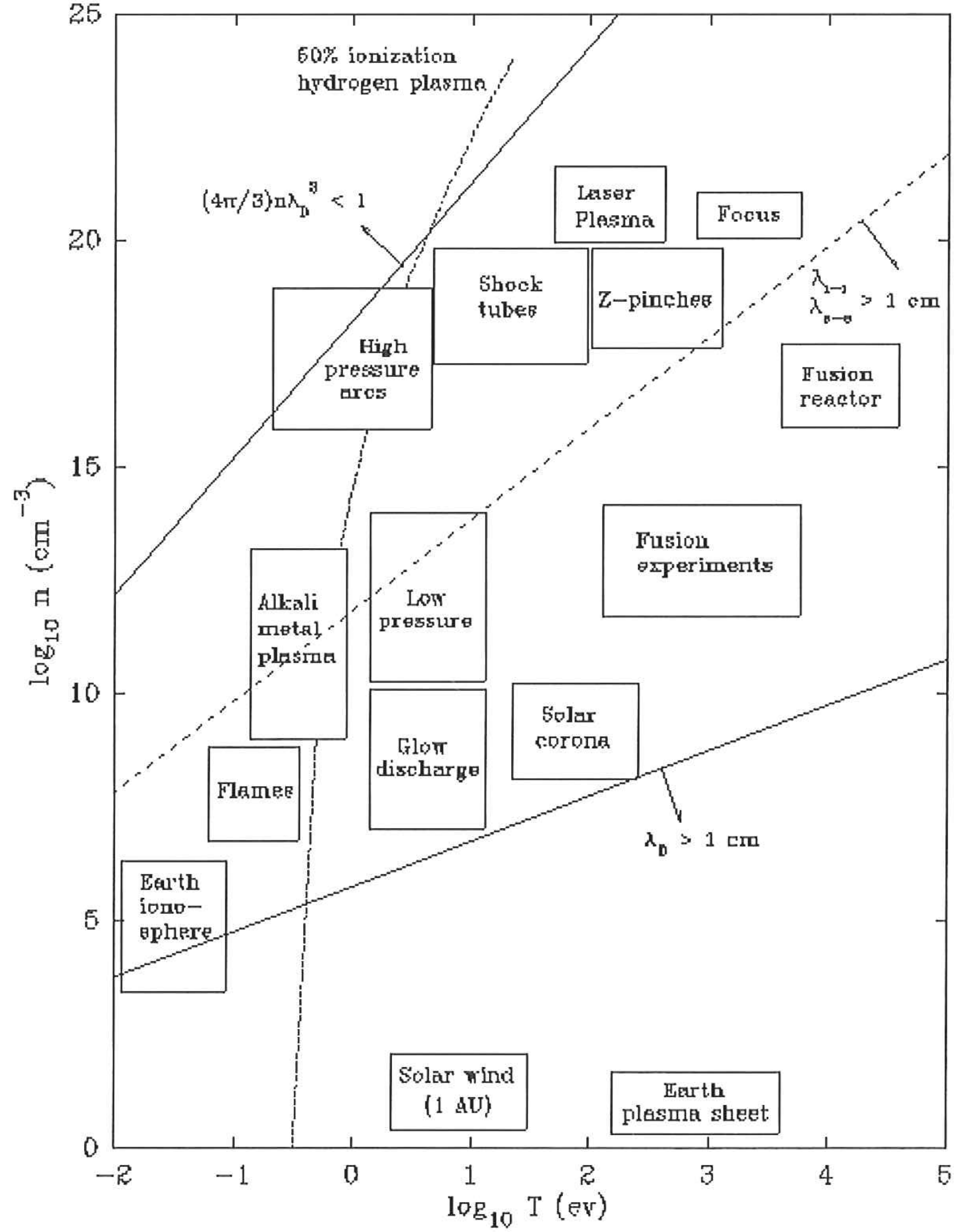


25
20
15
10
5
0
−2
−1
0
1
2
3
4
5
$\log_{10}$ n (cm^{-3})
$\log_{10}$ T (ev)
60% ionization hydrogen plasma
$(4\pi/3)n\lambda_D^3 < 1$
λ_{i-i}, λ_{e-e} > 1 cm
λ_D > 1 cm
Laser Plasma
Focus
Shock tubes
Z-pinches
High pressure arcs
Fusion reactor
Fusion experiments
Alkali metal plasma
Low pressure
Glow discharge
Solar corona
Flames
Earth iono-sphere
Solar wind (1 AU)
Earth plasma sheet

