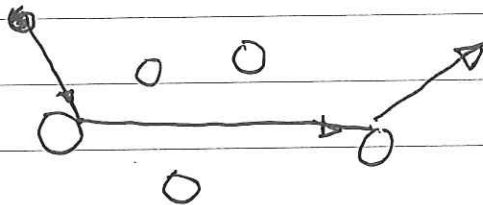


Collisions in Plasmas

General Comments

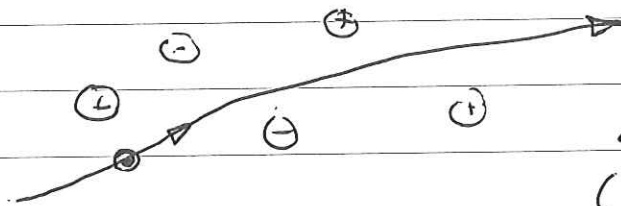
1 Collisions are not like colliding Billiard Balls

- Billiard Balls - collisions occur one ball at a time



- large angle deflection

- Plasma Particles - a given particle is colliding with all



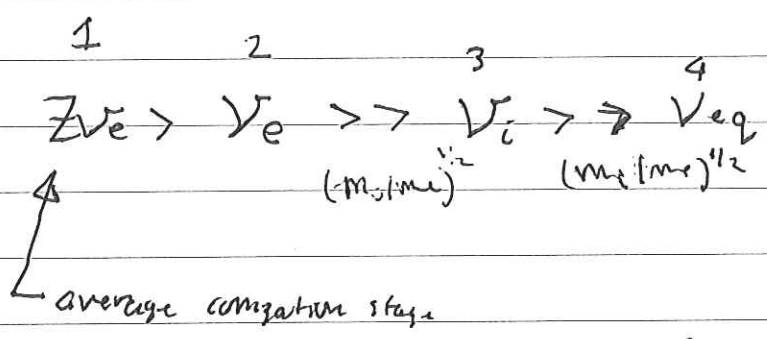
the other particles within a Debye sphere

(Coulomb Force is long range)

- many small angle deflection
- Diffusion and slowing down in velocity space

✍

2. Due to disparity in mass $M_i \gg m_e$
 collision occur at different rates
 for ions and electrons, different collisional
 processes occur at different rates



1. Electrons isotropize $f_e \rightarrow f_e(MI)$

2. Electrons thermalize with themselves $f_e(MI) \rightarrow \frac{n}{(2\pi m_e kT_e)^{3/2}} \exp(-\frac{1}{2} m_e v^2 / kT_e)$

3. Ions thermalize $f_i(V) \rightarrow \frac{n_i}{(2\pi m_i kT_i)^{3/2}} \exp(-\frac{1}{2} m_i v^2 / kT_i)$

4. electron and ion equilibrate $T_e \leftrightarrow T_i$

$v_e \sim \frac{D_w}{v^2}$ — diffusion coefficient

$\sim \frac{4\pi n e^2}{m^2 v^3} \frac{1}{v}$ — "Coulomb" logarithm

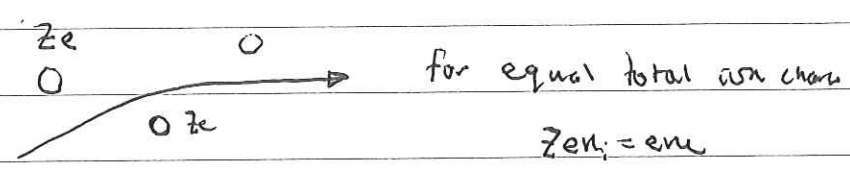
$\frac{1}{v^3}$

$v \sim \frac{\omega_p^4}{n v^3}$

$\lambda_D = v_{te} / \omega_{pe}$ Debye

$v \sim \left(\frac{v_{te}}{\lambda_D}\right) \left[\frac{a_{pp}}{\omega_{pe}}\right]$

Where does the Z come from?



