Summarize F-P results

Fokker - Planck

$$\frac{\partial f_n}{\partial t} = - \sum_\sigma \frac{\partial}{\partial \underset{\sim}{v}} \cdot \underset{\sim}{J}^{n/\sigma}$$

$\underset{\sim}{J}^{n/\sigma}$ = flux (in velocity space) of particles of type n due to collisions with type σ

$$\underset{\sim}{J}^{n/\sigma} = \cancel{\frac{4\pi\lambda q_n^2 q_\sigma^2}{m}} = f_n(\underset{\sim}{v}) \underset{\sim}{F}^{n/\sigma} - \frac{\partial}{\partial \underset{\sim}{v}} \cdot \left(\underset{\approx}{D}^{n/\sigma} f_n \right)$$

$$\underset{\sim}{F}^{n/\sigma} = - \frac{4\pi q_n^2 q_\sigma^2 \lambda^{n/\sigma}}{m_n^2} \left(\frac{m_n + m_\sigma}{m_\sigma} \right) \int d^3 v' \, \frac{\underset{\sim}{u}}{u^3} f_\sigma(\underset{\sim}{v}')$$

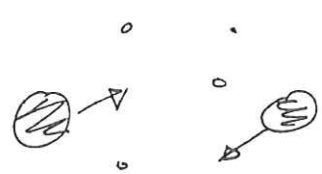
$\lambda = \ln\left(\frac{K_{max}}{K_{min}}\right)$

$$\underset{\approx}{D}^{n/\sigma} = \frac{2\pi q_n^2 q_\sigma^2 \lambda^{n/\sigma}}{m_n^2} \int d^3 v' f_\sigma(\underset{\sim}{v}') \frac{\underset{\approx}{I} u^2 - \underset{\sim}{u}\underset{\sim}{u}}{u^3}$$

$$\underset{\sim}{u} = \underset{\sim}{v} - \underset{\sim}{v}'$$

Process	Rate
electron pitch angle scattering on ions & electrons electron self equilibration	$\nu_{ee} = \dfrac{4\pi n_e e^4 \lambda}{m_e^2 \left(\frac{2T_e}{m_e}\right)^{3/2}}$ $\nu_{ei} = \dfrac{4\pi n_i e^2 Z^2 e^2 \lambda}{m_e^2 (2T_e/m_e)^{3/2}}$ $Z_{eff} = \dfrac{\sum_i Z_i^2 n_i}{n_e}$
ion pitch angle scattering on other ions ion-ion equilibration	$\nu_{ii} \sim \left(\frac{m_e}{m_i}\right)^{1/2} \nu_{ee}$
electron-ion equilibration ion pitch angle scattering on electrons	$\nu_{eq} \sim \dfrac{m_e}{m_i} \nu_{ee} \sim \left(\frac{m_e}{m_i}\right)^{1/2} \nu_{ii}$

think of plasma as a gas consisting of bowling balls (ions) and ping pong balls (electrons)



Fokker Planck Eqn

Let's describe collisions as a random process.

Due to collisions particles will change their velocity

$$t \qquad t+\Delta t$$

$$\underset{\sim}{V} \qquad \underset{\sim}{V}+\Delta\underset{\sim}{V}$$

$\Delta\underset{\sim}{V}$ = RANDOM VARIABLE

Define the probability distribution function

$$\cancel{P(\underset{\sim}{V}, \Delta\underset{\sim}{V})} \qquad P(\Delta\underset{\sim}{V}\,; \underset{\sim}{V}, \Delta t)$$

$P d^3\Delta\underset{\sim}{V}$ = probability that a particle with velocity $\underset{\sim}{V}$ at time t will have a velocity

$\underset{\sim}{V} + \Delta\underset{\sim}{V}$ at time $t+\Delta t$

$$\int P(\underset{\sim}{V}, \Delta\underset{\sim}{V})\, d^3\Delta V = 1$$

How does this effect the evolution of

$f(\underset{\sim}{V}, t)$?

$$\underbrace{f(\underset{\sim}{V},t)}_{\text{probability particle is at } \underset{\sim}{V}} = \int d^3\Delta V\, \overbrace{f(\underset{\sim}{V}-\Delta\underset{\sim}{V}, t-\Delta t)}^{\text{Prob. particle is at } f(\underset{\sim}{V}-\Delta\underset{\sim}{V}, t-\Delta t)}\, \overbrace{P(\underset{\sim}{V}-\Delta\underset{\sim}{V}, \Delta\underset{\sim}{V})}^{\text{Prob particle scatters by } \Delta\underset{\sim}{V} \text{ given its at } \underset{\sim}{V}-\Delta\underset{\sim}{V}}$$

NO ASSUMPTIONS MADE AT THIS POINT ABOUT SIZE OF $\Delta\underset{\sim}{V}$. OR Δt

This ~~equation~~ process should not create or destroy destroy particles.