~~In thermal equilibrium,~~
~~The average kinetic energy of a particle~~
~~in thermal equi.~~

$$\cancel{\langle K.E.\rangle = \left\langle \tfrac{1}{2}m\left(v_x^2+v_y^2+v_z^2\right)\right\rangle = \tfrac{3}{2}T}$$

~~3 degrees of Freedom~~

Maxwell - Boltzmann Distribution

In thermal equilibrium the probability of finding a particle ~~in~~ with energy E is proportional to the factor

$$\text{Const}\,\exp\left(-\frac{E}{T}\right)$$

temperature

Assume E is dominated by Kinetic Energy (Interactions are weak)

$$E = \tfrac{1}{2}m\left(v_x^2+v_y^2+v_z^2\right) + e\phi$$

local potential if present

What is constant of proportionality ?

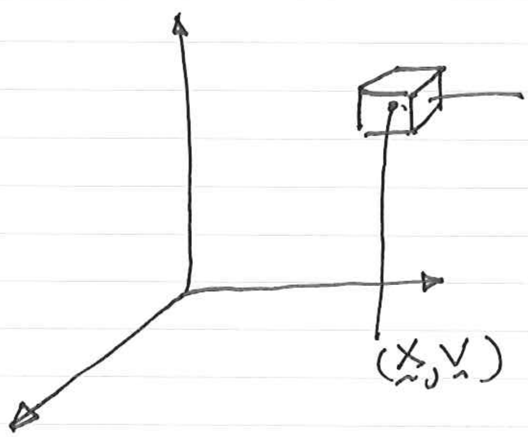
Particle distribution function

$$f(\underset{\sim}{x}, \underset{\sim}{v}, t) \qquad\qquad f_{s_i}(\underset{\sim}{x}, \underset{\sim}{v}, t)$$

(x, y, z) (v_x, v_y, v_z) t ← seven arguments

What ~~does it mean~~ is its definition ?

6-dimensional space $\underset{\sim}{x}, \underset{\sim}{v}$ phase space



small box with 6D volume

$d^3x\, d^3v$

The number of particles in the small box of volume $d^3x\, d^3v$ centered at the coordinates $(\underset{\sim}{x}, \underset{\sim}{v})$ is dN (a number)

$$dN = f(\underset{\sim}{x}, \underset{\sim}{v})\, d^3x\, d^3v$$

if I integrate over all phase space I get the total number of particles

$N_{e,i}$ = total number of particles of type e,i

$$= \int d^3x \underbrace{\int d^3v\, f_{e,i}(\underset{\sim}{x}, \underset{\sim}{v})}_{\text{this must be the local density } n_{e,i}}$$

why

$$N = \int d^3x \; n_{e,i}$$

density

volume

Thus, in thermal equilibrium

$$f = \frac{n_0}{(2\pi T/m)^{3/2}} \exp\left(-\frac{\frac{1}{2}m|v|^2 + q\phi}{T}\right)$$

a constant density

charge

$$\boxed{\text{local density} \qquad n = \int d^3v\, f = n_0 e^{-\frac{q\phi}{T}}}$$

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PRIVACY & SECURITY

- Plasma Crystals Home Page
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- Theoretical Treatment
- Gas Drag
- Determination of Crystal Parameters
- Collaboration Opportunities
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Long Range Particle Interactions and Collective Phenomena in Plasma Crystals

