DEPARTMENT OF
ELECTRICAL & COMPUTER ENGINEERING

ENEE681

Instructor: T. M. Antonsen Jr. antonsen@umd.edu

UNIVERSITY OF MARYLAND DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING
ENEE 681 Electromagnetic Theory II

INSTRUCTOR:	**T M. Antonsen Jr.**
	antonsen@umd.edu
	3339 A. V. Williams II
	405-1635
TA:	Zechuan Yin (Friday 11:00 – 12:00)
	(2302 ATL)
	zcyin@umd.edu
TIME:	**TuTh 9:30 AM – 10:45 PM**
LOCATION:	**AJC 2119**

OFFICE HOURS: **(TMA)** by appointment antonsen@umd.edu.

COURSE DESCRIPTION Continuation of ENEE 680. Theoretical analysis and engineering applications of Maxwell's equations. The homogeneous wave equation. Plane wave propagation. The interaction of plane waves and material media. Retarded potentials. The Hertz potential. Simple radiating systems. Relativisitic covariance of Maxwell's equations..

TEXT: Modern Electrodynamics by Andrew Zangwill, Cambridge University Press, ISBN 978-0-521-89697-9

Course Components

HOMEWORK: Assignments will be posted on ELMS. Assignments may involve computation.

GRADING: Your course grade will be computed on the basis of 600 points apportioned as follows:

EXAM1	**150**
EXAM2	**150**
Homework	**200**
	500

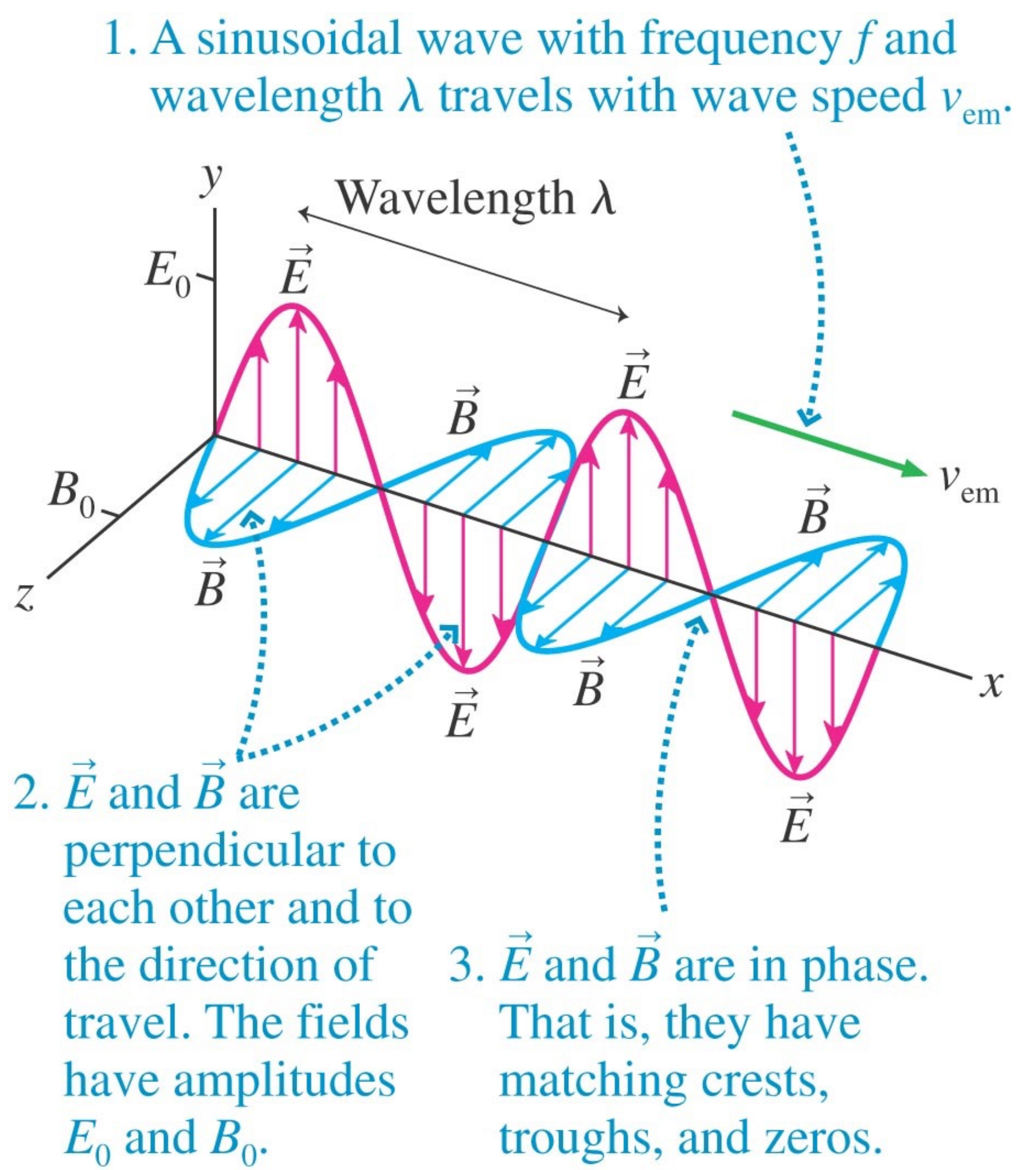
Tentative Schedule

Topic	Text Chapters	Lectures
Dynamic and Quasistatic Fields Faraday's Law, Magnetic Energy, Self and Mutual Inductance, Maxwell's Displacement Current	14	3
General Electromagnetic Fields Potentials, Conservation Laws, Gauge transformations	15	2
Waves in Vaccum Plane Waves, Polarization, Wave Packets, Diffraction	16	2
Waves in Simple Matter Reflection at Discontinuities, Radiation pressure, Anisotropic matter	17	3
Waves in Dispersive Matter Group velocity dispersion, attenuation, Foster's theorem	18	2
Guided and Confined Waves Transmission lines, conducting waveguides, optical waveguides, cavities	19	3
EXAM1	3/12/22	May Change
Retardation and Radiation, Radiation by given current distributuib, antennas, coherent/incoherent	20	3
Scattering and Diffraction Thomson and Rayleigh scattering	21	2
Special Relativity, transformations, Energy and Momentum, Charged Particle Motion in Strong Fields, Lagrangian Densitty	22	3
Radiation from moving charges Cherenkov radiation, Bremstralung and Synchrotron radiation	23	2
	Final Exam	

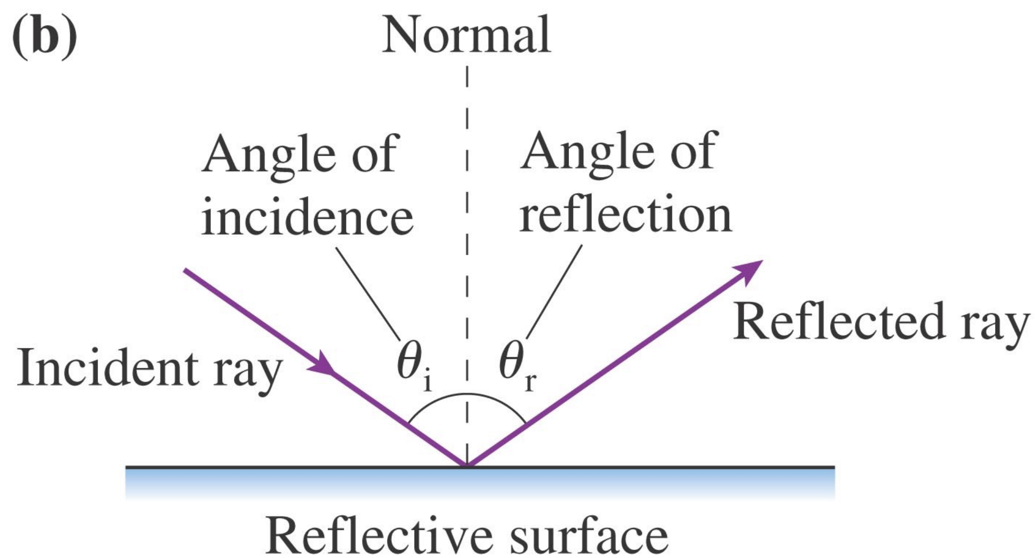
Overview

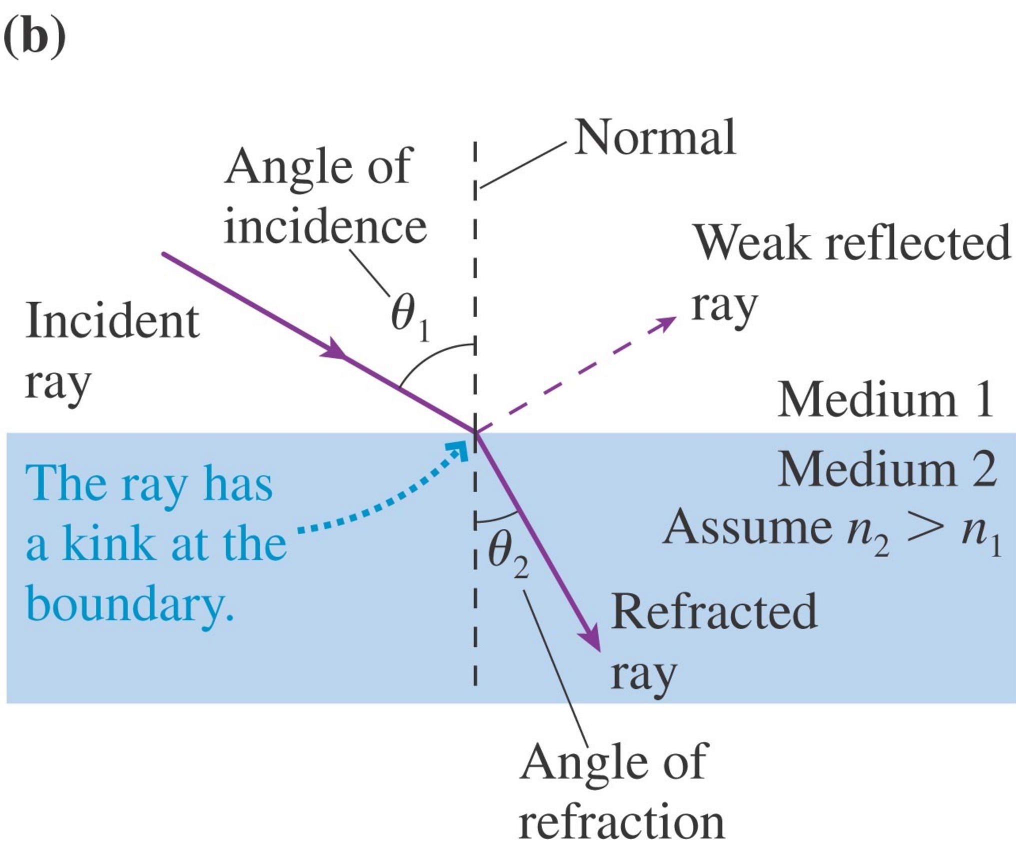
The goal of the course is to:
Introduce the phenomena of wave of wave propagation
Develop an understanding of the properties of Electromagnetic waves
Learn how to solve problems involving wave propagation