i,e $\quad \int d^3v\, f(\mathbf{v},t) = \int d^3v\, f(\mathbf{v}, t-\Delta t)$

$$\int d^3v\, f(\mathbf{v},t) = \int d^3v \int d^3\Delta v\, f(\mathbf{v}-\Delta\mathbf{v}, t-\Delta t)\, P(\mathbf{v}-\Delta\mathbf{v}, \Delta\mathbf{v})$$

let $\quad \mathbf{v}-\Delta\mathbf{v} = \mathbf{v}'$

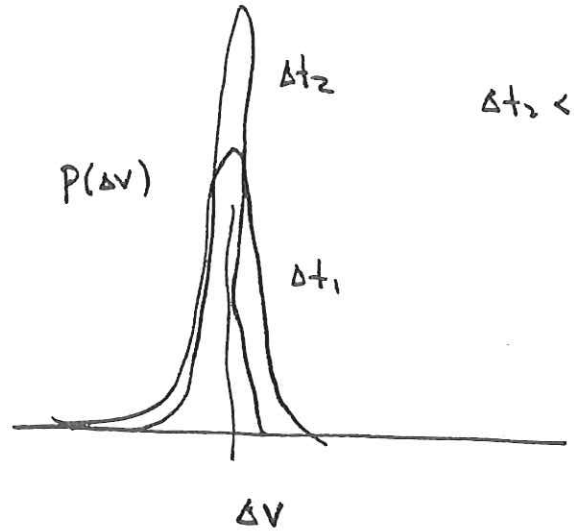
$$d^3v\, d^3\Delta v = d^3v'\, d^3\Delta v$$

$$\int d^3v\, f(\mathbf{v},t) = \int d^3v'\, d^3\Delta v\, f(\mathbf{v}', t-\Delta t)\, P(\mathbf{v}', \Delta\mathbf{v})$$

$$= \int d^3v'\, f(\mathbf{v}', t-\Delta t) \underbrace{\int d^3\Delta v\, P(\mathbf{v}', \Delta\mathbf{v})}$$

$$= \int d^3v\, f(\mathbf{v}', t-\Delta t)$$

as $\Delta t \to 0$



$\Delta t_2 < \Delta t_1$

NOW ASSUME THAT ~~ΔV & Δt~~

~~are small.~~ as $\Delta t \to 0$ $\Delta V \to 0$

(small angle collisions). See picture.

Taylor Expand

$$f(\underset{\sim}{V},t) = \int d^3\Delta V \Big\{ f(\underset{\sim}{V},t) P(\underset{\sim}{V},\Delta \underset{\sim}{V}) - \Delta t \frac{\partial}{\partial t} f(\underset{\sim}{V},t) P(\underset{\sim}{V},\Delta \underset{\sim}{V})$$

$$- \Delta \underset{\sim}{V} \cdot \frac{\partial}{\partial \underset{\sim}{V}} f(\underset{\sim}{V},t) P(\underset{\sim}{V},\Delta \underset{\sim}{V}) + \frac{1}{2} \Delta \underset{\sim}{V} \Delta \underset{\sim}{V} : \frac{\partial^2}{\partial \underset{\sim}{V} \partial \underset{\sim}{V}} f(\underset{\sim}{V},t) P(\underset{\sim}{V},\Delta \underset{\sim}{V})$$

$$+ \Delta t \Delta \underset{\sim}{V} \cdot \frac{\partial}{\partial \underset{\sim}{V}} \frac{\partial}{\partial t} f(\underset{\sim}{V},t) P(\underset{\sim}{V},\Delta \underset{\sim}{V}) \cdots \Big\}$$

written out to second order in small quantities.

take limit as $\Delta t \to 0$

$$\int d^3\Delta V \; P(\underset{\sim}{V},\Delta \underset{\sim}{V}) = 1$$

WE CAN DO THE INTEGRALS

$$\int d^3\Delta v \; P(\underset{\sim}{v}, \underset{\sim}{\Delta v}) = 1$$

divide by Δt

$$0 = - \frac{\partial f(\underset{\sim}{v}, t)}{\partial t} - \frac{\partial}{\partial \underset{\sim}{v}} \cdot \left(f(\underset{\sim}{v}, t) \int d^3\Delta v \; P(\underset{\sim}{v}, \underset{\sim}{\Delta v}) \frac{\underset{\sim}{\Delta v}}{\Delta t} \right)$$

$$+ \frac{\partial^2}{\partial \underset{\sim}{v} \partial \underset{\sim}{v}} : \left(f(\underset{\sim}{v}, t) \int d^3\Delta v \; P(\underset{\sim}{v}, \underset{\sim}{\Delta v}) \frac{\underset{\sim}{\Delta v}\underset{\sim}{\Delta v}}{2\Delta t} \right)$$

$$+ \frac{\partial}{\partial \underset{\sim}{v}} \cdot \left(\underbrace{\frac{\partial f}{\partial t} \int d^3\Delta v \; P(\underset{\sim}{v}, \underset{\sim}{\Delta v}) \underset{\sim}{\Delta v}}_{\Delta t F \to 0 \text{ as } \Delta t \to 0} + \cdots \right)$$