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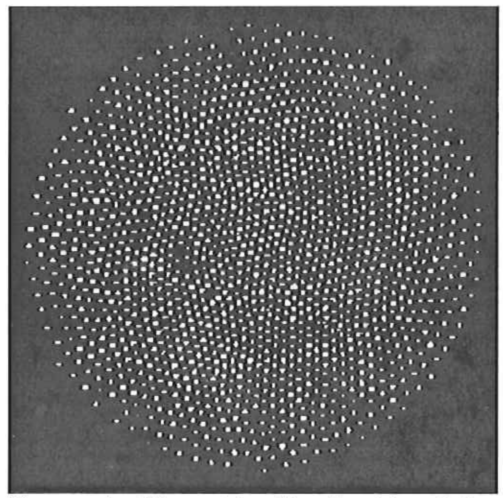
- The macroscopic size of the crystal offers a unique vehicle:
  - to investigate the fundamental principles underlying long-range multi-particle interactions and
  - to investigate collective phenomena in macro-crystals, and to probe crystal structure and dynamics
- Recent literature has begun to document basic phenomena such as influence of discharge geometry and characteristic wave propagation, mach cone models, and experimental measurements of interaction potential effects due to gravity.
- Funded by Division of Material Sciences, Office of Science, US DOE and Sandia National Laboratories.



Very regular 2D crystals with noticeable radial compression are producedin the parabolic well. For this crystal 8.3 μm diameter Melamine spheres are suspended in a 100 mTorr, 1.8 W Argon plasma above a 0.5 m radius of curvature lower electrode. The crystal contains 1155 particles and is 21.8 mm in diameter.

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  - Attractive forces?
  - Can we identify and manipulate the forces?
- Identifying the crystal dynamic response, stability criterion, "defect" propagation.
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- Developing new diagnostic techniques, particle mapping methodologies, statistics, and novel techniques to measure the electric fields between the particles. (All under consideration) as are methods to characterize the surface charge.
- Addressing concerns, such as, step changes in the plasma Debye length by electron heating.
- Combining experiments and models to benchmark first-principle models, to describe the charged particle interaction potentials.

(6')

APPROXIMATE MAGNITUDES IN SOME TYPICAL PLASMAS

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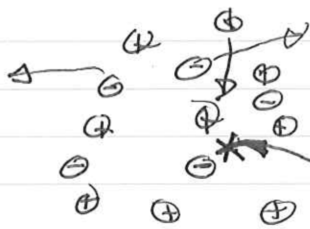
The diagram (facing) gives comparable information in graphical form.[22]

Debye shielding

1) Suppore we have an electron-ion plasma which is overall charge neutral

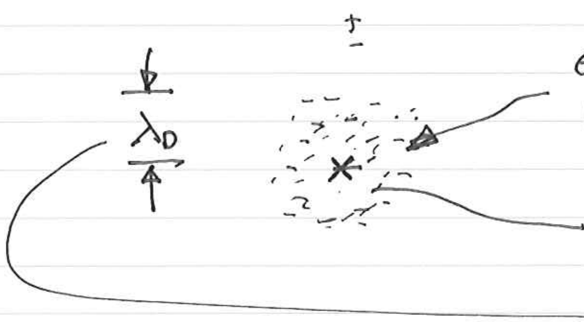
$q_e n_e \cong q_i n_i$



(2) Introduce a fixed test charge

q_+ (suppose q_+ is positive)

Negative charges will be attracted
positive charges reppelled



effective cloud of negative charge

q_+

extent of cloud $\sim \lambda_D$
Debye length

How to describe this?

Poisson Equation

$$-\nabla^2 \phi = 4\pi \rho$$

ρ = charge density

$$\rho = q_i n_i + q_e n_e + q_t \delta^3(\underset{\sim}{x} - \underset{\sim}{x}_t)$$

$\uparrow$ test charge

$$q_i n_i = q_i n_{i0} \exp\left(-\frac{q_i \phi}{T}\right)$$

$$q_e n_e = q_e n_{e0} \exp\left(-\frac{q_e \phi}{T}\right)$$

if $\phi > 0$

$q_i \phi > 0$ $\quad n_i \downarrow$

$q_e \phi < 0$ $\quad n_e \uparrow$

Let's assume $\Gamma \ll 1$ (weakly coupled)

This implies $|q_{i,e} \phi / T| \ll 1$ if $q_t \sim q_i, q_e$

$$-\nabla^2 \phi = 4\pi q_i n_{i0} \left(1 - \frac{q_i \phi}{T}\right) + 4\pi q_e n_{e0} \left(1 - \frac{q_e \phi}{T}\right)$$
$$+ 4\pi q_t \delta^3(\underset{\sim}{x} - \underset{\sim}{x}_t)$$