Propagation
Attenuation
Polarization
Reflection
Refraction
Dispersion
Diffraction
Interference



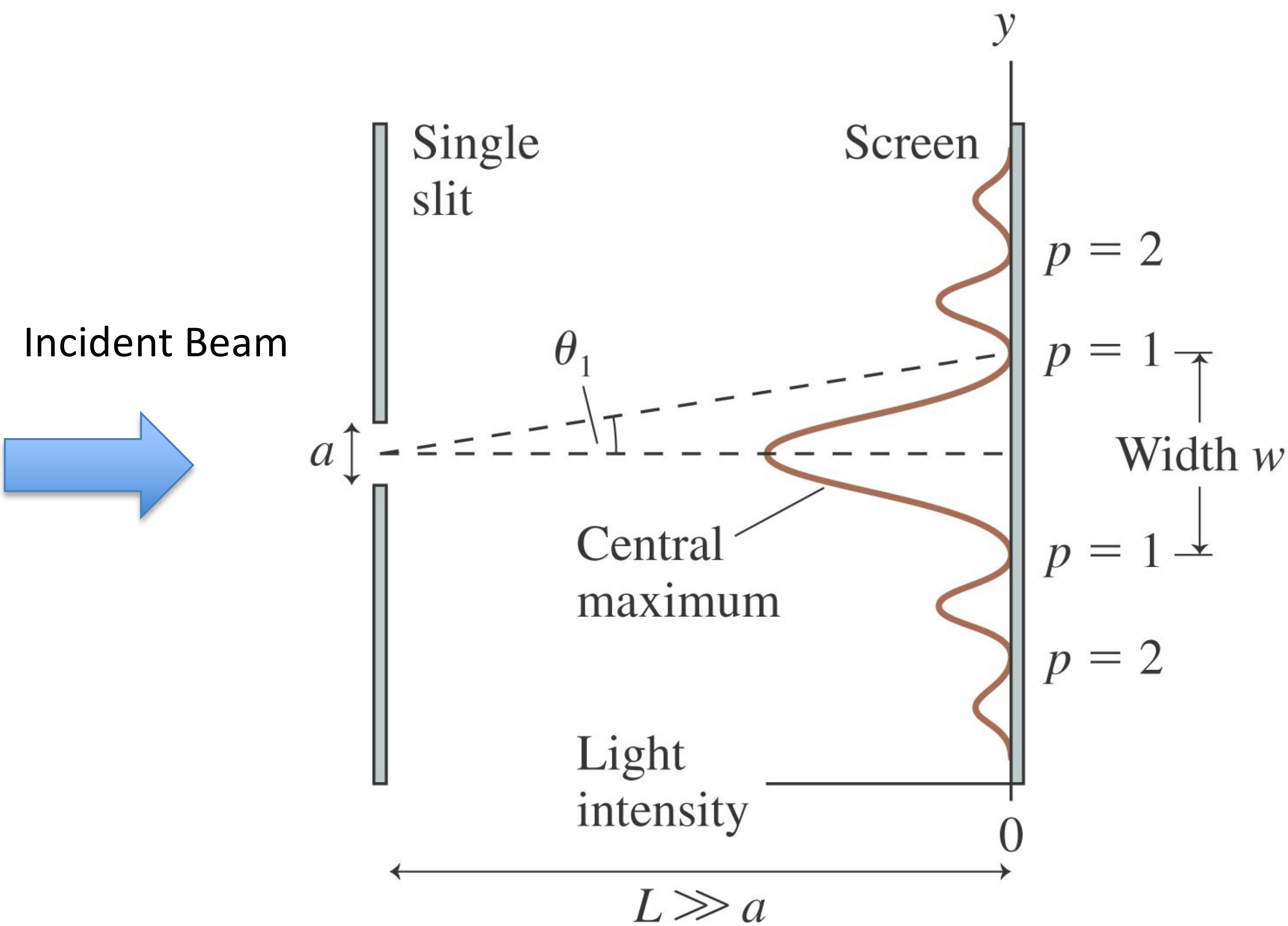
Reflection and Refraction





Diffraction and Interference

Incident Beam



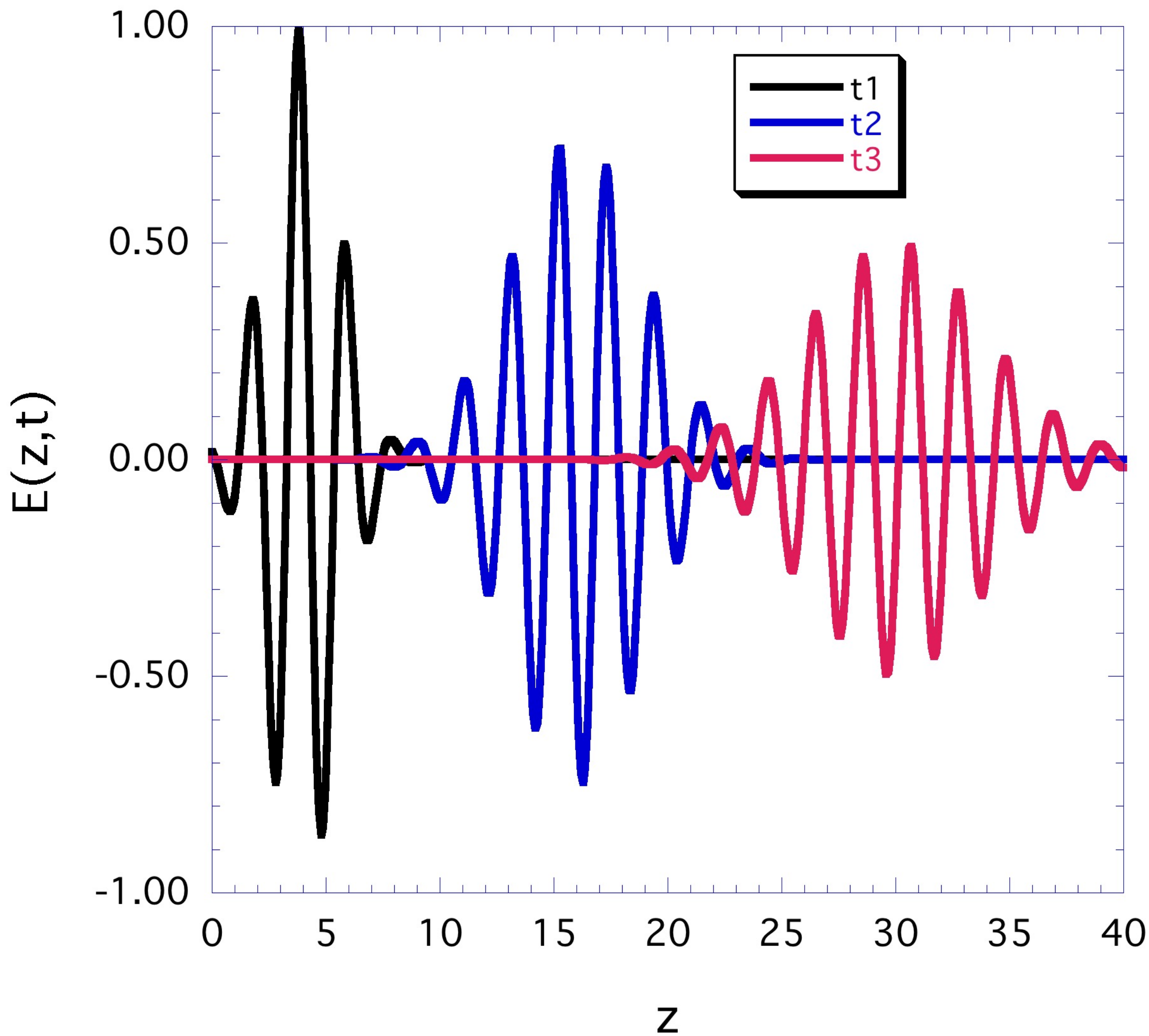
Dispersion and Attenuation

Pulses contain a spectrum of frequencies.