For the type of process we are considering

"DIFFUSION"

$$\lim_{\Delta t \to 0} \int d^3\Delta v \; P(\underset{\sim}{v}, \underset{\sim}{\Delta v}) \frac{\underset{\sim}{\Delta v}\underset{\sim}{\Delta v}}{2\Delta t} \equiv \underset{\approx}{D}(\underset{\sim}{v})$$

independent of Δt

"FRICTION"

$$\lim_{\Delta t \to 0} \int d^3\Delta v \; P(\underset{\sim}{v}, \underset{\sim}{\Delta v}) \frac{\underset{\sim}{\Delta v}}{\Delta t} = \underset{\sim}{F}(\underset{\sim}{v})$$

why isn't $\underbrace{\int d^3\Delta v\, P(\Delta \underset{\sim}{v}) \frac{\Delta \underset{\approx}{v}}{\Delta t}}_{\underset{\sim}{F}} >> \underbrace{\int d^3\Delta v\, P(\Delta \underset{\sim}{v}) \frac{\Delta \underset{\sim}{v} \Delta \underset{\sim}{v}}{\Delta t}}_{\underset{\approx}{D}}$

both finite as $\Delta t \to 0$

$\Delta \underset{\sim}{v}$ due to ~~small~~ microscopic fields

discrete particles $\tilde{E}$

$\int d^3\Delta v\, P \Delta \underset{\sim}{v} \Delta \underset{\sim}{v} \sim \tilde{E}^2$

$\Delta \underset{\sim}{v} \sim \tilde{E}$

$\int d^3\Delta v\, P(\Delta \underset{\sim}{v})$, $\tilde{E}$ average is zero thus as $\Delta t \to 0$

$$\frac{\partial f}{\partial t} = -\frac{\partial}{\partial \underset{\sim}{v}} \cdot f \underset{\sim}{F}(\underset{\sim}{v}) + \frac{\partial}{\partial \underset{\sim}{v}} \frac{\partial}{\partial \underset{\sim}{v}} \underset{\approx}{D} f$$

Fokker - Planck Eqn.

why do we call $\underset{\sim}{F}$ friction and $\underset{\approx}{D}$ diffusion.

$$\frac{\partial}{\partial t} \overset{\text{expected value}}{E}(\underset{\sim}{v}) = \int d^3v\, \underset{\sim}{v} \frac{\partial f}{\partial t} = \int d^3v\, \underset{\sim}{v} \left\{ -\frac{\partial}{\partial \underset{\sim}{v}} \cdot f\underset{\sim}{F} + \frac{\partial^2}{\partial \underset{\sim}{v} \partial \underset{\sim}{v}} \underset{\approx}{D} f \right\}$$

do by parts $$= \int d^3v\, f\left(\underset{\sim}{F} \cdot \frac{\partial}{\partial \underset{\sim}{v}} \underset{\sim}{v} + \underset{\approx}{D} \cdot \cancelto{0}{\frac{\partial^2}{\partial \underset{\sim}{v} \partial \underset{\sim}{v}} \underset{\sim}{v}}\right)$$

$\frac{\partial}{\partial \underset{\sim}{v}} \underset{\sim}{v} = \underset{\approx}{I}$

$$= \int d^3v\, f \underset{\sim}{F}(\underset{\sim}{v})$$

$\frac{\partial}{\partial \underset{\sim}{v}} \underset{\approx}{I} = 0$

$$\cancel{\frac{\partial}{\partial t} E(\underset{\sim}{v}\underset{\sim}{v}) =}$$

$$\frac{\partial}{\partial t} E(v^2) = \int d^3v \, f \left[\underset{\sim}{F} \cdot \frac{\partial}{\partial \underset{\sim}{v}} v^2 + \underset{\approx}{D} \cdot \frac{\partial^2}{\partial \underset{\sim}{v} \partial \underset{\sim}{v}} v^2 \right]$$

$$= \int d^3v \, f \left[2\underset{\sim}{v} \cdot \underset{\sim}{F} + 2(\underset{\approx}{D} \cdot \underset{\approx}{I}) \right]$$

suppose $\underset{\sim}{F}$ & $\underset{\sim}{D}$ independent of $\underset{\sim}{v}$

f normalized to unity

$$\frac{\partial}{\partial t} E(v^2) = 2 \underset{\sim}{F} \cdot E(\underset{\sim}{v}) + \cancel{2D_{tr}} \int d^3v f$$

$$2(\underset{\approx}{D} \cdot \underset{\approx}{I})$$

$$E(v^2) = E\left(\left(\underset{\sim}{v} - \bar{\underset{\sim}{v}} + \bar{v}\right)^2\right) = E\left(\left(\underset{\sim}{v} - E(\underset{\sim}{v})\right)^2\right)$$