Approx
Charge neutrality $\quad q_i n_{i0} + q_e n_{e0} = 0$

$$-\nabla^2 \phi = -\sum_{e,i} \frac{4\pi q_{e,i}^2 n_{i0}}{T} \phi + 4\pi q_t \delta^3(\underset{\sim}{x} - \underset{\sim}{x}_t)$$

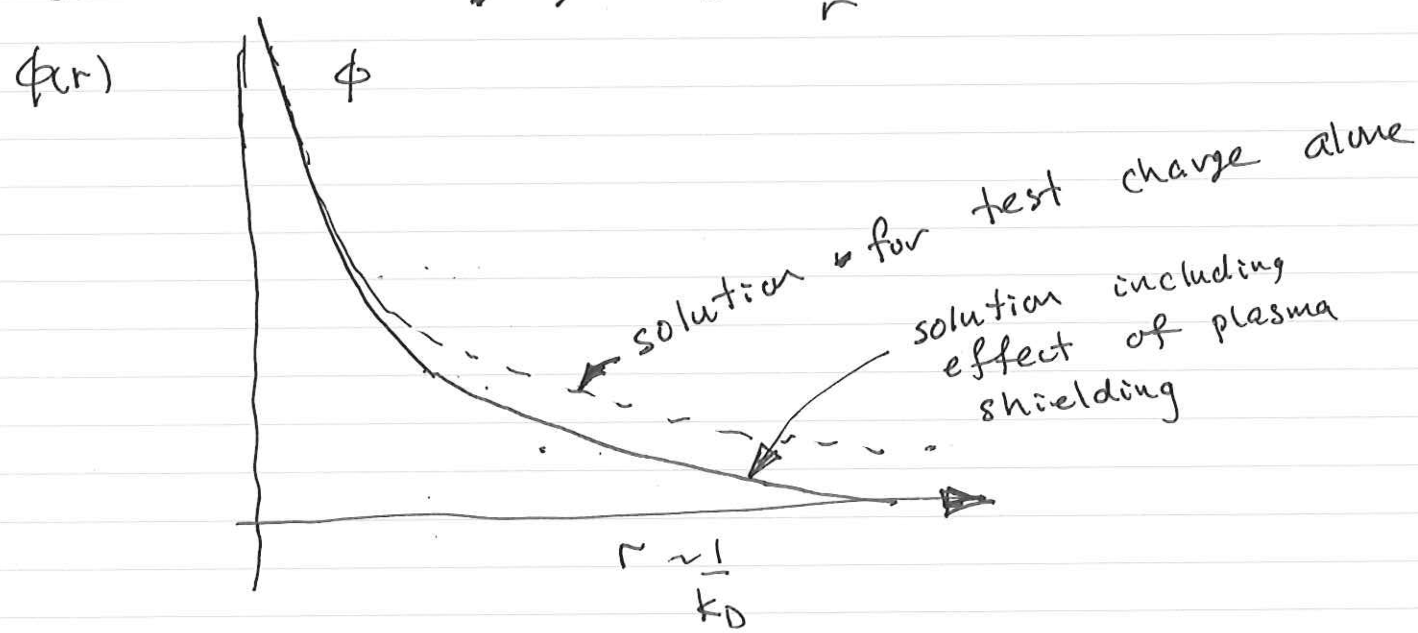
call $\displaystyle \sum_{e,i} \frac{4\pi q_{e,i}^2 n_{i0}}{T} = k_D^2 > 0$

* Assume test charge is at origon ~~in sphere~~

* Solution is spherically symmetrix

$$-\frac{1}{r^2}\frac{\partial}{\partial r} r^2 \frac{\partial \phi}{\partial r} + k_D^2 \phi(r) = 4\pi q_t \frac{\delta(r)}{4\pi r^2}$$

Solution $\qquad \phi(r) = q_t \frac{\exp(-k_D r)}{r}$



$$k_D^2 = \sum_{ei} \frac{4\pi n q_{ei}^2 n_{ei0}}{T} = 2\,\frac{4\pi n e^2}{T}$$ for hydrogen plasma