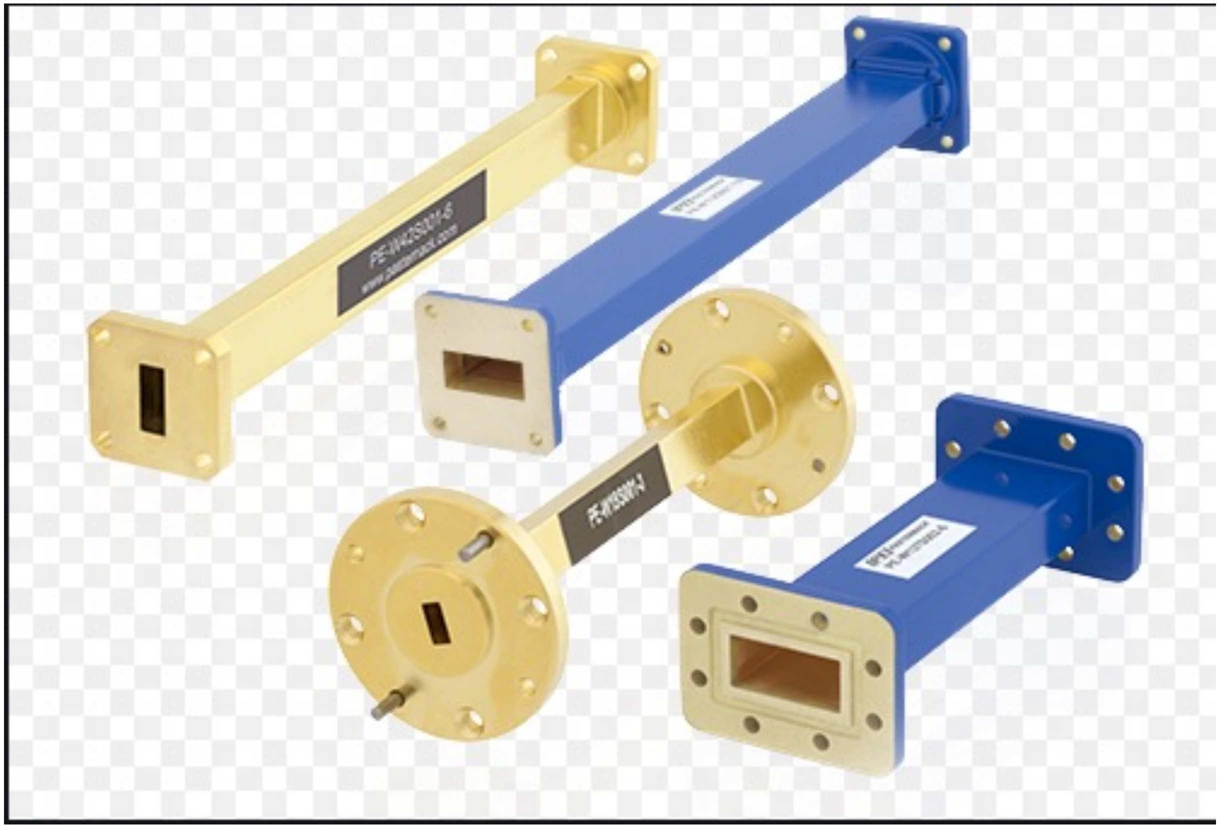
In dispersive media different frequency components propagate with different speeds.

Pulses spread out.

Losses lead to attenuation



Guided Waves



Pasternak Enterprizes
https://www.pasternack.com/

A bundle of optical fibers

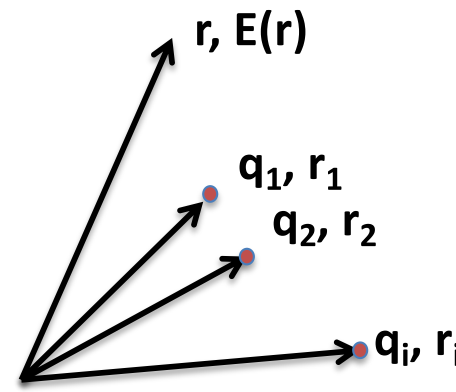
Wikipedia

Radiation and Antennas

Review of Static Fields

Static: not changing in time
For us: changing sufficiently slowly

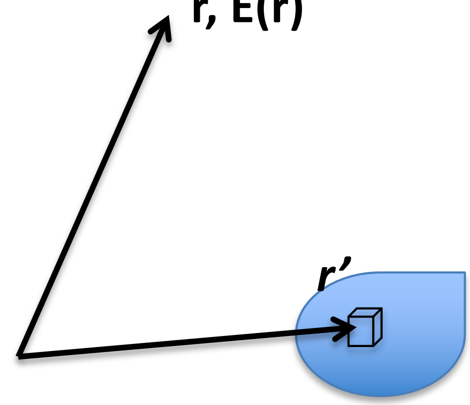
Start with Coulomb's Law for the electric field



Point Charges

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \sum_{charges-i} \frac{q_i(\mathbf{r}-\mathbf{r}_i)}{|\mathbf{r}-\mathbf{r}_i|^3}$$

Force on charge q $\quad \mathbf{F} = q\mathbf{E}(\mathbf{r})$



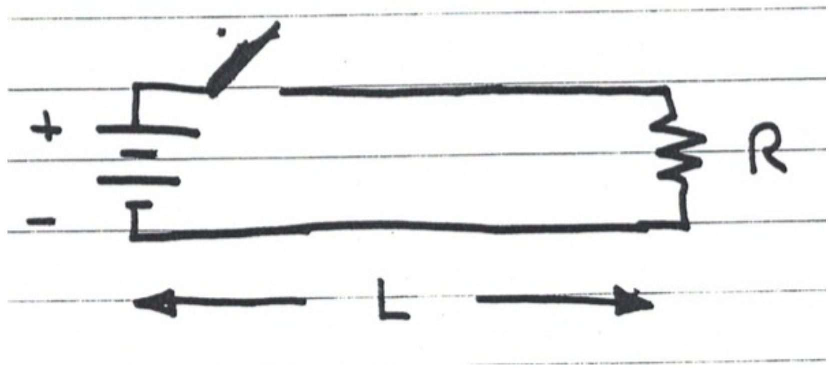
Continuous charge distributions

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \int_V \frac{\rho(\mathbf{r}')(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3} d^3r'$$

$$dq' = d^3r'\,\rho(\mathbf{r}')$$

Electrostatic or not?

Circuit vs Transmission line?
When the switch is closed how long until current flows in R?



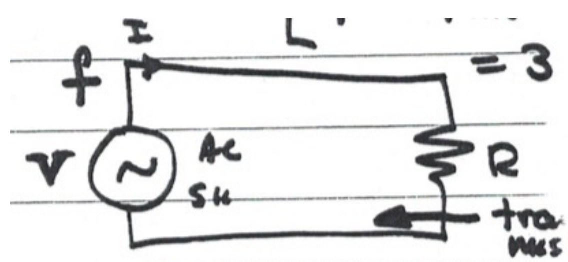
$$\Delta t = L / c$$

L=1m, c = 3 x10^8 m/s

$$\Delta t = 3.3 \times 10^{-9} s \quad = 3.3 ns$$

How long until current reaches steady state? Depends on reflections.

AC source, What load does source see?



If L < < wavelength = c/f then R
However, once L = wavelength/4 load is transformed.

Some Examples

Comcast signal: 55.25 **MHz** to 553 **MHz** Wavelength at 553 MHz = 0.54 m

Verizon 5G signal: 28 GHz Wavelength at 28 GHz = 0.01 m

Infrared laser: 3×10^{14} Hz Wavelength = 1 micron = 10^{-6} m

Bohr Radius = 5.29×10^{-11} m << 1 micron wavelength

Laser field in atom is electrostatic

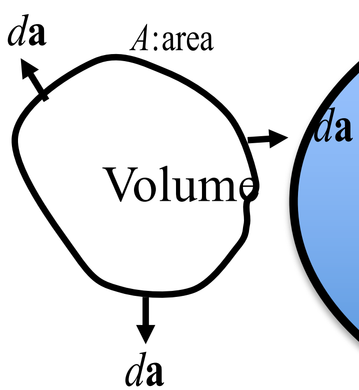
Fork in microwave oven: f = 2 GHz Wavelength = 0.15 m >> fork prong

Three ways to say the same thing

Coulomb's Law:

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\int_V \frac{\rho(\mathbf{r}')\left(\mathbf{r}-\mathbf{r}'\right)}{\left|\mathbf{r}-\mathbf{r}'\right|^3} d^3r'$$

Gauss' Law:



$$\oint_S \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \frac{Q_{in}}{\varepsilon_0} = \frac{1}{\varepsilon_0}\int_V \rho d^3r$$

$$\oint_{loop} \vec{\mathbf{E}}\cdot d\vec{\mathbf{l}} = 0$$

Poisson Equation:

$$\nabla\cdot\vec{\mathbf{E}} = \frac{\rho}{\varepsilon_0}$$

$$\nabla\times\vec{\mathbf{E}} = 0$$

Magnetostatics

Biot-Savart Law:

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int_V \frac{\mathbf{J}(\mathbf{r}')\times(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}\, d^3r'$$

Ampere's Law:

$$\oint_{loop} \vec{\mathbf{B}}\cdot d\vec{\mathbf{S}} = \mu_0 I_{enclosed}$$

$$= \mu_0 \oint_S \vec{\mathbf{J}}\cdot d\vec{\mathbf{A}}$$

$$\nabla\times\vec{\mathbf{B}} = \mu_0\vec{\mathbf{J}}$$

Gauss' Law:

$$\oint_S \vec{\mathbf{B}}\cdot d\vec{\mathbf{A}} = 0$$

$$\nabla\cdot\vec{\mathbf{B}} = 0$$

MKS-SI Units

E	Volts/meter
Q	Coulombs
B	Tesla
I	Amperes

$$\oint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{in}}{\varepsilon_0}$$

$[\varepsilon_0]$ = Coulombs/ Volts-Meters

$$[\varepsilon_0] = 8.8542 \times 10^{-12} \quad \text{Farads/meter}$$

Force on a moving charge q

$$\vec{\mathbf{F}} = q\left(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}\right)$$

[B] = Volts-seconds/meter^2

Ampere's Law

$$\oint_{loop} \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \mu_0 I_{enclosed}$$

[B.dl] = Volts-seconds/meter = Amperes $[\mu_0]$

$$\mu_0 = 4\pi \times 10^{-7} \quad \text{Volt-seconds/Ampere-meters} \quad = \quad \text{Henry's/meter}$$

What to remember:

$$1/\sqrt{\varepsilon_0 \mu_0} = c = 3 \times 10^8 \quad \text{m/s} \qquad \sqrt{\mu_0 / \varepsilon_0} \quad = 377 \text{ Ohms} = \text{impedance of free space}$$

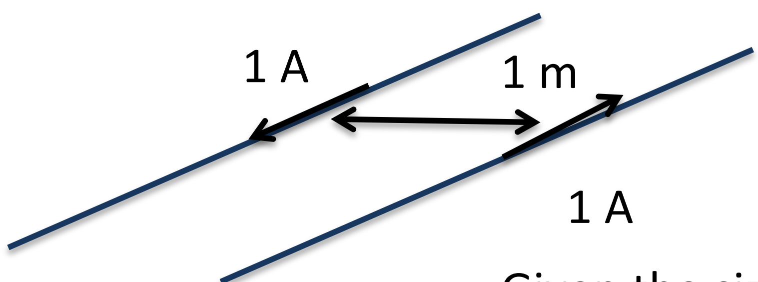
Why such funny numbers?

