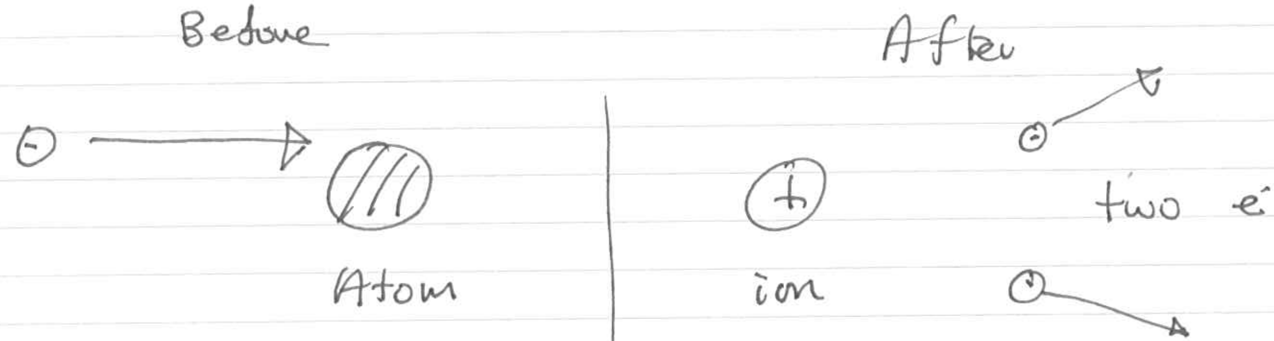


Photo Ionization

Inverse processes

Before | After

Three body recombination

Radiative recombination

In thermal equilibrium these processes balance and determine the fraction of atoms that are ionized (consider H for simplicity)

n_I = density of ions

n_a = density of atoms

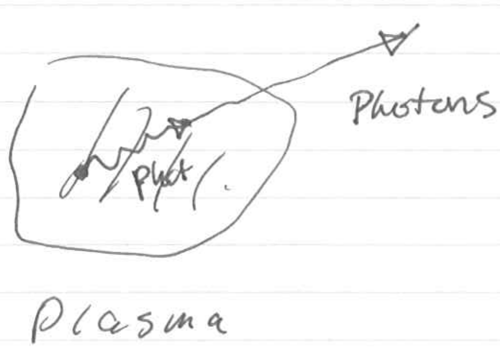
fraction of ionized

$$f = \frac{n_I}{n_I + n_a} = f(n_I, T)$$

depends on T & density

⊛ Plasma's in Equilibrium are rare.

Photon's are not confined



Executive summary

1. For $T \gtrsim 1\,eV$ almost all atoms are ionized ~~fully~~

$E_H = 13.6\,eV$

we'll see why

2. Unless $T \lesssim$ several eV
and $n \lesssim 10^{16}\,cm^{-3}$

Radiative Recombination > Three body recombination
proportional to n^2 propor n^3

~~(assumes photons not confined)~~

No photon confinement

Coronal Equilibrium

electron impact ionization } balances { radiative recombination

$$\frac{n_I}{n_{Tot}} = g(T) < f(n,T)$$ depends only on temp.

Radiative Processes see ~~the~~

"Principles of Plasma Spectroscopy"
Hans R. Griem
Cambridge Univ Press.

Reaction Cross-Sections

Various atomic and nuclear processes are described by reaction rates and cross-sections

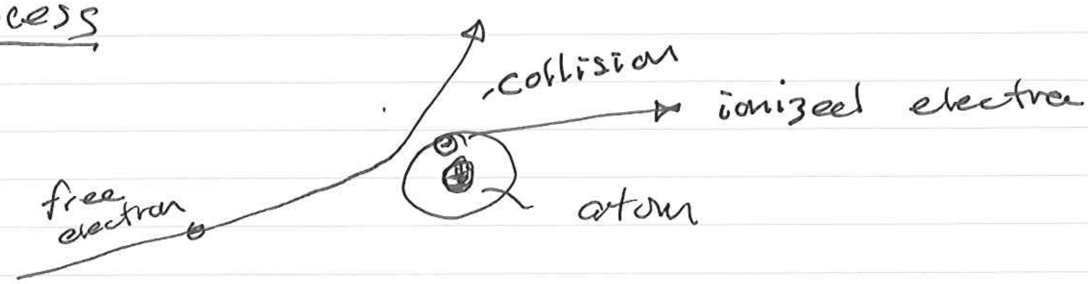
What does this mean?