Electric field of test charge is "screened" out at distances $r > \lambda_D$

IF $n\lambda_D^3 = \frac{1}{\Gamma} \gg 1$ as we have ~~assumed~~ here

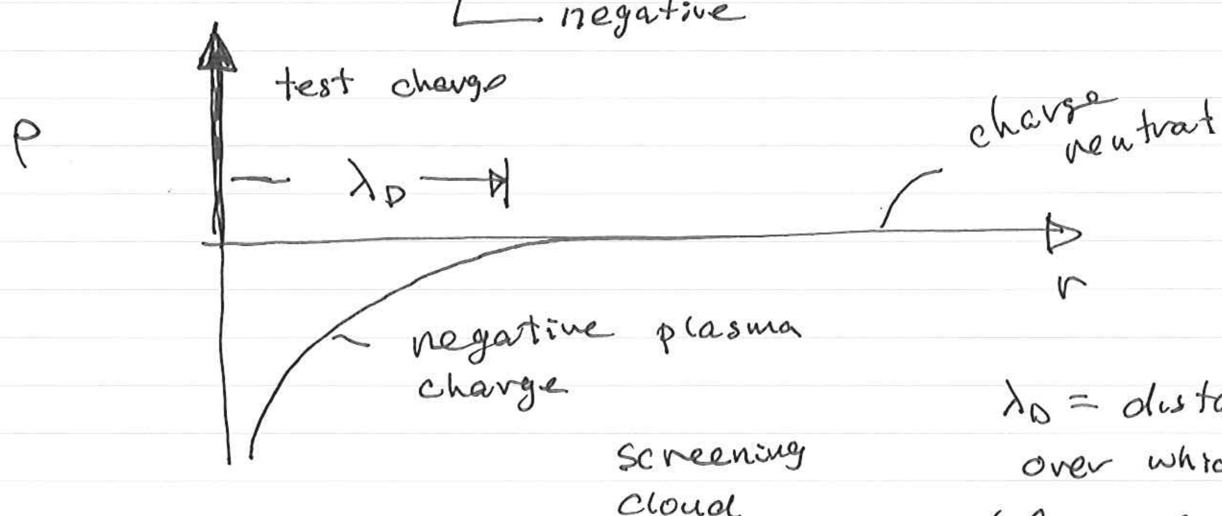
There is a large # of charged particles in screening cloud justifies using distribution function

VIEW TABLE

Quasineutrality

charge density

$$4\pi\rho = \underbrace{-k_D^2\phi}_{\text{negative}} + \overbrace{4\pi q_t \frac{\delta(r)}{4\pi r^2}}^{\text{positive test charge}}$$



λ_D = distance over which $(q_e n_e + q_i n_i) \neq 0$

charge neutrality not maintained

Summary of Lecture # 1

Plasma's in TE

density: $n_{e,i}$ and Temperature

1) plasma parameter

$$\Gamma = \frac{e^2}{T} n^{1/3}$$

$\Gamma > 1$ strong coupling
strong correlations

- liquid, crystal

$\Gamma < 1$ weak coupling
interactions with neighbors weak

2) Debye length

$$\lambda_D = \left[\frac{T}{4\pi n e^2} \right]^{1/2}$$

distance over which charge imbalance is shielded

screening

$$\Gamma = \frac{1}{4\pi} \frac{1}{(n \lambda_D^3)^{2/3}}$$

$n \lambda_D^3 \gg 1$ many particles in sreening cloud

High Frequency dielectric constant

$$\epsilon = \left(1 - \frac{\omega_p^2}{\omega^2}\right)$$

Suppose we have an oscillating electric field in the plasma

$$\underset{\sim}{E} = \mathrm{Re}\{\cancel{\ } \hat{\underset{\sim}{E}}_0 e^{-i\omega t}\}$$

Oscillation frequency ω

This will produce an oscillating current density

$$\underset{\sim}{J} = \mathrm{Re}\{\cancel{\ } \hat{\underset{\sim}{J}}_0 e^{-i\omega t}\}$$

$\longleftrightarrow$ Oscillating E

oscillating velocity

$\longleftrightarrow$ Oscillating J

~~Can Show~~ $\underset{\sim}{V} = \mathrm{Re}\{\cancel{\ } \underset{\sim}{V}_{osc} e^{-i\omega t}\}$

$$\cancel{\frac{4\pi}{e} J_0 = -\sum_e \frac{4\pi n_{e,i} q_{e,i}^2}{m_{e,i}\,\omega^2}}$$