$$\varepsilon_0 = 8.8542 \times 10^{-12} \quad \text{Farads/meter}$$

$$\mu_0 = 4\pi \times 10^{-7} \quad \text{Henry's/meter}$$

The size of the Ampere is set by the requirement that two infinitely long parallel wires separated by 1 meter and each carrying 1 Ampere of current feel a force of

$$\mu_0 = 4\pi \times 10^{-7} \quad \text{Newtons/meter}$$



Given the size of an Ampere and the unit of time, 1 second, the unit of charge is defined,

1 Coulomb = 1 Ampere X 1 second

Statics to Dynamics

Integrals over closed surfaces

Poisson: $\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = Q / \varepsilon_0$

Gauss' Law: $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$

Integrals around closed loops

Faraday's Law:

$$\oint_{loop} \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} = -\int_S d\vec{\mathbf{A}} \cdot \frac{\partial \vec{\mathbf{B}}}{\partial t}$$

Ampere's Law:

$$\oint_{Loop} \vec{\mathbf{B}}(\vec{\mathbf{r}}) \cdot d\vec{\mathbf{l}} = \mu_0 \int_S d\vec{\mathbf{A}} \cdot \left[\vec{\mathbf{J}} + \varepsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial t} \right]$$

Dynamic Fields

Faraday's Law

Maxwell's Displacement Current

Integrals around closed loops

Faraday' s Law:

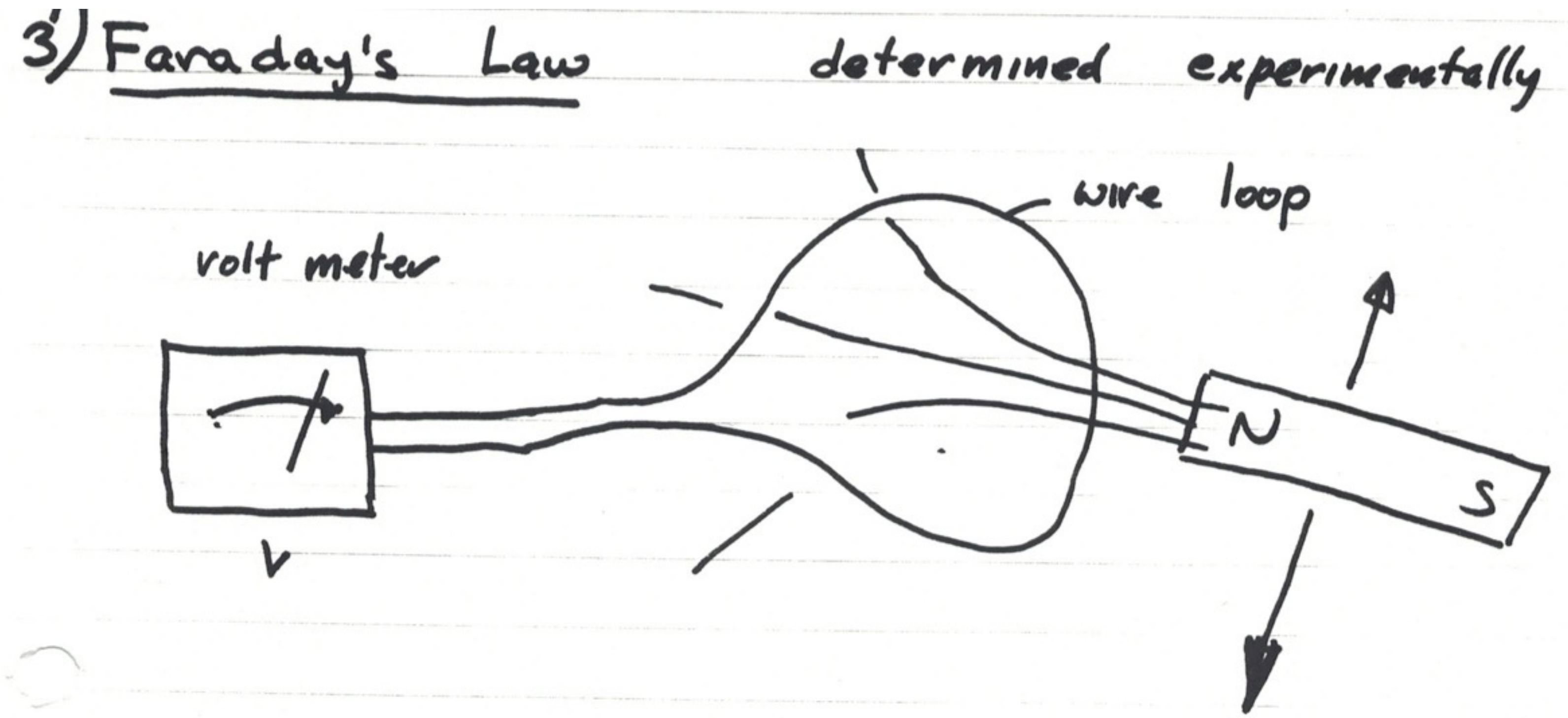
$$\oint_{loop} \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} = -\int_S d\vec{\mathbf{A}} \cdot \frac{\partial \vec{\mathbf{B}}}{\partial t}$$

$$\nabla \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial t}$$

Ampere' s Law:

$$\oint_{Loop} \vec{\mathbf{B}}(\vec{\mathbf{r}}) \cdot d\vec{\mathbf{l}} = \mu_0 \int_S d\vec{\mathbf{A}} \cdot \left[\vec{\mathbf{J}} + \varepsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial t} \right]$$

$$\nabla \times \vec{\mathbf{B}}(\vec{\mathbf{r}}) = \mu_0 \left[\vec{\mathbf{J}} + \varepsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial t} \right]$$



As the magnet was moved a voltage appeared on the meter.

The polarity of the voltage depended on whether the magnetic flux threading the loop was increasing or decreasing

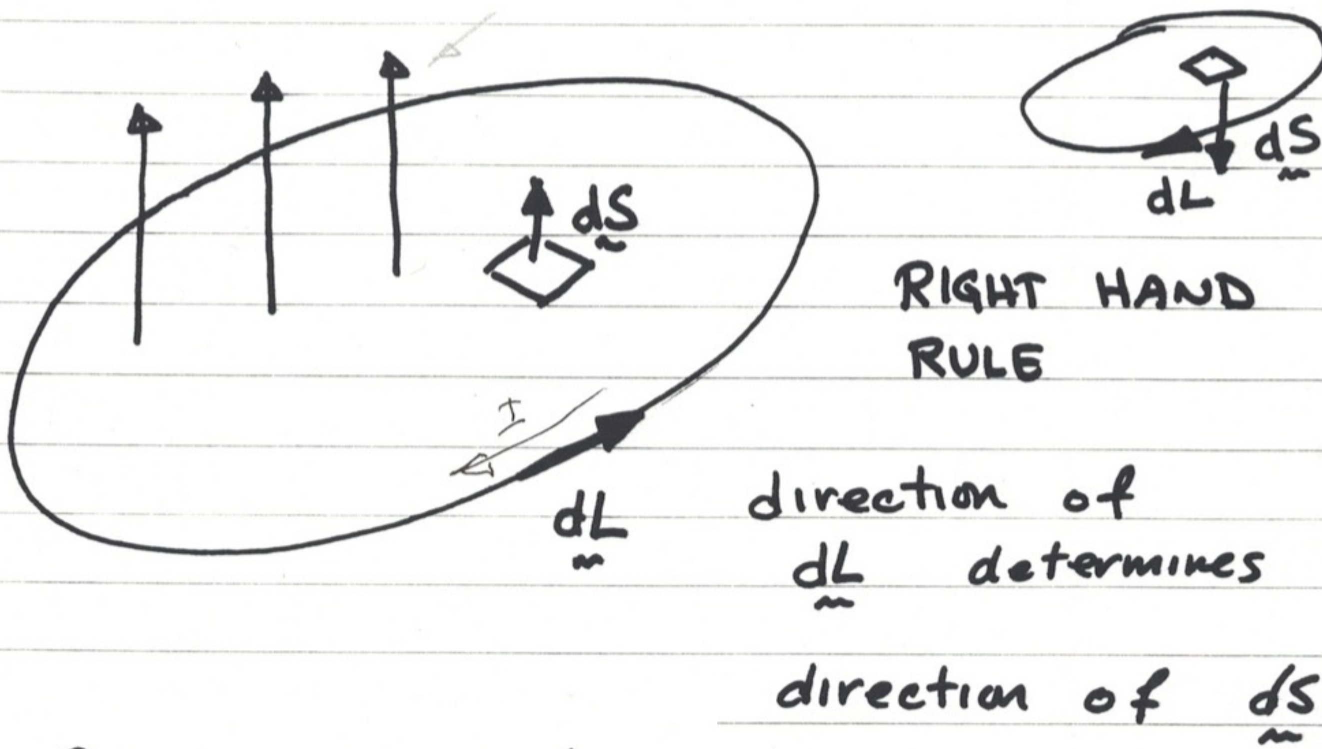
Experimentally deduced relation

$$V = -\int_C \underline{E} \cdot d\underset{\sim}{L} = \frac{d\psi}{dt}$$

where $\psi = \int_S \underset{\sim}{B} \cdot d\underset{\sim}{S}$

For stationary loops .

Sign determined by right hand rule



$$V = -\oint_C \underline{E}\cdot d\underline{L} = \frac{d\psi}{dt}$$

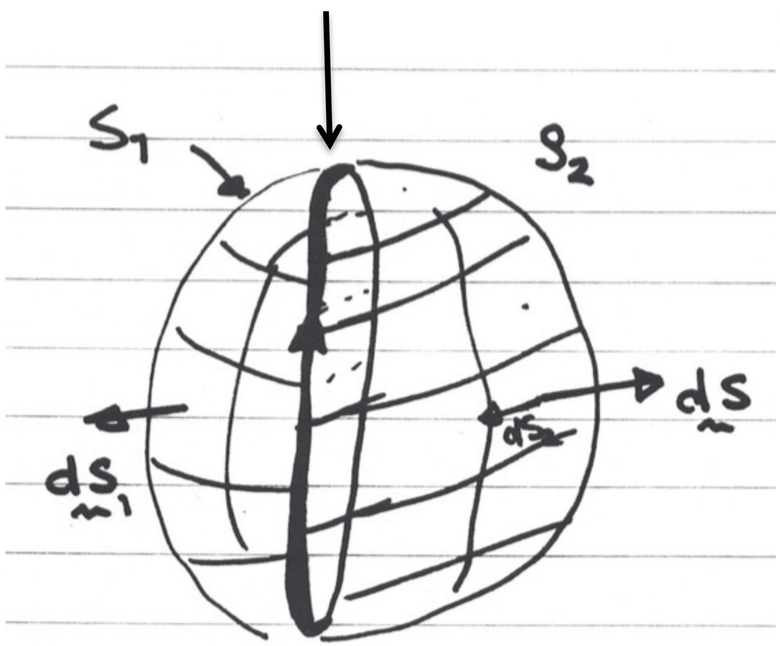
where $\psi = \int_S \underline{B}\cdot d\underline{S}$

Lenz Law
E would induce I to cancel change in B

$$V = -\int_C \underline{E} \cdot d\underline{L} = \frac{d\psi}{dt}$$

where $\psi = \int_S \underline{B} \cdot d\underline{S}$

Loop C



Which surface S_1 or S_2?