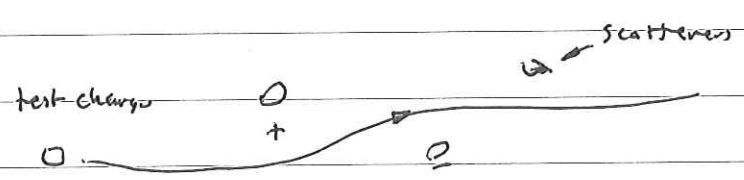
Putting more charges in each scatterer even though there are fewer scatterers results in higher diffusion. Think in terms of random walk in angle

step size $\Delta\theta \sim Z$

time between collision $\tau \sim Z$

$D \sim \frac{\Delta\theta^2}{\tau} \sim Z$ collisions are electrons

Where does the factor $(m_i/m_e)^{1/2}$ come from?



Suppose for the moment the scatterers are fixed.

$$\frac{d}{dt} \underline{v}_e = \frac{q}{m_e} \underline{B}(z(t))$$

$$\frac{dx}{dt} = v_e$$

let $\underline{u} = \lambda \underline{v}$ $\tau = t/\lambda$ scattering factor

$\frac{1}{m_i} = \frac{\lambda^2}{m_e}$

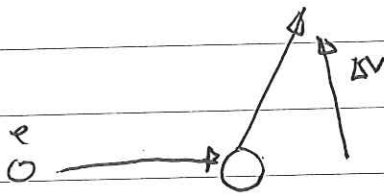
$$\frac{dx}{dt} = v_e \quad \frac{dx}{d\tau} = \lambda \frac{dx}{dt} = \lambda v_e = u$$

$$\frac{dx}{d\tau} = u \quad \lambda = \sqrt{\frac{m_e}{m_i}}$$

$$\underline{du} = \lambda^2 \frac{d\underline{v}}{d\tau} = \lambda^2 \underline{B}(z(t))$$

4) equilibration (OM) are bowling balls
 electron is ping-pong balls

to lowest order there is no energy exchange in
 a collision



FROM momentum conservation

$$m_e \Delta v_e = -m_i \Delta v_i$$

$$\text{thus } \frac{\langle \Delta v_i^2 \rangle}{\tau} = \frac{m_e^2 \langle \Delta v_e^2 \rangle}{m_i^2 \tau} \quad \frac{m_i \langle \Delta v_i^2 \rangle}{\tau} \sim \frac{m_e n_e \langle v_e^2 \rangle}{\tau}$$

rate of ion collision in energy $\sim \frac{m_e}{m_i}$ rate of electron collision

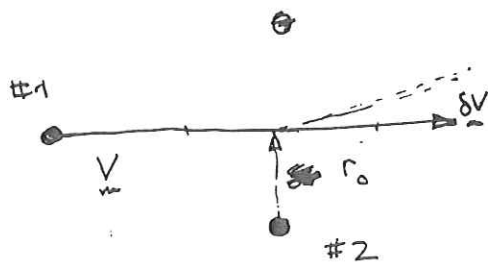
Collisions ~~Using~~ ~~Using Vlasov Egn~~ In deriving Liouville from Vlasov

$$F_N(x_1, v_1; x_2, v_2 \dots \dots, t) \cong F_1(x_1, v_1, t) F_1(x_2, v_2, t) \dots$$

Assumption of iid eliminates detailed effects of individual particles on each other. i.e. it eliminates collisions.

Collisions may still be important over long time scales.

To estimate effect of collisions consider effect of one stationary charge on another moving charge



#1 is deflected by #2

$$\delta v \approx \delta t \cdot \frac{E}{m} \cdot e$$

$\delta t =$ time during which field is felt

$$\delta t \sim \frac{r_0}{v} \quad E \sim \frac{e}{r_0^2}$$

$$\delta v = \frac{e^2}{m v r_0}$$

LETS CONSTRUCT A DIFFUSION COEFFICIENT IN VELOCITY SPACE USING RANDOM WALK ARGUMENTS

$\frac{\partial f}{\partial t} = \cancel{\frac{\partial f}{\partial x} v}$ $D \nabla_v^2 f$