NEWTON'S LAW

$$-i\omega m_{e,i} \underset{\sim}{V}_0 = q_{e,i} \underset{\sim}{E}_0$$

current density
~~From Newtons~~

$$\underset{\sim}{J}_{osc} = q \underset{\sim}{V}_{o\,q_i} n_{e,i} = \frac{i}{\omega} \frac{q_{q_i}^2 n_{e,i}}{m_{e,i}} E_o$$

$$\nabla \times \underset{\sim}{B} = \frac{4\pi}{c} \underset{\sim}{J} + \frac{1}{c}\frac{\partial}{\partial t} \underset{\sim}{E} = Re\left\{ -\frac{i\omega}{c} \left(1 - \sum_{q_i} \frac{\omega_{pe,i}^2}{\omega^2}\right) E_o \, e^{-i\omega t} \right\}$$

conduction current

displacement current

$$\omega_{pe,i}^2 = \frac{4\pi n_{e,i} q_q^2}{m_{e,i}}$$

$$\epsilon = \left(1 - \sum_{e,i} \frac{\omega_{pe,i}^2}{\omega^2}\right)$$

$\omega_{pe}^2 >> \omega_{pi}^2$

$\boxed{\text{main contribution is from electrons}}$

if $\omega >> \omega_{pe}$ electron response is unimportant
~~$\omega \lesssim \omega_{pe}$ electron response is VI.~~

$\epsilon = 0$ corresponds to a normal mode

$\underset{\sim}{E}_o$ but no $\underset{\sim}{B}$ (electrostatic mode)

displacement current cancells conduction current

$\omega = \omega_{pe}$

Determines ~~frequency~~ time scale for restoration of charge imbalance

How far does a typical ~~particle~~ electron go in one plasma period?

$$\epsilon \cong 1 - \frac{\omega_p^2}{\omega^2}$$

What's wrong with this picture?

Newton's Law 

$$m_e \frac{d\mathbf{v}_e}{dt} = q_e \mathbf{E}_0(\mathbf{x}, t)$$

we assume electron excursion is unimporta

Wave

$$\mathbf{E}_0(\mathbf{x},t) = \mathrm{Re}\left\{ \hat{E}_0 e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)} \right\}$$

excursion must satisfy $\mathbf{k}\cdot\delta\mathbf{x} \ll 1$

$\delta \mathbf{x} \sim \frac{\mathbf{V}_{osc}}{\omega}$ $\quad \frac{k}{\omega} V_{osc} \ll$ motion is nonrelativistic

for light waves $\frac{\omega}{k} \sim c$

what ~~about~~ about thermal motion?

electrons zipping about with typical speed V_{th} $\quad$ th → thermal

$\frac{1}{2} m V_{th}^2 \sim T$ — temper

Require $kV_t < \omega$ $\quad$ no problem for light waves

What about plasma waves?

$\omega \sim \omega_{pe}$ $\quad k^2 < \dfrac{\omega_{pe}^2}{V_{the}^2} = \dfrac{4\pi n e^2}{m_e \frac{2T}{m_e}} = \dfrac{1}{2\lambda_D^2}$

wavelength $\lambda > \lambda_D$ can neglect thermal motion

In general

$\epsilon = \epsilon(\omega, \underset{\sim}{k})$

we will derive this in the next few weeks!