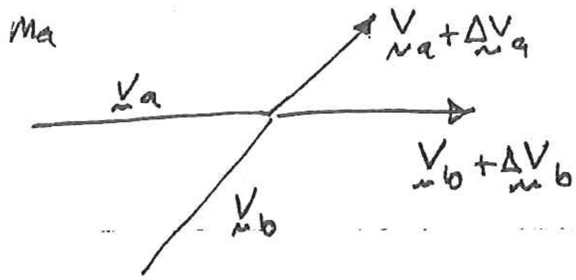
$$+ \left(E(\underset{\sim}{v})\right)^2$$

$$\frac{\partial}{\partial t} E\left(\left(\underset{\sim}{v} - E(\underset{\sim}{v})\right)^2\right) = 2 \underset{\approx}{D} \cdot \underset{\approx}{I}$$

work out

Properties OF Friction Force and DIFFUSION

collision between particle a and particle b



conservation OF momentum

$$m_a(\underset{\sim}{V}_a + \Delta\underset{\sim}{V}_a) + m_b(\underset{\sim}{V}_b + \Delta\underset{\sim}{V}_b) = m_a\underset{\sim}{V}_a + m_b\underset{\sim}{V}_b$$

$$m_a \Delta\underset{\sim}{V}_a + m_b \Delta\underset{\sim}{V}_b = 0$$

$$\boxed{\Delta\underset{\sim}{V}_b = -\frac{m_a}{m_b}\Delta\underset{\sim}{V}_a}$$

conservation of energy

$$m_a(\underset{\sim}{V}_a + \Delta\underset{\sim}{V}_a)^2 + m_b(\underset{\sim}{V}_b + \Delta\underset{\sim}{V}_b)^2 = m_a\underset{\sim}{V}_a^2 + m_b\underset{\sim}{V}_b^2$$

$$\cancel{\sum_i \frac{\partial}{\partial x_i} M_{ij} x_j}$$

$$\cancel{\nabla\cdot\underset{\sim}{M}\cdot\underset{\sim}{A} = \sum_{ij} M_{ij}\delta_{ij}}$$

$$m_a \Delta \underset{\sim}{V}_a^2 + 2m_a \underset{\sim}{V}_a \cdot \Delta \underset{\sim}{V}_a + m_b \Delta \underset{\sim}{V}_b^2 + 2m_b \underset{\sim}{V}_b \cdot \Delta \underset{\sim}{V}_b = 0$$

substituting $\Delta \underset{\sim}{V}_b = -\frac{m_a}{m_b} \Delta \underset{\sim}{V}_a$

$$\left(m_a + \frac{m_a^2}{m_b}\right)(\Delta \underset{\sim}{V}_a)^2 + 2m_a(\underset{\sim}{V}_a - \underset{\sim}{V}_b) \cdot \Delta \underset{\sim}{V}_a = 0$$

so $$\Delta \underset{\sim}{V}_a^2 = -\frac{2m_b}{(m_a+m_b)}(\underset{\sim}{V}_a - \underset{\sim}{V}_b) \cdot \Delta \underset{\sim}{V}_a$$

$$\left\langle \frac{\Delta V_a^2}{\Delta t} \right\rangle = -\frac{2m_b}{(m_a+m_b)}(\underset{\sim}{V}_a - \underset{\sim}{V}_b) \cdot \left\langle \frac{\Delta \underset{\sim}{V}_a}{\Delta t} \right\rangle$$

Relation between diffusion due to scattering of a by b and friction

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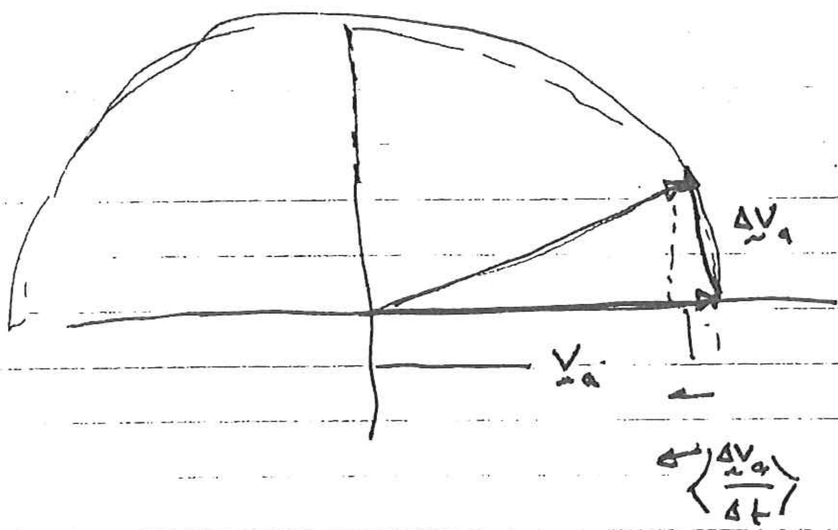
FROM symmetry we can assume $\left\langle \frac{\Delta \underset{\sim}{V}_a}{\Delta t} \right\rangle$ is in the $(\underset{\sim}{V}_a - \underset{\sim}{V}_b)$ direction.

in center of mass frame $\underset{\sim}{V}_a - \underset{\sim}{V}_b$ gives the perfered direction