D = velocity space diffusion coefficient

$D = \frac{(\delta v)^2}{\Delta T}$

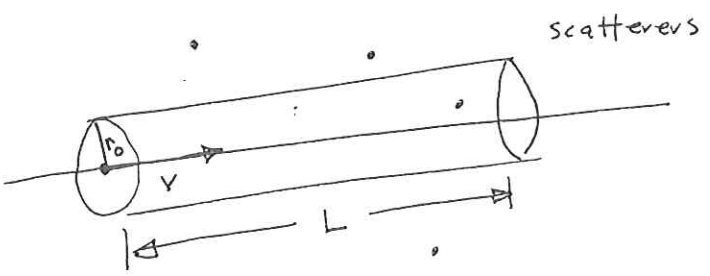
$\delta v =$ deflection in a collision

$\Delta T =$ time between collisions

$\delta v = \frac{e^2}{m v r_0}$

collision is when electron passes within r_0 of scatterer

How long between "collisions" depends on density



$L = v \Delta T$

we can expect another collision when volume = $\pi r_0^2 L = \frac{1}{n}$

$$L = 1 / \pi n r_0^2$$

$$\Delta T = 1 / \sqrt{v} \pi n r_0^2$$

$$\frac{(\delta V)^2}{\Delta T} = \frac{\pi e^4 n r_0^2 V}{m^2 V^2 r_0^2} = \frac{\pi e^4 n}{V m^2} = D$$

notice r_0 disappears

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial V} \cdot D \frac{\partial f}{\partial V}$$

"collision frequency"

$$(D/V^2) = \frac{\pi e^4 n}{m^2 V^3}$$

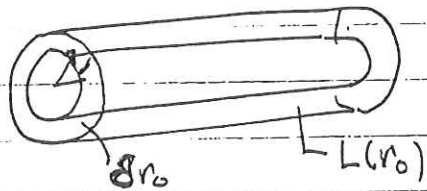
$$V \propto n \quad \cancel{V} \propto V^{-3}$$

LET TRY TO BE more careful

LETS SUM UP THE EFFECTS OF COLLISIONS OCCURRING WITH DIFFERENT r_0

$$D = \sum_{r_0} \frac{e^4}{m^2 v^2 r_0^2 \Delta T(r_0)} \quad D = \sum_{r_0} \frac{\delta v(r_0)^2}{\Delta T(r_0)}$$

$T(r_0)$ = Time between collisions with ~~the~~ a ~~particle~~ particular r_0



$$\pi r_0^2 \delta r_0 L = \frac{1}{N}$$

$$L = V \Delta T$$

$$\Delta T = \frac{1}{\pi r_0^2 \delta r_0 V N}$$

$$D = \sum_{r_0} \pi r_0^2 \delta r_0 \delta v(r_0)^2 V N$$

Volume = $\pi r_0^2 \delta r_0 L(r_0) = \frac{1}{N}$

$L(r_0) = \Delta T(r_0) V$

$$\delta v(r_0) = \frac{e^2}{m v r_0}$$

$$\frac{1}{\Delta T(r_0)} = \frac{V}{L(r_0)} = \pi r_0^2 \delta r_0 N V$$

$$D = \int \frac{e^4}{m^2 v^2 r_0^2} \cdot \pi r_0^2 \delta r_0 N V = \pi n e^4 \int_{r_{min}}^{r_{max}} \frac{dr_0}{r_0}$$

Integral does not converge

Estimate of D_{VV}

$$D = \sum_{\text{all } r_0} \frac{1}{\Delta T} (SV)^2$$

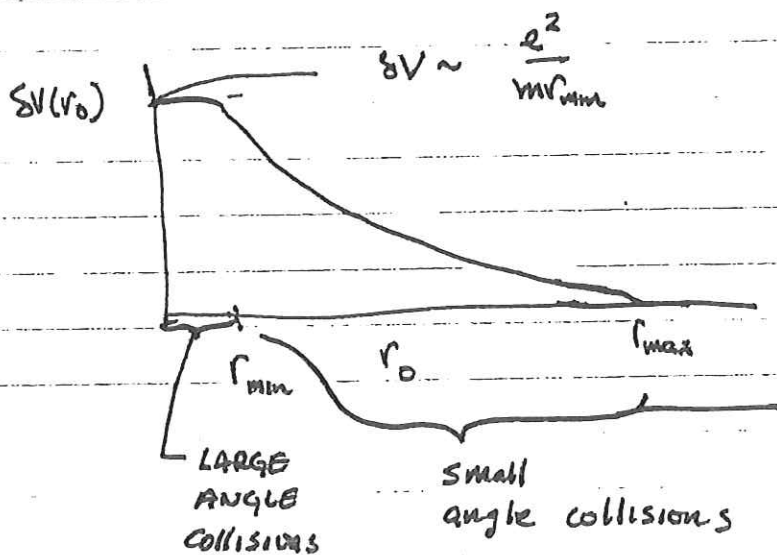
~~$$D_{VV} = \int_0^{\infty} \frac{dr_0}{r_0} \frac{\pi n e^4}{m^2 V}$$~~