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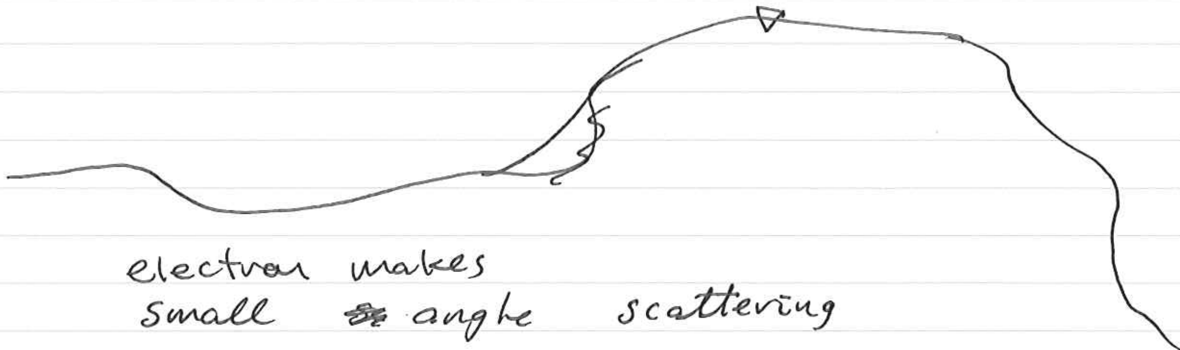
The diagram (facing) gives comparable information in graphical form.[22]

<u>Collisions</u> We have already said that ~~collisions~~ inter particle interactions are weak