~~Suppose we have an~~

* To be concrete let's consider an example, collisional ionization

We would like to know the rate at which new electrons are ~~created~~ released in a plasma by collisions of existing free electrons ~~by ne~~ with neutral atoms.

Process



The rate at which ~~a~~ the atom is ionized is given by

$$\sigma n v = \frac{\#\text{ of ionizations}}{\text{sec}}$$

σ depends on the energy of the beam, viz $\frac{1}{2}mv^2$

~~Note~~ σ has units of area

$$\text{Area} \times \frac{\#\text{ of electrons}}{\text{Area se}} = \frac{\#}{\text{sec}}$$

σ = ionization cross-section

This is the rate if we have an incident beam with velocity v. For a distribution of electrons the rate at which the atom is ionized is

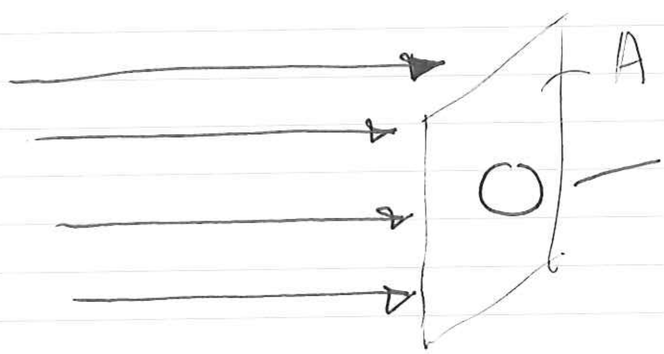
$$\langle \sigma n v \rangle = \int d^3v\, f_e(\underset{\sim}{x}, \underset{\sim}{v})\, \sigma v$$

~speed

First, the basic process is characterized by a ~~cross~~ quantity called the cross-section
the cross section represents the size of the target

Suppose we have an infinite beam of electrons with speed v

sec = FA



The flux of electrons is $F = n_e v$ (density, speed)

$$F = \frac{\#\text{ of electrons}}{\text{cm}^2\ \text{sec}}$$

~~The rate at~~

Assume that each electron encounters ~~the~~ a single ~~same~~ neutral atom

Some electrons miss atom
some electrons excite the electron to a higher state but don't ionize

some electrons ionize the atom

a_0 = Bohr radius

$E_H = \frac{e^2}{2a_0}$ · ergs $\qquad E_H \sim 13.6$ eV

E_∞ = energy needed to ionize = E_H in hydrogen

$\sigma \sim \frac{1}{E}$ for high energy

~~Other ionization processes~~

~~Photo ionization~~ ~~look-up~~

Ionization rate

electron collision

$$\cancel{\frac{dn_e}{dt} \cong n_e \left(\frac{n_a}{n_e}\right) \nu_e\, f\!\left(\frac{E_\infty}{T_e}\right)}$$

$$\frac{dn_e}{dt} = n_a \langle \sigma n_e v \rangle$$

ν_I

ν_I = Ionization rate $\propto n_e$

function of T_e

$\sim \nu_I \sim \nu_e$ collision

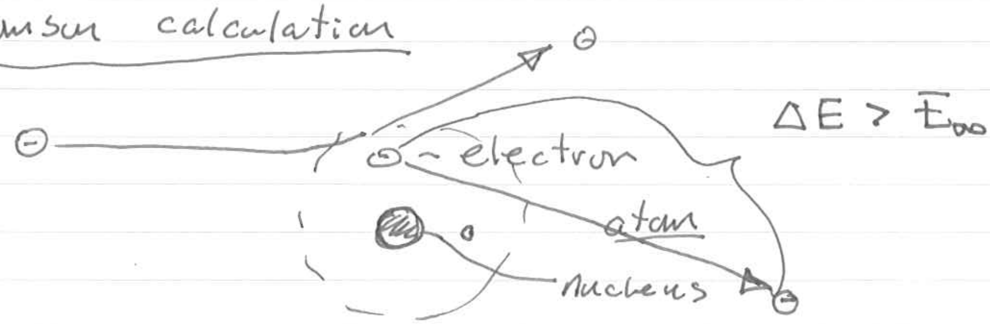
Note this is proportional to electron density and depends on ~~elect~~ plasma electron temperature

~~Fina~~ Finally the number of ionizations per unit volume is obtained by multiplying by the atomic density.

$$\dot{n}_e = n_a \langle \sigma n_e v \rangle$$

But what is σ?