IF $m_b \to \infty$ $\quad$ $\underset{\sim}{v}_b \to 0$

$$\left\langle \frac{\Delta v_a^2}{\Delta t} \right\rangle = -2\, \underset{\sim}{v}_a \cdot \left\langle \frac{\Delta \underset{\sim}{v}_a}{\Delta t} \right\rangle$$



scattering ~~causes~~ does not change the magnitude of velocity

start here

$$Tr\{\underset{\approx}{D}\} = -\frac{m_b}{m_a+m_b}\left(\underset{\sim}{v}_a - \underset{\sim}{v}_b\right) \cdot \underset{\sim}{F}$$

$$\underline{\underline{D}} = \frac{1}{2}\left\langle \frac{\Delta \underset{\sim}{v}_a \Delta \underset{\sim}{v}_a}{\Delta T} \right\rangle$$

$$Tr\{\underline{\underline{D}}\} = \frac{1}{2}\frac{|\Delta \underset{\sim}{v}_a|^2}{\Delta T} = -$$

$$\Delta \underset{\sim}{V}(\Delta t) = \int_0^{\Delta t} dt' \frac{q}{m} \underset{\sim}{E}(\underset{\sim}{X}_0+\underset{\sim}{V}_0 t', t')$$

$$\Delta \underset{\sim}{V} \Delta \underset{\sim}{V} = \int_0^{\Delta t} dt' \frac{q}{m} \underset{\sim}{E}(\underset{\sim}{X}_0+\underset{\sim}{V}_0 t', t') \int_0^{\Delta t} dt'' \frac{q}{m} \underset{\sim}{E}(\underset{\sim}{X}_0+\underset{\sim}{V}_0 t'', t'')$$

Now ~~⇔~~ imagine averaging this equation over some ensemble of fields (positions and velocities of scattering particles

$$\langle \Delta \underset{\sim}{V} \Delta \underset{\sim}{V} \rangle = \frac{q^2}{m^2} \int_0^{\Delta t}\int_0^{\Delta t} \langle \underset{\sim}{E}(\underset{\sim}{X}_0+\underset{\sim}{V}_0 t', t') \underset{\sim}{E}(\underset{\sim}{X}_0+\underset{\sim}{V}_0 t'', t'') \rangle dt' dt''$$

call this the correlation function

$$\underset{\approx}{C} = \langle \underset{\sim}{E}(\underbrace{\underset{\sim}{X}_0+\underset{\sim}{V}_0 t'}_{X'}, t') \underset{\sim}{E}(\underbrace{\underset{\sim}{X}_0+\underset{\sim}{V}_0 t''}_{\underset{\sim}{X}''}, t'') \rangle$$

for time stationary, homogeneous, isotropic fluctuations

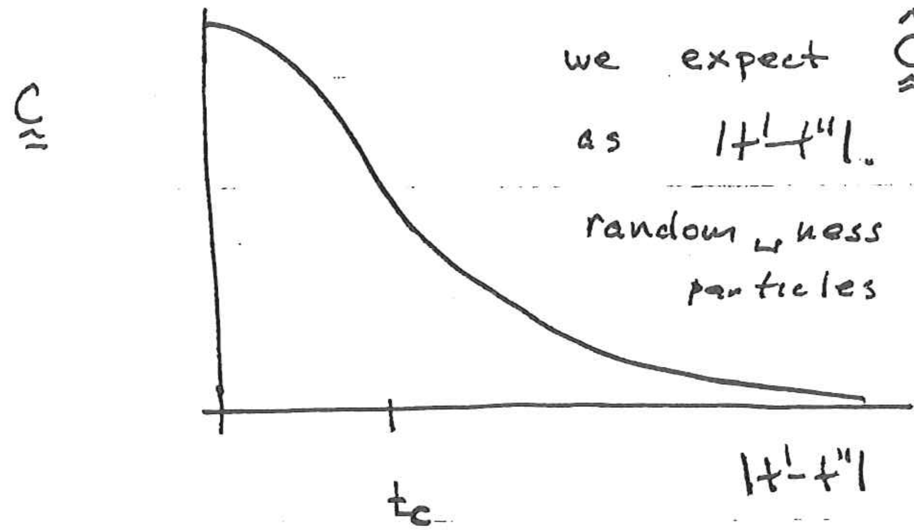
$$\underset{\approx}{C} = \underset{\approx}{C}(|\underset{\sim}{x}' - \underset{\sim}{x}''|, |t'-t''|)$$

statistical properties independent of [space, time, direction < assumes no magnetic field

$$\langle \underset{\sim}{E}(\underset{\sim}{x}_0+\underset{\sim}{v}_0 t', t')\, \underset{\sim}{E}(\underset{\sim}{x}_0+\underset{\sim}{v}_0 t', t'')\rangle$$

$$\doteq \underset{\approx}{C}\left(|\underset{\sim}{v}_0|\,|t'-t''|,\ |t'-t''|\right) \equiv \hat{\underset{\approx}{C}}\left(\underset{\sim}{v}_0,\ |t'-t''|\right)$$