electron makes
small angle scattering

Time to be deflected by $90^{\circ} = \frac{1}{\nu_e}$

ν_e = collision rate

the coulomb logarithm

$$\nu_{e} = \frac{4\sqrt{2\pi}}{3} \frac{n e^4}{m_e^{1/2} T^{3/2}} \Lambda$$

$\Lambda \sim 20$

$$\cancel{\nu_{ei} = \frac{4\pi n e^4}{m_e}}$$

USE

$$T = \lambda_D^2 \, 4\pi n e^2$$

$$\nu_{e} = \frac{n e^4}{m_e^{1/2} \lambda_D^3 (4\pi n e^2)^{3/2}}$$

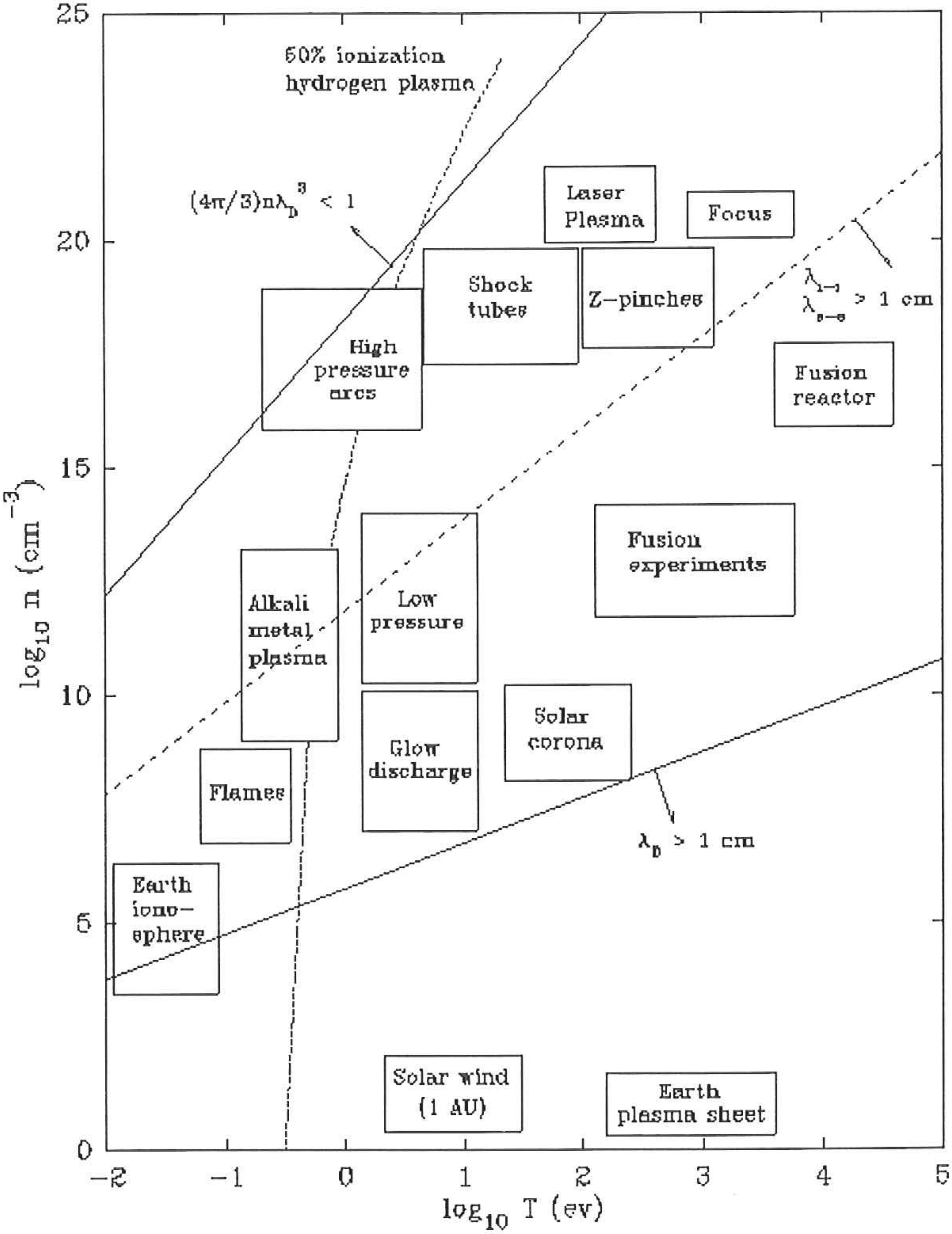
~~ν_{ei}~~

$$\nu_{ei} = \frac{4\sqrt{2\pi}}{(4\pi)^2 3} \; \frac{(4\pi)^2 n^2 e^4}{m_e^{1/2} n \lambda_D^3 (4\pi n e^2)^{3/2}} \lambda$$

$$\nu_{ei} = \frac{4\sqrt{2\pi}}{(4\pi)^2 3} \; \frac{\omega_{pe}}{(n \lambda_D^3)} \lambda$$

if $n\lambda_D^3 \gg 1$ $\quad \nu_{ei} \ll \omega_{pe}$

Collisions are ~~small~~ infrequent in a weakly coupled plasma



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λ_{i-i}
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arcs
Fusion
reactor
Fusion
experiments
Alkali
metal
plasma
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Solar
corona
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discharge
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Earth
iono-
sphere
Solar wind
(1 AU)
Earth
plasma sheet
$\log_{10} n\ (cm^{-3})$
$\log_{10} T$ (ev)
0
5
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1
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3
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5

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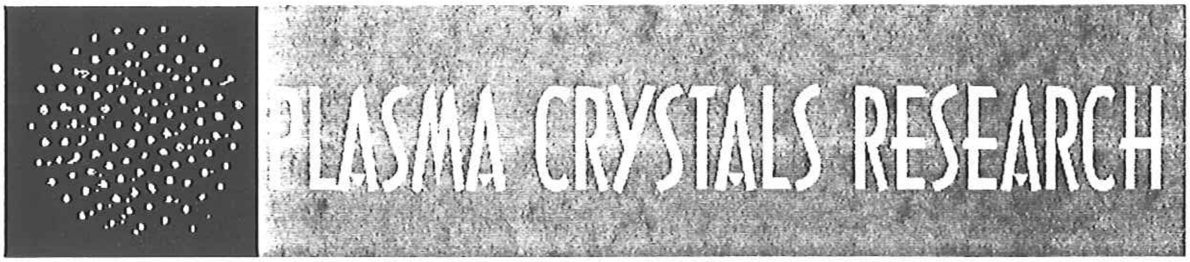

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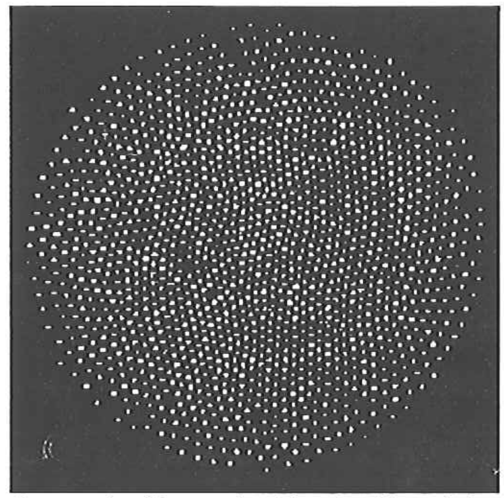

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