Thomson calculation



electron scatters from rest $\qquad a_0 = \dfrac{\hbar^2}{me^2}$

if it ~~[illegible]~~ picks up enough energy it is free

$$\sigma = 4\pi a_0^2 \left(\frac{E_H}{E}\right)^2 \left(\frac{E}{E_\infty} - 1\right)$$ not quantum

Recombination

before:

Three body recombination

after:

$$\frac{dn_e}{dt} = \underset{\sim n_e}{n_a \nu_I} - n_I \nu_R$$

ν_R is the recombination rate

depends on n_e, T_e

Principle of detailed balance

In thermal equilibrium rate of inoization = rate of recombination

$$\nu_R = \left(\frac{n_a}{n_I}\right)_{\text{th. eq.}} \nu_I^{\sim n_e}$$

we calc this last lecture

where $\left(\frac{n_a}{n_I}\right)_{th}$ = ratio of neutrals to ions in TE $\sim f(T, (n_a + n_I))$

$$\left(\frac{n_a}{n_I}\right)_{th} = 8\pi^{3/2} a_0^3 n_T \left(\frac{E_H}{T}\right)^{3/2} e^{E_H/T}$$

$$\frac{dn_e}{dt} = \underbrace{n_a \nu_I}_{\text{two powers of density}} - n_I \nu_I \, 8\pi^{3/2} a_0^3 n_T \left(\frac{E_H}{T}\right)^{3/2} e^{E_H/T}$$

$n_I \; n_e \; n_T$

three powers of density

Thermal equilibrium (Saha Equilibrium)

Suppose we have $n_T = n_a + n_I$ atoms

n_a = number of neutral atoms

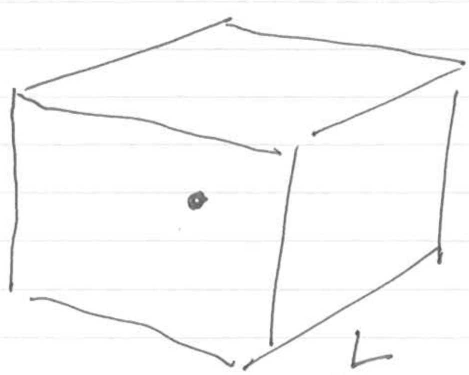
n_I = number of ions

Consider case of hydrogen singly ionized only (assume molecules H_2 dissociated)

Q: At a given temperature and density what fraction of atoms are ionized?

* bottom line $\longrightarrow$

* probability that system is in state of energy E is proportional to exp(−E/T)



consider box containing 1 atom

side $L = n_T^{-1/3}$

Bottom line

$$\left(\frac{n_I}{n_T}\right)_{Th} = \frac{1}{8\pi^{3/2}\, a_0^3\, n_T \left(\frac{E_H}{T}\right)^{3/2} e^{E_H/T} + 1}$$

Bohr radius — total density

when $E_H \sim T$ & $\underbrace{a_0^3 n_T \ll 1}_{\text{dilute gas}}$

almost all atoms are ionized

$E_H = 13.6\ eV$

with temperature fixed $n_T \uparrow$ $\frac{n_I}{n_T} \downarrow$

with density fixed $T \uparrow$ $\frac{n_I}{n_T} \to 1$

Ionization fraction

$$\left(\frac{n_I}{n_T}\right)_{th} = \frac{\sum_{\text{states } E>0} e^{-E/T}}{\sum_{\text{states } E<0} e^{-E/T} + \sum_{\text{states } E>0} e^{-E/T}}$$

TREAT jus one state (ground state)

lets evaluate states $E>0$, ~~elas~~ neglect atomic potential (assume plasma parameter is small)