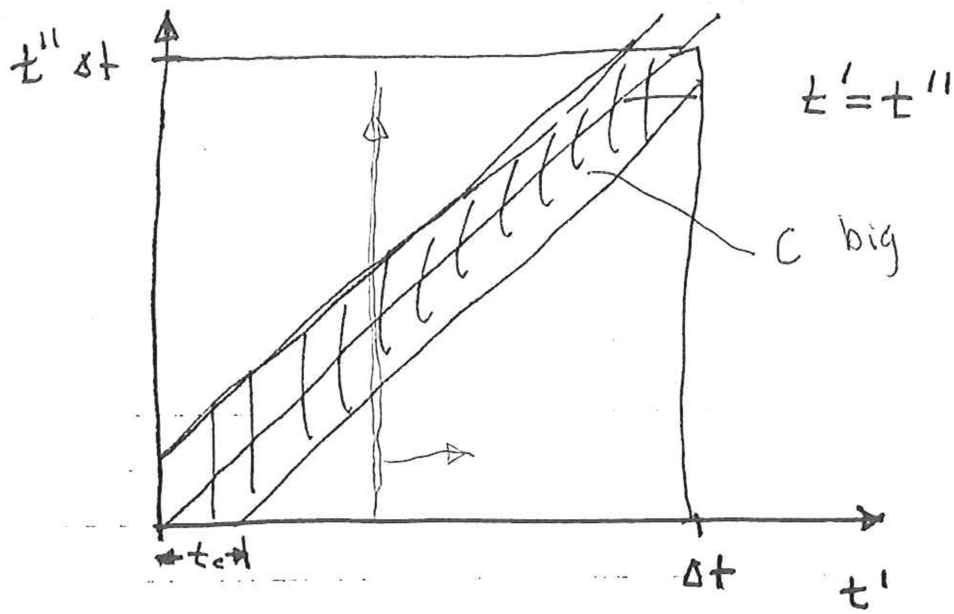
~~because~~



we expect $\hat{\underset{\approx}{C}}$ to decrease as $|t'-t''|$. Because of randomness of scattering particles values of E far away or much ~~later~~ different ~~in~~ times are indepent.

$$\langle \underset{\sim}{E}(\underset{\sim}{x}', t')\, \underset{\sim}{E}(\underset{\sim}{x}, t)\rangle = 0$$

if values of $\underset{\sim}{x}', t'$ / $\underset{\sim}{x}, t$ are well separated thus, in t'', t' plane



the integrand is peaked about $t' = t''$

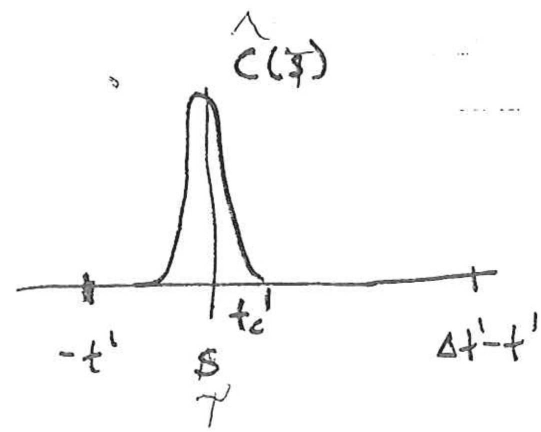
so for $\Delta t > t_c$

$$\int_0^{\Delta t} dt' \int_0^{\Delta t} dt'' \; \underset{\approx}{\hat{C}}\left(|t' - t''| \right)$$

call that s

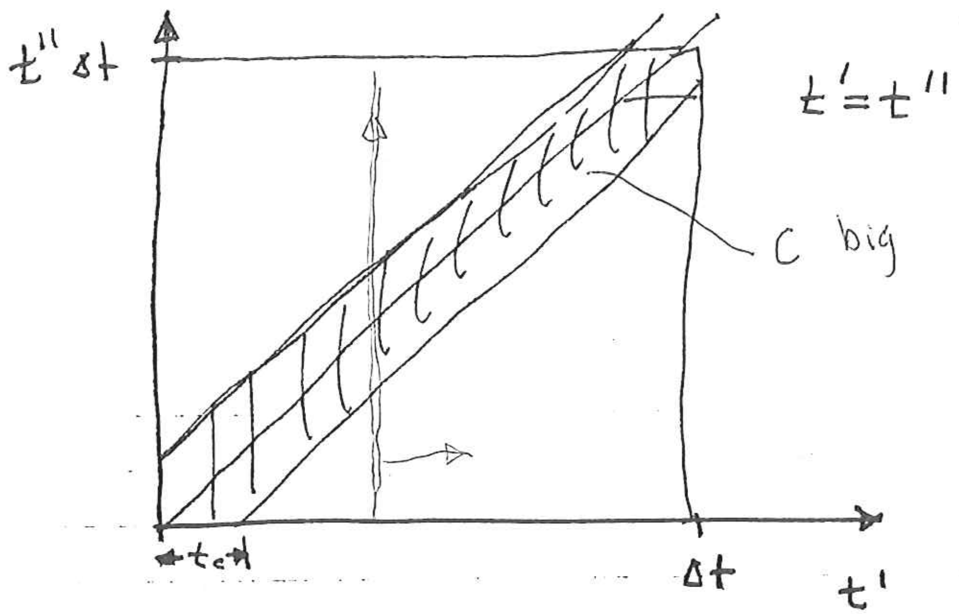
$\tau \; \cancel{s} = t'' - t'$

$$\int_0^{\Delta t} dt' \int_{-t'}^{\Delta t - t'} d\tau \; \underset{\approx}{\hat{C}}(\tau)$$