Answer: Either one

From Gauss' Law

$$\int_{S_1+S_2} \vec{\mathbf{B}} \cdot d\vec{\mathbf{S}} = 0$$

$$d\vec{\mathbf{S}}_1 = d\vec{\mathbf{S}}$$

$$d\vec{\mathbf{S}}_2 = -d\vec{\mathbf{S}}$$

$$\int_{S_1+S_2} \vec{\mathbf{B}} \cdot d\vec{\mathbf{S}} = 0 \Rightarrow \int_{S_1} \vec{\mathbf{B}} \cdot d\vec{\mathbf{S}}_1 = \int_{S_2} \vec{\mathbf{B}} \cdot d\vec{\mathbf{S}}_2$$

Using Stokes' Theorem

$$\int_C \underline{E} \cdot d\underline{L} = \int_S d\hat{S} \cdot \nabla \times \underline{E} = -\frac{\partial}{\partial t} \int_S \underline{B} \cdot d\underline{S}$$

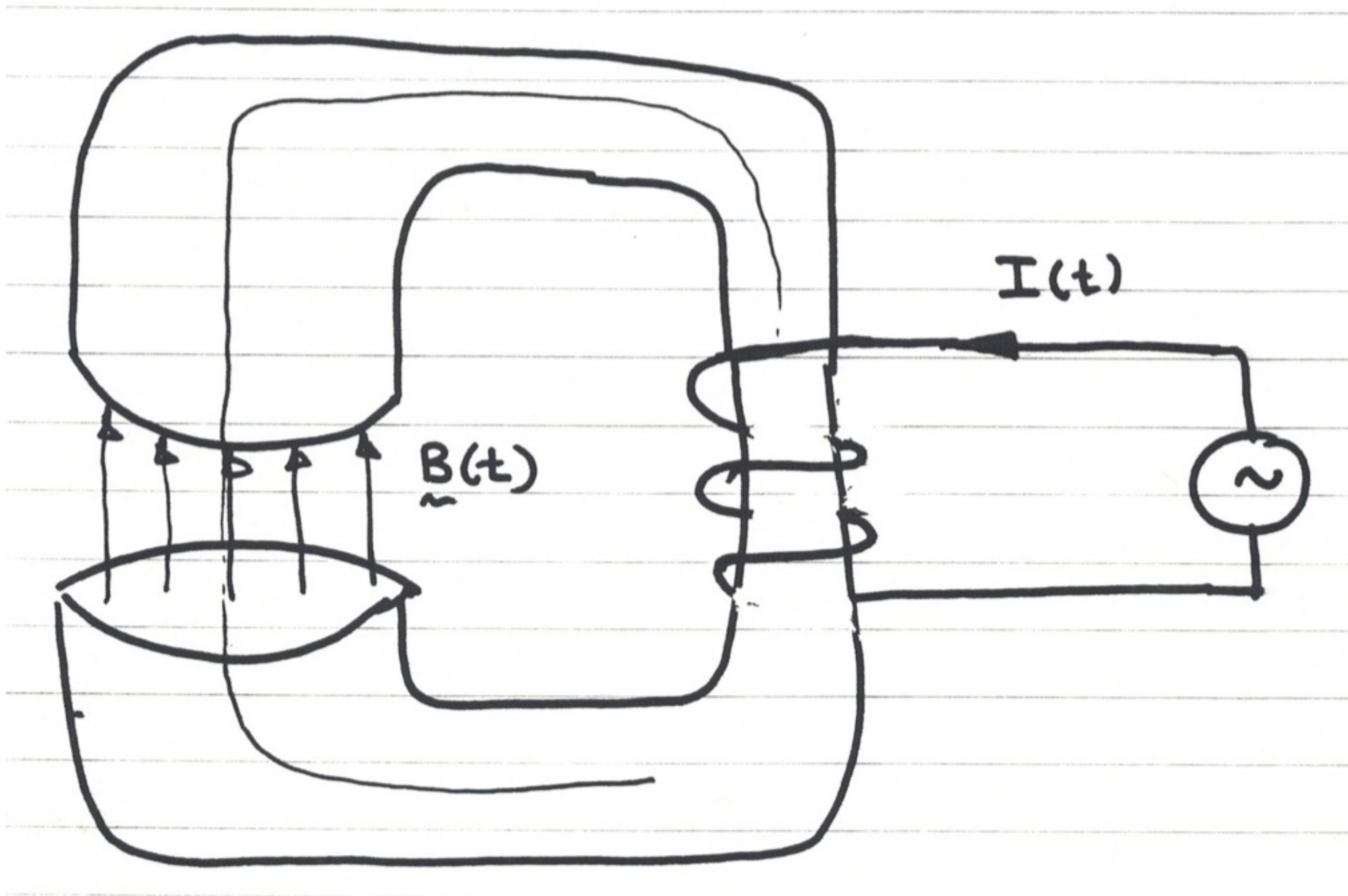
True for any loop and any surface

THUS

$$\nabla \times \underline{E} = -\frac{\partial}{\partial t} \underline{B}$$

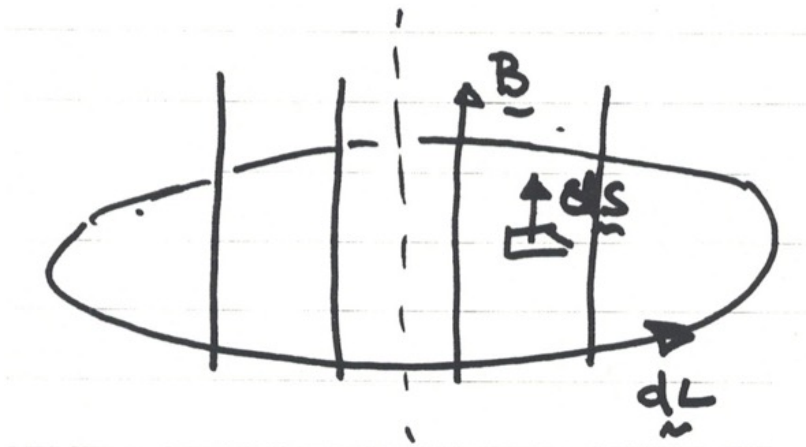
Faraday's Law in differential form

Time varying B induces E



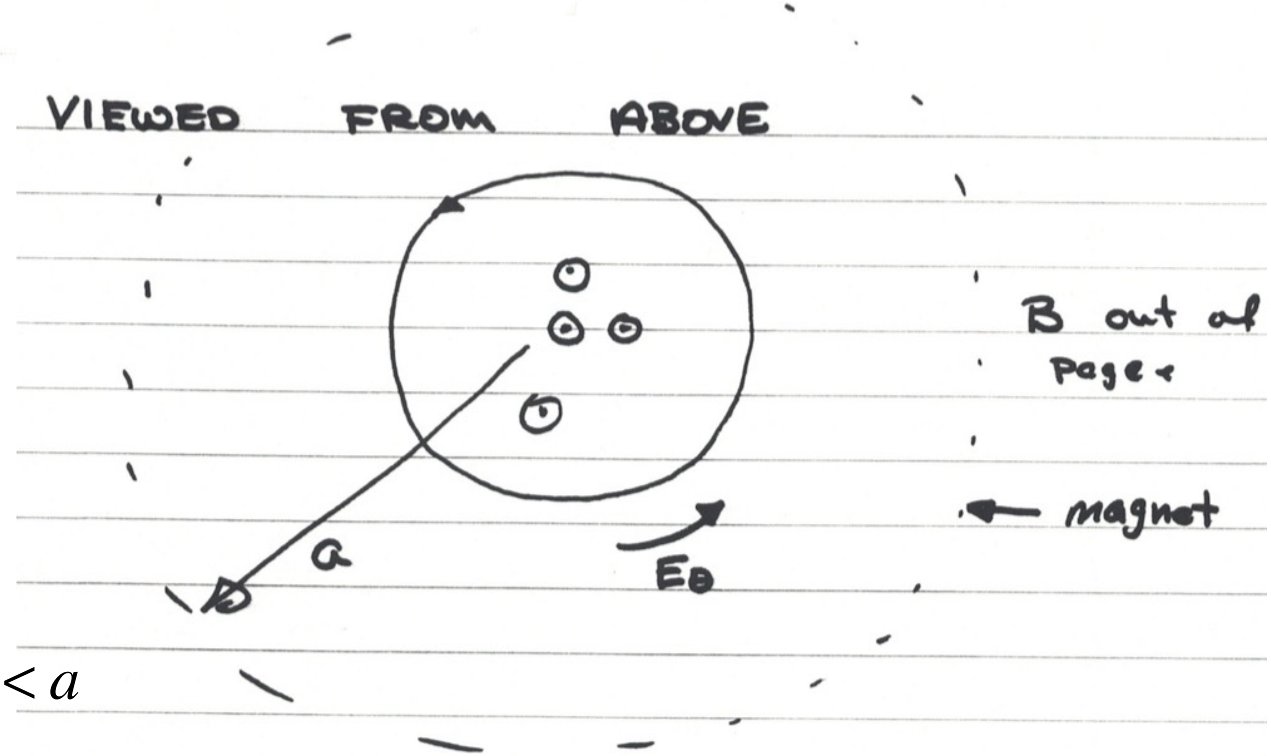
Find E in the gap

Gap Field



Evaluate on a loop of radius *r*.