Box of size L $\qquad \psi \sim e^{i\underset{\sim}{k}\cdot\underset{\sim}{x}}$

wave numbe $\qquad \underset{\sim}{k} = \frac{2\pi}{L}\left(\underset{\sim}{e}_x n_x + \underset{\sim}{e}_y n_y + \underset{\sim}{e}_z n_z\right)$

$$n_{x,y,z} = -\infty \cdots 0 \cdots \infty$$

$$p = \hbar k$$

$$E = \frac{\hbar^2 k^2}{2m}$$

$$\sum_{\substack{\text{States}\\E>0}} e^{-E/T} = \sum_{n_x, n_y n_z} \exp\left[-\left(\frac{h}{L}\right)^2 \frac{1}{2mT}\left(n_x^2 + n_y^2 + n_z^2\right)\right]$$

For large box $\quad \sum_{n_x n_y n_z} = \int dn_x\, dn_y\, dn_z$

$$\sum_{E>0} e^{-E/T} = \left[\int dn \, \exp\left[-\left(\frac{h}{L}\right)^2 \frac{n^2}{2mT} \right] \right]^3$$

$$= (2mT\pi)^{3/2} \left(\frac{L}{h}\right)^3$$

$$= \frac{L^3}{h^3} \left(2\pi m T \right)^{3/2}$$

Bohr radius $a_0 = \frac{\hbar^2}{me^2}$

$$E_H = \frac{e^2}{2a_0} = 13.6\,\mathrm{eV}$$

$\boxed{\hbar = h/2\pi}$

$$e^2 = 2a_0 E_H$$

$$\hbar^2 = a_0 m e^2$$

skip

$$\hbar^2 = m \; 2 a_0^2 E_H$$

$$\hbar = (2mE_H)^{1/2} a_0 \qquad h = 2\pi\hbar$$

$$\sum_{E>0} e^{-E/T} = \frac{L^3 \, (2\pi m T)^{3/2}}{(2\pi)^3 \, a_0^3 \, (2mE_H)^{3/2}} = \frac{L^3}{a_0^3} \frac{\pi^{3/2}}{2^3} \left(\frac{T}{E_H}\right)^{3/2}$$

now put $L^3 = \frac{1}{n_T}$

$$\sum_{E>0} e^{-E/T} = \frac{1}{a_0^3 n_I} \frac{1}{8\pi^{3/2}} \left(\frac{T}{E_H}\right)^{3/2}$$

very big factor for dilute gas

what that means is if $T \sim E_H$ almost all atoms are ionized

(much more phase space volume available to free electrons)

Result for a dilute gas ~~equal~~ comparable fractions of ~~neutralized~~ neutral and ionized atoms occur for $E_H/T \gg T$

$\sum_{E<0} e^{-E/T}$ is dominated by ground state

$$\sum_{E<0} e^{-E/T} = e^{E_H/T}$$

$$\left(\frac{n_I}{n_T}\right)_{TH} = \frac{\frac{1}{a_0^3 n_T}\frac{1}{8\pi^{3/2}}\left(\frac{T}{E_H}\right)^{3/2}}{e^{E_H/T} + \frac{1}{a_0^3 n_T}\frac{1}{8\pi^{3/2}}\left(\frac{T}{E_H}\right)^{3/2}}$$

$$\left(\frac{n_a}{n_I}\right)_{th} = \frac{e^{E_H/T}}{\frac{1}{8\pi^{3/2}}\frac{1}{a_0^3 n_T}\left(\frac{T}{E_H}\right)^{3/2}} \propto \underset{\sim n_e}{n_T}$$

$$\boxed{\frac{dn_e}{dt} = \overset{\propto n_e}{n_a}\nu_I - \underbrace{\overset{n_T}{n_I}\left(\frac{n_a}{n_I}\right)_{th}\overset{\sim n_e}{\nu_I}}_{n^3 \text{ three body process}}}$$

skip ~~an~~ fill later

50% ionized

$$\boxed{\left(\frac{E_H}{T}\right)^{3/3} e^{\frac{E_H}{T}} = \frac{1}{8\pi^{3/2} a_0^3 n_T}}$$

$\frac{E_H}{T} > 1$

See picture