replace limits by $\pm\infty$

thus, in t'', t' plane



the integrand is peaked about $t' = t''$

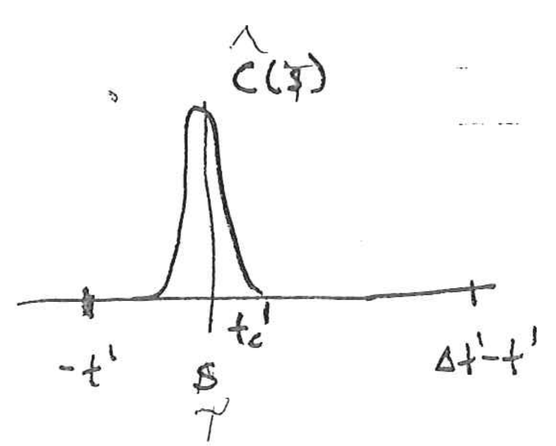
so for $\Delta t > t_c$

$$\int_0^{\cancel{\Delta t}} dt' \int_0^{\Delta t} dt'' \; \underset{\approx}{\hat{C}}\left(\underbrace{|t' - t''|}_{s} \right)$$

call that s

$\tau \; \cancel{s} = t'' - t'$

$$\int_0^{\Delta t} dt' \int_{-t'}^{\Delta t - t'} d\tau \; \underset{\approx}{\hat{C}}(\tau)$$



replace limits by $\pm\infty$

$$\langle \Delta V \Delta V \rangle = \frac{q^2}{m^2} \Delta t \int_{-\infty}^{\infty} d\xi \, \hat{C}(|\xi|)$$

$$\cancel{= \frac{2q^2}{m^2} \Delta t \int_{0}^{\infty} d\xi \, \hat{C}(\xi)}$$

$$D_n = \frac{\langle \Delta V \Delta V \rangle}{2\Delta t} = \frac{1}{2} \frac{q_n^2}{m_n^2} \int_{-\infty}^{\infty} d\xi \, \hat{C}(\xi, V_n)$$

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where $\hat{C}(\tau, V_0)$

$$= \langle E(x_0, 0) \, E(x_0 + V_R s, s) \rangle$$

①

$$\hat{\underline{\underline{C}}}(\tau) = \langle \underline{E}(0,0)\,\underline{E}(\underline{V}_n \tau, \tau) \rangle$$

$$\underline{E} = -\nabla\phi \qquad \nabla^2\phi = -4\pi q_0 \sum_i \delta(\underline{x} - \underline{x}_i)$$

N particles in box L

Solve by Fourier Series in a periodic box of side L, then let $L \to \infty$

$$\phi = \sum_k \bar{\phi}(\underline{k}, t)\exp(i\underline{k}\cdot\underline{x}) \qquad \bar{\phi} = \int \frac{d^3x}{L^3} e^{-i\underline{k}\cdot\underline{x}}\phi(\underline{x})$$

$$\underline{E} = -i\sum_k \underline{k}\bar{\phi}\exp(i\underline{k}\cdot\underline{x})$$

$$-k^2\bar{\phi} = -\frac{4\pi q_0}{L^3}\sum_i \exp[-i\underline{k}\cdot\underline{x}_i(t)]$$

$$\hat{\underline{\underline{C}}}(\tau) = \sum_{\underline{k}\underline{k}'}\sum_{ii'}\left(\frac{4\pi q_0}{L^3}\right)\left(\frac{-\underline{k}\underline{k}'}{k^2k'^2}\right)$$

$x_{i'}(\tau) = x_{i'}(0) + V_{ni'}\tau$

$$\langle \exp[-i\underline{k}\cdot\underline{x}_i(0)]\exp[i\underline{k}'\cdot\underline{V}_n\tau - i\underline{k}'\cdot\underline{x}_{i'}(\tau)] \rangle$$

there are $N(N-1)$ terms with $i \neq i'$

there are N terms with $i = i'$

We will say that particles are distributed ~~unif~~ randomly with a uniform pdf in Box $-L^3$