Assume: $\frac{\partial E_\theta}{\partial \theta} = 0$

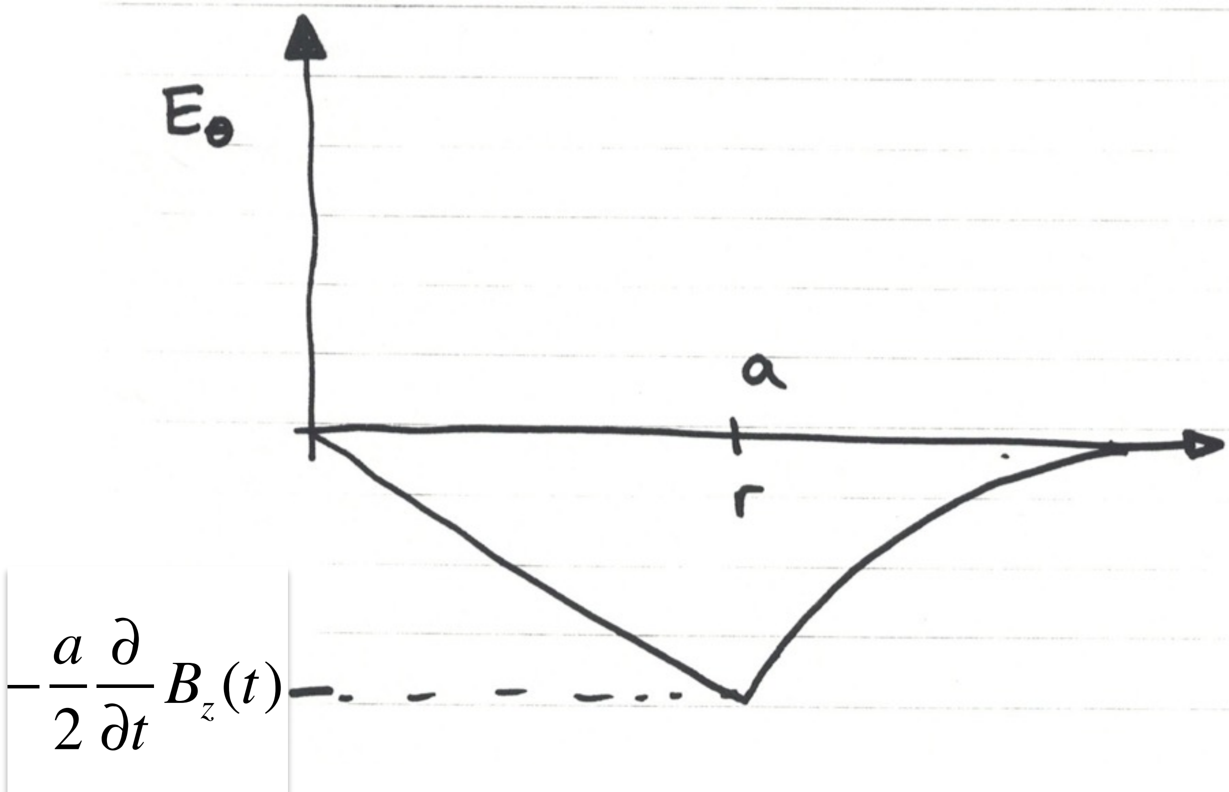


$$\int_C \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} = 2\pi r E_\theta(r)$$

$$\int_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{S}} = \begin{cases} \pi r^2 B_z, & r < a \\ \pi a^2 B_z, & r > a \end{cases}$$

$$2\pi r E_\theta(r) = -\frac{\partial}{\partial t} \begin{cases} \pi r^2 B_z(t), & r < a \\ \pi a^2 B_z(t), & r > a \end{cases}$$

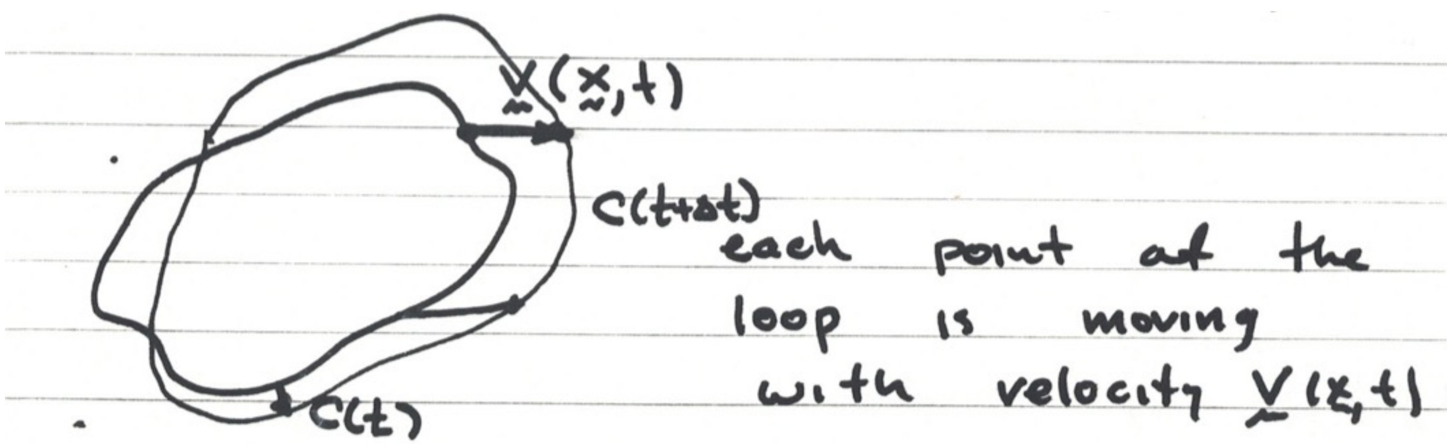
$$E_\theta(r) = -\begin{cases} \dfrac{r}{2}\dfrac{\partial}{\partial t}B_z(t), & r < a \\[2ex] \dfrac{a^2}{2r}\dfrac{\partial}{\partial t}B_z(t), & r > a \end{cases}$$



Moving Loops

So far we have considered stationary loops.

What is the rate of change of flux through a moving loop?

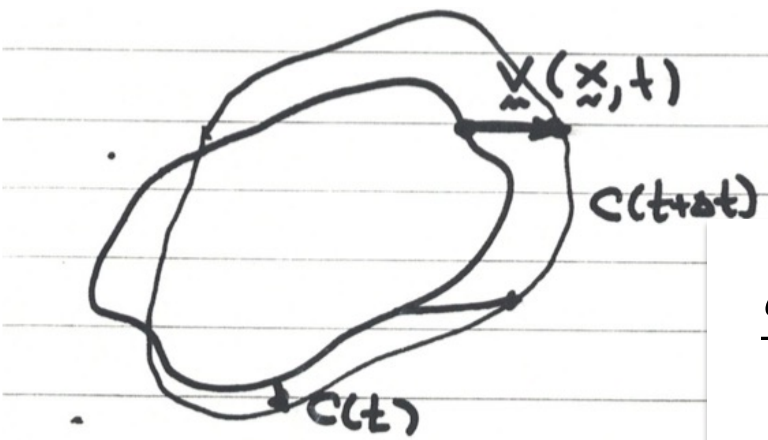


It can be shown

$$\frac{d\psi}{dt} = \int_{S(t)} \frac{\partial \underset{\sim}{B}}{\partial t} \cdot d\underset{\sim}{S} - \oint_{C(t)} d\underset{\sim}{l} \cdot \underset{\sim}{v} \times \underset{\sim}{B}$$

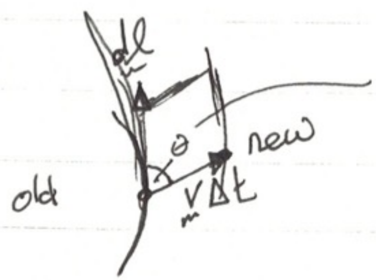
Contribution from time changing B, Contribution from moving loop.

Rate of change of flux



$$\frac{d\psi}{dt} = \lim_{\Delta t \to 0} \frac{\psi(t+\Delta t) - \psi(t)}{\Delta t}$$

Derivation



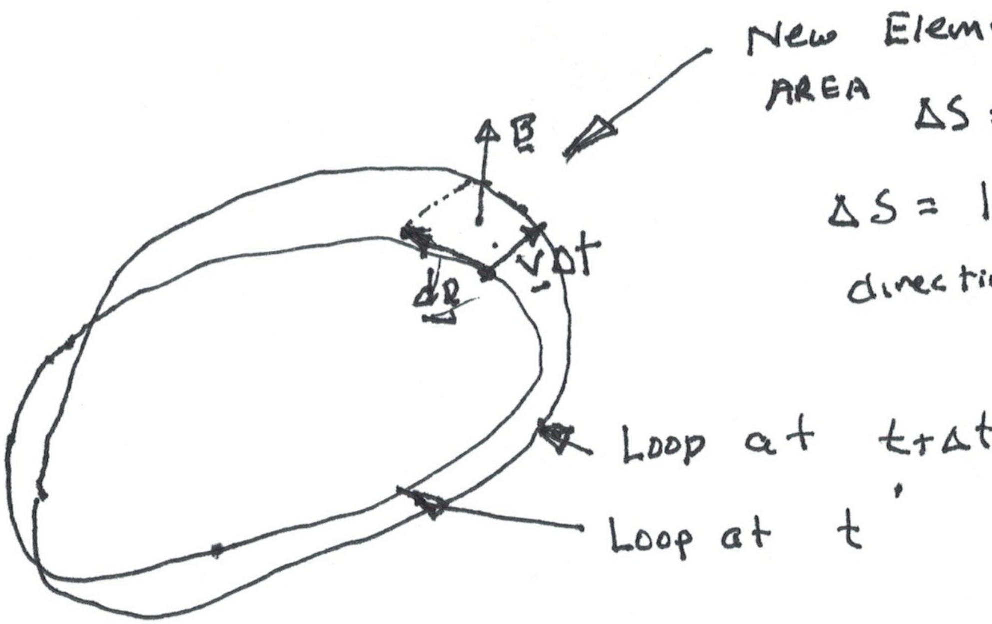
new area

$|\Delta S| = |\mathbf{v}\Delta t||d\mathbf{l}|\sin\theta \quad = |\mathbf{v}\times d\mathbf{l}|\Delta t$

direction $\mathbf{v}\times d\mathbf{l}$

$\int_{C(t)} \mathbf{B}\cdot\mathbf{v}\times d\mathbf{l} \quad = -\int d\mathbf{l}\cdot\mathbf{v}\times\mathbf{B}$