$$\frac{1}{\Delta T} = \pi n d r_0 n V$$

~~$$D_{VV} = \int \pi n d r_0 n V (SV(r_0))^2$$~~

$$SV(r_0) = \frac{e^2}{mVr_0}$$

estimate



~~$$D_{\Delta T} = \int_0^{r_{\min}}$$~~

~~$$D_{\Delta T} = \int \pi r_{\min}^2 n V \frac{e^2}{m^2 V^2 r_{\min}^2} = \frac{n \pi e^2}{m^2 V} \frac{1}{\Delta T}$$~~

$$D_{SA} = \int_{r_{\min}}^{r_{\max}} \pi r_0 dr_0 n V \frac{e^2}{m^2 V^2 r_0^2} = \frac{\pi n e^2}{m^2 V} \ln\left(\frac{r_{\max}}{r_{\min}}\right)$$

EXTRA PHYSICS REQUIRED

$$D = \frac{\pi n e^4}{m^2 v} \ln \left| \frac{r_{0max}}{r_{0min}} \right|$$

r_{0min} is easy ~~suppose we were to~~

We assumed small deflections ~~Formula's~~ Formula's
For small deflections require $\delta V \ll V$

$$\delta V \approx \frac{e^2}{r_0 v m} \approx V$$

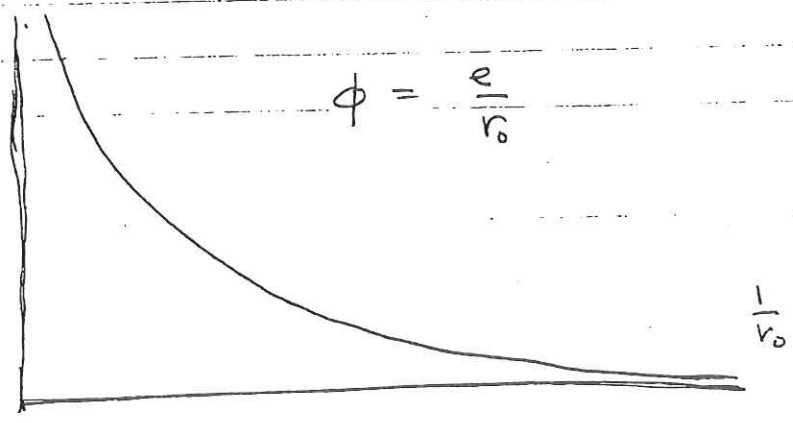
$$r_{0min} = \frac{m v^2}{e^2}$$

$$\frac{e^2}{r_{0min}} = m v^2$$

closest approach

r_{0max}

$\phi(r_0)$



Force gets weak very slowly as $r \rightarrow \infty$

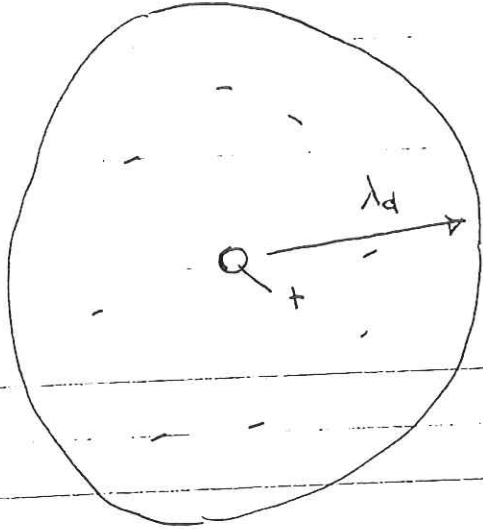
$$F \sim \frac{1}{r_0^2}$$

Other particles will shield

$$\lambda_d^2 = \frac{T}{4\pi n e^2}$$

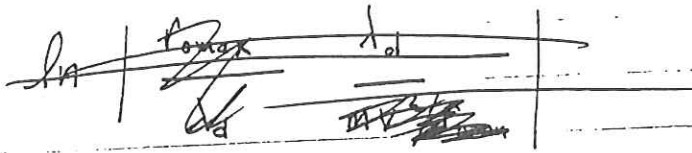
$r > \lambda_d$
~~the~~

Field goes to zero



$$r_{\text{max}} = \lambda_d$$

$$r_{\text{min}} = \frac{\frac{1}{2} e^2}{\frac{1}{2} m v^2}$$