$\therefore$ Terms with $i \neq i'$ average to zero

There are N terms with $i = i'$

$$X_i(\tau) = \underline{x}_i(0) + \underline{v}_{i\sigma}\tau$$

$$\left\langle \exp\left[-i\underline{k}\cdot\underline{x}_i(0) + i\underline{k}'\cdot\underline{v}_\sigma\tau - i\underline{k}'\cdot\underline{x}_i(\tau)\right]\right\rangle = \int \frac{d^3x_i(0)}{L^3}\int d^3v_{\sigma i}\, f(\underline{v}_{\sigma i})$$

note: on averaging over $\underline{x}_i(0)$ $\langle\ldots\rangle$

only when $\underline{k} = -\underline{k}'$ term survives

$$\underline{\underline{\hat{C}}}(\tau) = N\sum_{k}\int d^3v_{\sigma}\, f_\sigma(\underline{v}_{\sigma})\left(\frac{4\pi q_\sigma}{L^3}\right)^2 \frac{\underline{k}\underline{k}}{k^4}\exp[-i\underline{k}\cdot\underline{u}\tau]$$

$$\underline{u} = \underline{v}_n - \underline{v}_{\sigma}$$

Replace sum by integral $L \to \infty$ $\quad \dfrac{N}{L^3} \to n_\sigma$

$$\sum_k \to \int \frac{d^3k}{(\delta k)^3} \qquad \delta k = \frac{2\pi}{L}$$

$$\underline{\underline{\hat{C}}} = (4\pi q_\sigma)^2 n_\sigma \int \frac{d^3k}{(2\pi)^3}\int d^3v_\sigma\, f_\sigma(\underline{v}_\sigma)\frac{\underline{k}\underline{k}}{k^4}\exp[-i\underline{k}\cdot\underline{u}\tau]$$

$$\int_{-\infty}^{\infty} d\tau\, \underline{\underline{\hat{C}}} = (4\pi q_\sigma)^2 n_\sigma \int \frac{d^2k_\perp dk_\parallel}{(2\pi)^3}\int d^3v_\sigma\, f_\sigma \frac{\underline{k}\underline{k}}{k^4}\, 2\pi\,\delta(k_\parallel u)$$

$$\int_{-\infty}^{\infty} d\tau\, \hat{C} = (4\pi q_\sigma)^2 n_\sigma \int \frac{d^2 k_\perp}{(2\pi)^2} \int d^3 v_\sigma f_\sigma \frac{\underline{k}_\perp \underline{k}_\perp}{k_\perp^4 u}$$

$$\underline{e}_1 \qquad \underline{e}_2$$

$$d^2 k_\perp = k_\perp dk_\perp d\theta_k \qquad \underline{k}_\perp = k_\perp \cos\theta_k,\ k_\perp \sin\theta_k$$

$$\int_{-\infty}^{\infty} d\tau\, \hat{C} = (4\pi q_\sigma)^2 n_\sigma \left(\frac{1}{2}\right) \int d^3 v_\sigma f_\sigma \int_0^\infty \frac{k_\perp dk_\perp}{(2\pi)} \frac{1}{k_\perp^2} \left(\frac{\underline{\underline{1}} u^2 - \underline{u}\underline{u}}{u^3}\right)$$

$\langle \cos^2\theta_k \rangle$

$$= \frac{16\pi^2 n_\sigma}{2 \cdot 2\pi} q_\sigma^2 \ln\left(\frac{k_{max}}{k_{min}}\right) \int d^3 v_\sigma f_\sigma \frac{\underline{\underline{1}} u^2 - \underline{u}\underline{u}}{u^3}$$

$$= 4\pi q_\sigma^2 \ln\left(\frac{k_{max}}{k_{min}}\right) \int d^3 v_\sigma f_\sigma^{n_\sigma} \frac{\underline{\underline{1}} u^2 - \underline{u}\underline{u}}{u^3}$$

$D = \frac{1}{2}$

$$\underline{\underline{D}} = \frac{1}{2} \frac{q_n^2}{m_n^2} \int_{-\infty}^{\infty} d\tau\, \underline{\underline{\hat{C}}}(\tau) = \frac{2\pi q_n^2 q_\sigma^2}{m_n^2} \ln\left(\frac{k_{max}}{k_{min}}\right) \int d^3 v_\sigma f_\sigma(v) \frac{\underline{\underline{1}} u^2 - \underline{u}\underline{u}}{u^3}$$

discuss shield

~~Define~~ Consider

$$F^{n/\sigma} = C^{n/\sigma}\left(\frac{m_n + m_\sigma}{m_\sigma}\right)\int d^3v'\, f_\sigma(\underline{v}')\, \frac{\underline{u}}{u^3}$$

$$\underline{u} = \underline{v} - \underline{v}' \qquad \frac{\underline{u}}{u^3} = -\frac{\partial}{\partial \underline{v}}\frac{1}{u}$$

$$F^{n/\sigma} = C^{n/\sigma}\frac{\partial}{\partial \underline{v}} H^{n/\sigma} \qquad \boxed{\nabla_v^2 H = -4\pi\, \frac{m_n + m_\sigma}{m_\sigma} f_\sigma}$$

$$\underline{\underline{D}} = \frac{\partial^2 G}{\partial \underline{v}\underline{v}} \qquad \boxed{\nabla_v^2 G = \frac{2}{1 + m_n/m_\sigma} H}$$