Contribution from moving loop



$\Delta S = V \Delta t \, dl \sin\theta$

$\Delta S = |\underline{V} \times d\underline{l}| \Delta t$

direction $\underline{n} \propto \underline{V} \times d\underline{l}$

$$\Delta \psi = \Delta t \int \underline{B} \cdot (\underline{V} \times d\underline{l}) = -\Delta t \int d\underline{l} \cdot \underline{V} \times \underline{B}$$

EMF – electromotive force

$$\frac{d\psi}{dt} = \int_{S(t)} \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S} - \int_{C(t)} d\mathbf{l} \cdot \mathbf{v} \times \mathbf{B}$$

Convert surface integral to line integral

$$\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S} = - \int_S d\mathbf{S} \cdot \nabla \times \mathbf{E} = - \int_{C(t)} d\mathbf{l} \cdot \mathbf{E}$$

$$\frac{d\psi}{dt} = - \int_{C(t)} d\mathbf{l} \cdot (\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Two ways to compute EMF

$$EMF = -\frac{d}{dt}\psi = -\frac{d}{dt}\int_{S(t)} d\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}$$

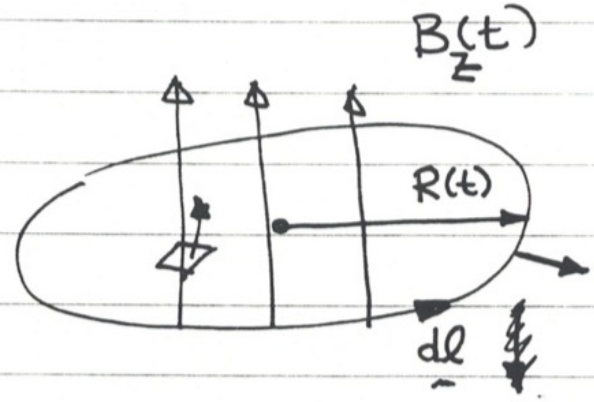
$$EMF = \oint_{loop} d\vec{\mathbf{l}} \cdot \left(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}\right)$$

Both are always true. One may be easier to determine than the other.

Note: same combination of E, B, and v appears in force

$$\vec{\mathbf{F}} = q\left(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}\right)$$

Example



constant in space But varying in time

R(t) Varyies in time

$$EMF = -\frac{d}{dt}\psi(t) = -\frac{d}{dt}\pi R^2(t)B_z(t)$$

$$= -\pi R^2(t)\frac{d}{dt}B_z(t) - 2\pi R(t)B_z(t)\frac{d}{dt}R(t)$$

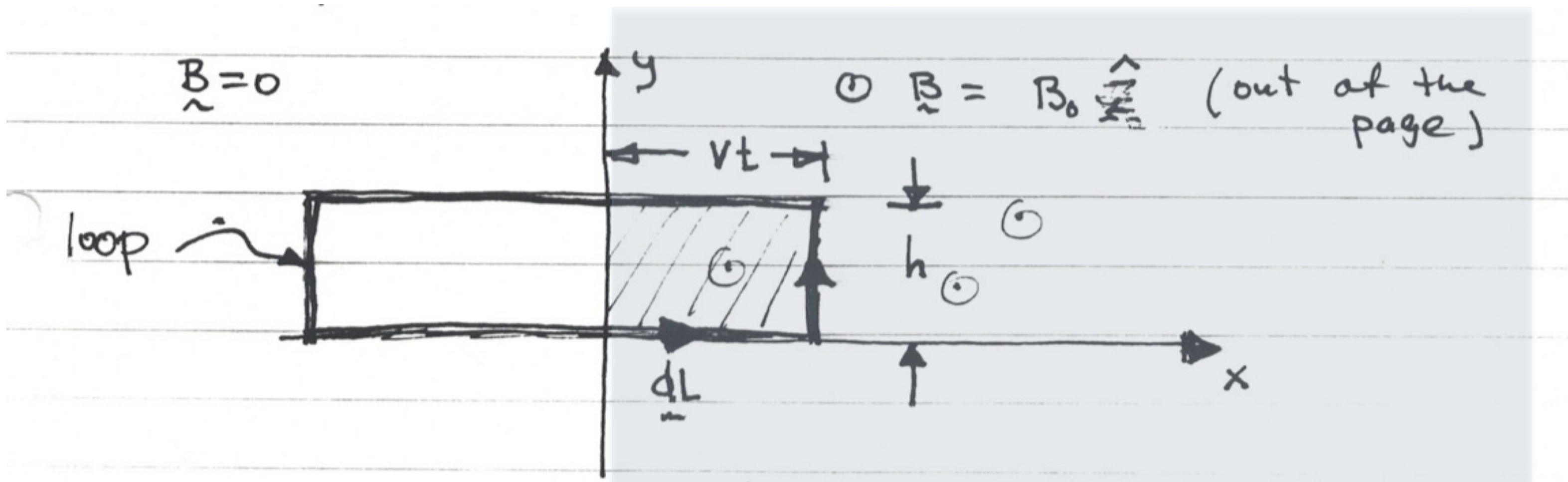
$$EMF = -\int_S d\vec{\mathbf{A}}\cdot\frac{\partial\vec{\mathbf{B}}}{\partial t} + \int_C d\vec{\mathbf{l}}\cdot\vec{\mathbf{v}}\times\vec{\mathbf{B}}$$

$$+\int_C d\vec{\mathbf{l}}\cdot\vec{\mathbf{v}}\times\vec{\mathbf{B}} = -2\pi R\frac{dR}{dt}B_z$$

$$\hat{\theta}\cdot\hat{r}\times\hat{z} = -1$$

Calculate the EMF

$$B(x,y,z,t)=\begin{cases} B_0\hat{z}, & x>0 \\ 0, & x<0 \end{cases}$$

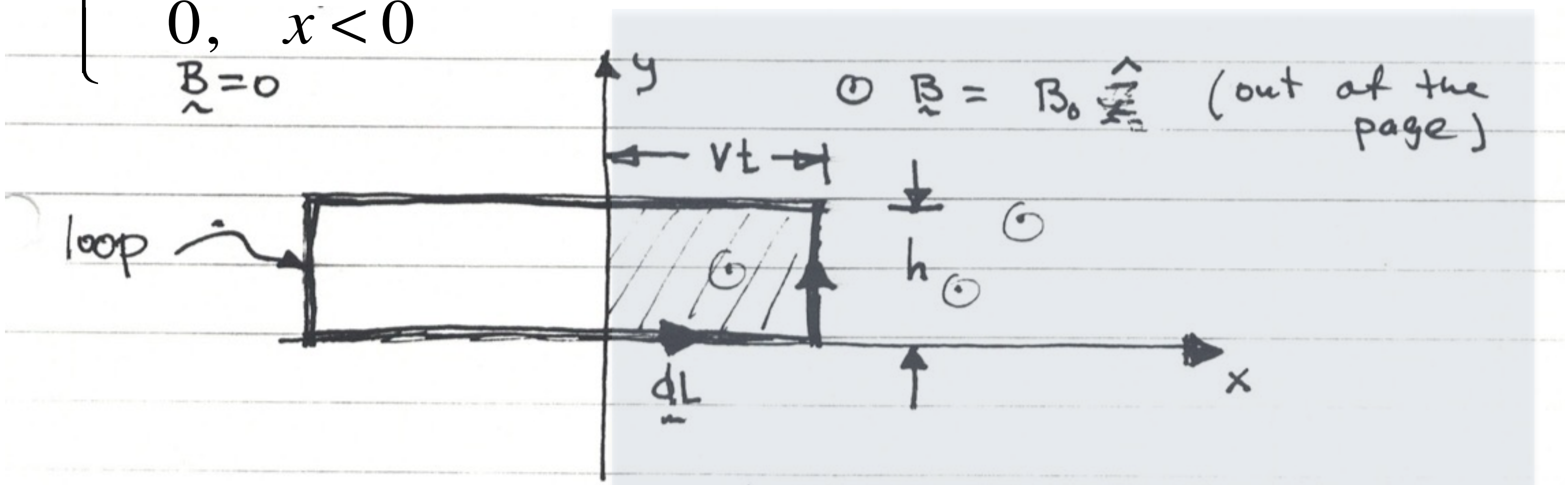


Three cases:

1. Loop is nonconducting
2. Loop is partially conducting
3. Loop is fully conducting

Case #1 non-conducting

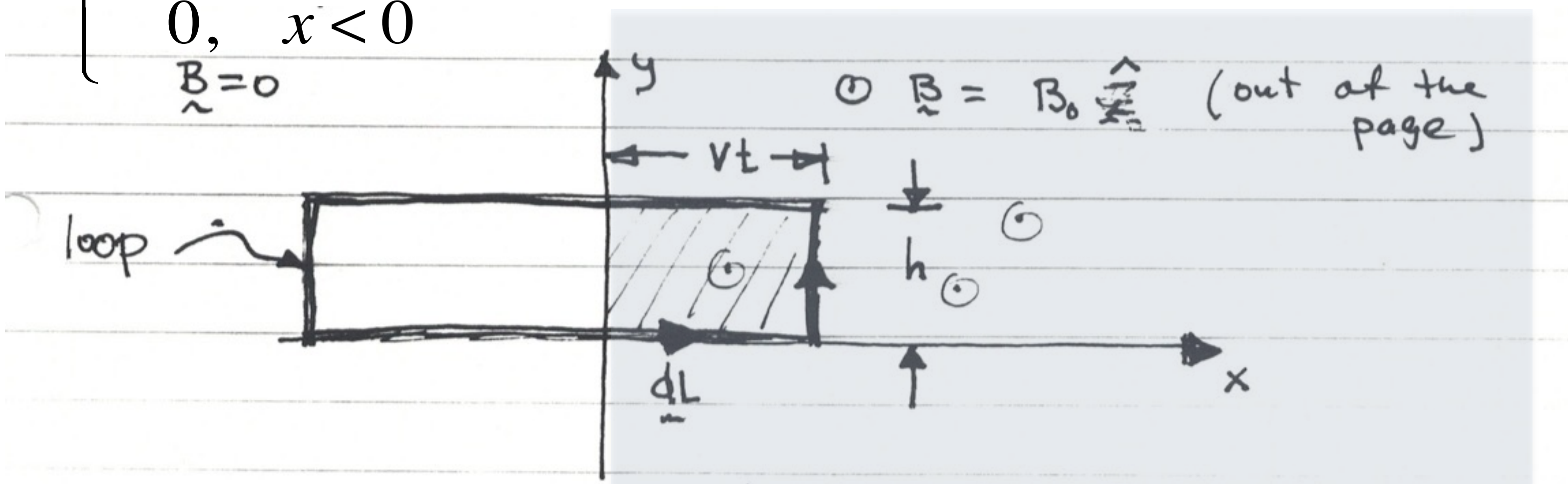
$$B(x,y,z,t) = \begin{cases} B_0\hat{z}, & x>0 \\ 0, & x<0 \end{cases}$$