~~$$\ln \left| \frac{r_{\text{max}}}{r_{\text{min}}} \right| = \frac{1}{2} \ln \left| \frac{\lambda_d^2}{\frac{1}{2} \frac{e^2}{m v^2}} \right| = \frac{1}{2} \ln \left| \frac{T}{4\pi n e^2} \frac{m^2 v^4}{e^4} \right|$$~~

$$\frac{r_{\text{min}}}{\lambda_d} = \frac{4\pi e^2 n}{T 4\pi n} = \frac{\frac{1}{2}}{\frac{4\pi n \lambda_d^3}{3}}$$

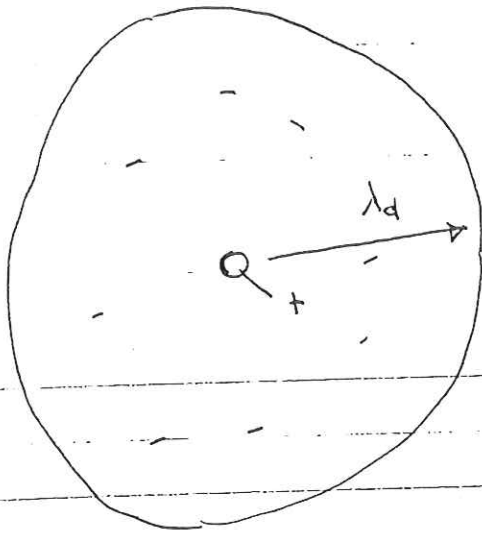
$$= \frac{1}{\# \text{ particles in debye sphere}}$$

other particles will shield

$$\lambda_d^2 = \frac{T}{4\pi n e^2}$$

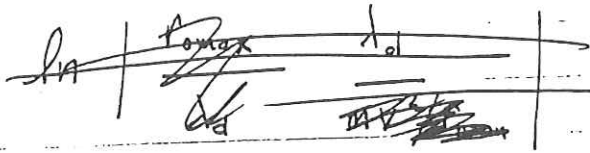
$r > \lambda_d$
~~Field~~

Field goes to zero



$$r_{\text{max}} = \lambda_d$$

$$r_{\text{min}} = \frac{\frac{1}{2} e^2}{\frac{1}{2} m v^2}$$



~~$$\ln \left| \frac{r_{\text{max}}}{r_{\text{min}}} \right| = \frac{1}{2} \ln \left| \frac{\lambda_d^2}{\frac{1}{2} r_{\text{min}}} \right| = \frac{1}{2} \ln \left| \frac{T}{4\pi n e^2} \frac{m^2 v^4}{e^4} \right|$$~~

$$\frac{r_{\text{min}}}{\lambda_d} = \frac{4\pi e^2 n}{T 4\pi n} = \frac{\frac{1}{2}}{\frac{4\pi n \lambda_d^3}{3}}$$

$$= \frac{1}{\# \text{ particles in debye sphere}}$$

SIZE OF

$$\frac{r_{\max}}{r_{\min}} = \frac{\lambda_d}{e^2/mv^2} \approx \frac{\lambda_d T}{e^2} \times \frac{4\pi n}{4\pi n}$$

↑
T

$$\approx 4\pi n \lambda_d^3$$

=, # of particles in
Debye sphere $\gg 1$

If # particles \gg is LARGE small angle collisions dominate.