$$EMF = -\frac{d}{dt}\psi = -\frac{d}{dt}\left[h(vt)B_0\right] = -hvB_0$$

Case #1 non-conducting

$$B(x,y,z,t)=\begin{cases} B_0\hat{z}, & x>0 \\ 0, & x<0 \end{cases}$$



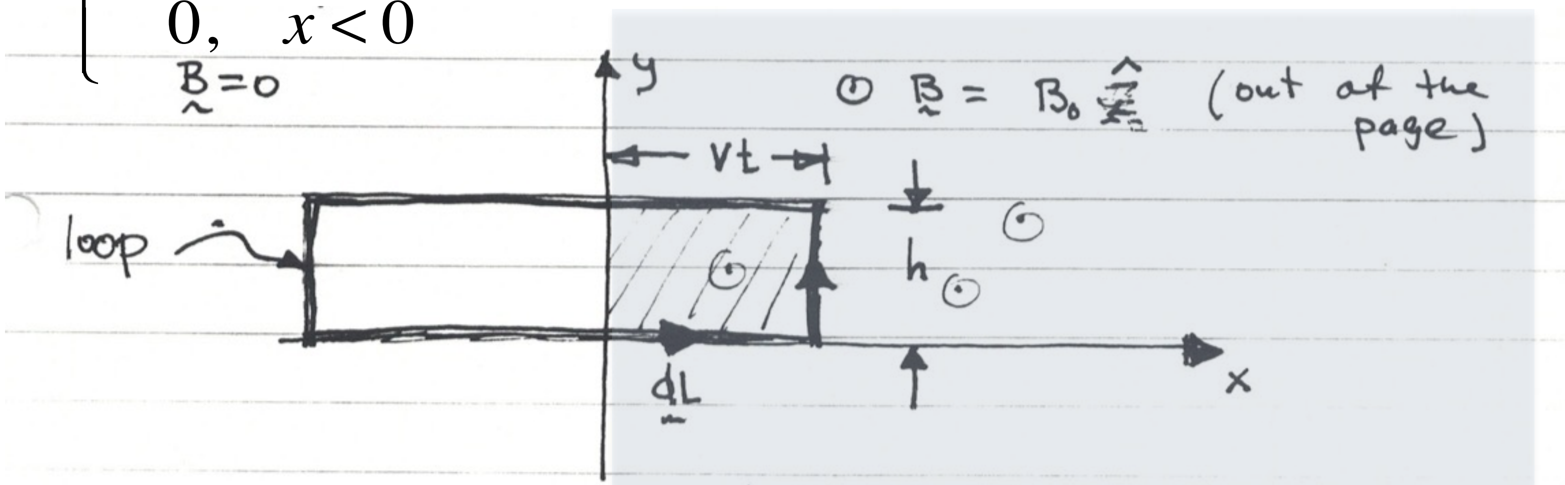
$$\frac{\partial \vec{\mathbf{B}}}{\partial t}=0 \rightarrow \nabla\times\mathbf{E}=0 \rightarrow \mathbf{E}=-\nabla\phi \rightarrow \oint_{loop} d\vec{\mathbf{l}}\cdot\mathbf{E}=0$$

$$EMF=\oint_{loop} d\vec{\mathbf{l}}\cdot\left(\mathbf{E}+\vec{\mathbf{v}}\times\vec{\mathbf{B}}\right)=\oint_{loop} d\vec{\mathbf{l}}\cdot\left(\vec{\mathbf{v}}\times\vec{\mathbf{B}}\right)$$

$$=\int dy v B_0(\hat{\mathbf{y}}\cdot\hat{\mathbf{x}}\times\hat{\mathbf{z}})=-hvB_0$$

Case #1 non-conducting

$$B(x,y,z,t)=\begin{cases} B_0\hat{z}, & x>0 \\ 0, & x<0 \end{cases}$$

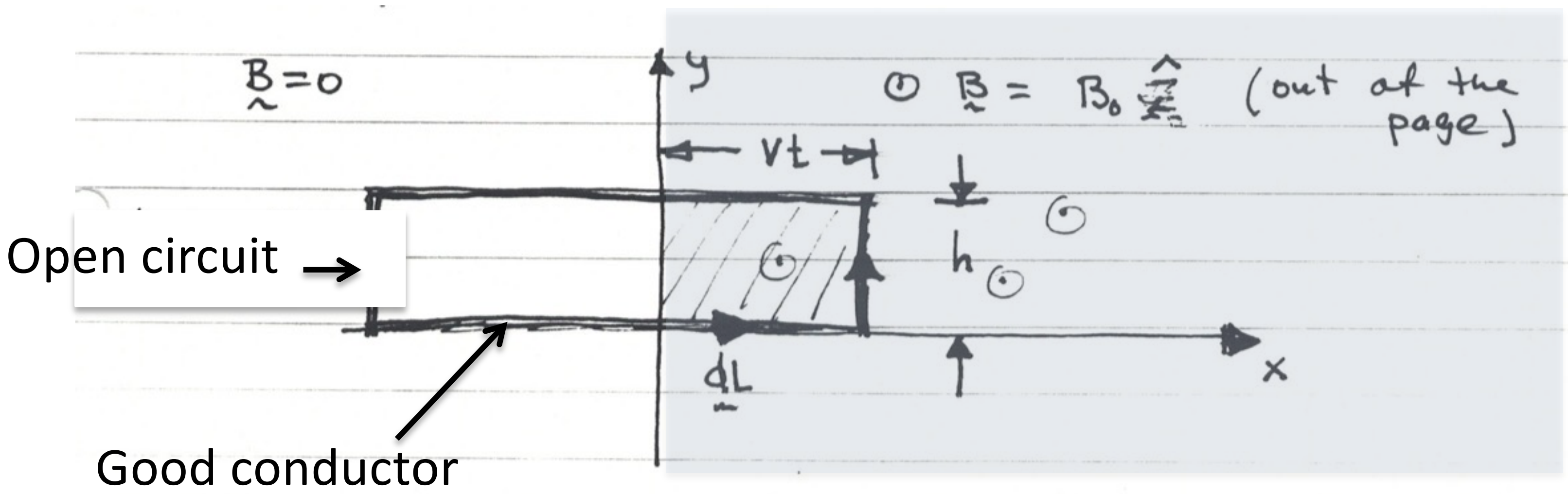


$$EMF=-\frac{d}{dt}\psi=-\frac{d}{dt}\left[h(vt)B_0\right]=-hvB_0$$

$$EMF=\oint_{loop}d\vec{\mathbf{l}}\cdot\left(\vec{\mathbf{E}}+\vec{\mathbf{v}}\times\vec{\mathbf{B}}\right)=$$

$$\oint_{loop}d\vec{\mathbf{l}}\cdot\left(\vec{\mathbf{0}}+\vec{\mathbf{v}}\times\vec{\mathbf{B}}\right)=\int dyvB_0(\hat{\mathbf{y}}\cdot\hat{\mathbf{x}}\times\hat{\mathbf{z}})=-hvB_0$$

Case #2: partially conducting

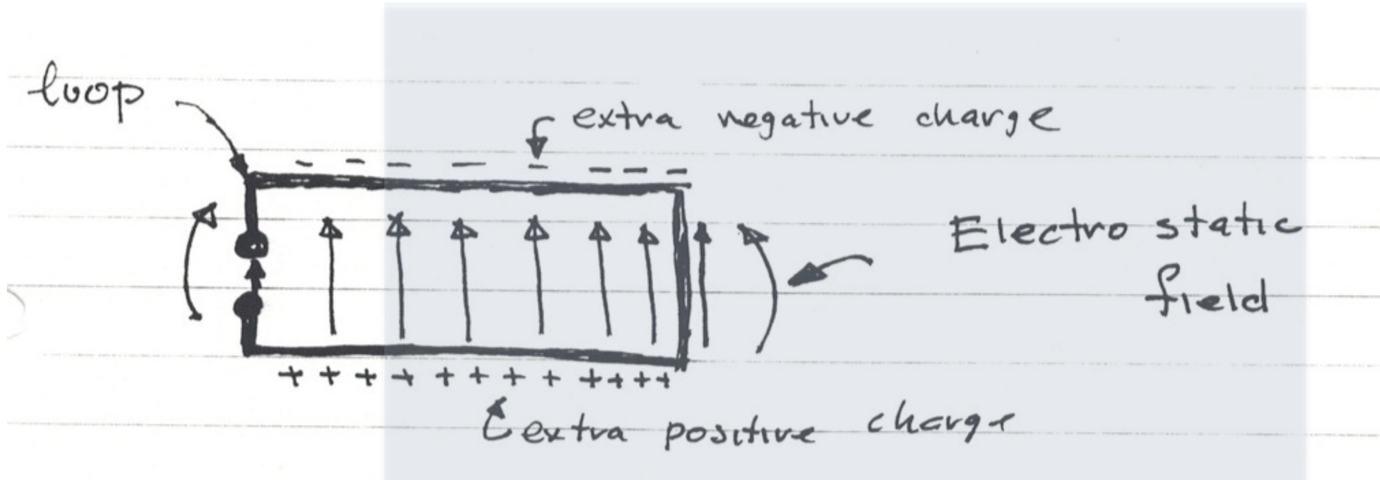


No current flows in conductor, B is unchanged,

$$EMF = -\frac{d}{dt}\psi = -hvB_0$$

$$EMF = \oint_{loop} d\vec{\mathbf{l}} \cdot \left(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}\right)$$

In conductor $\left(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}\right) = 0$



On right end of moving loop

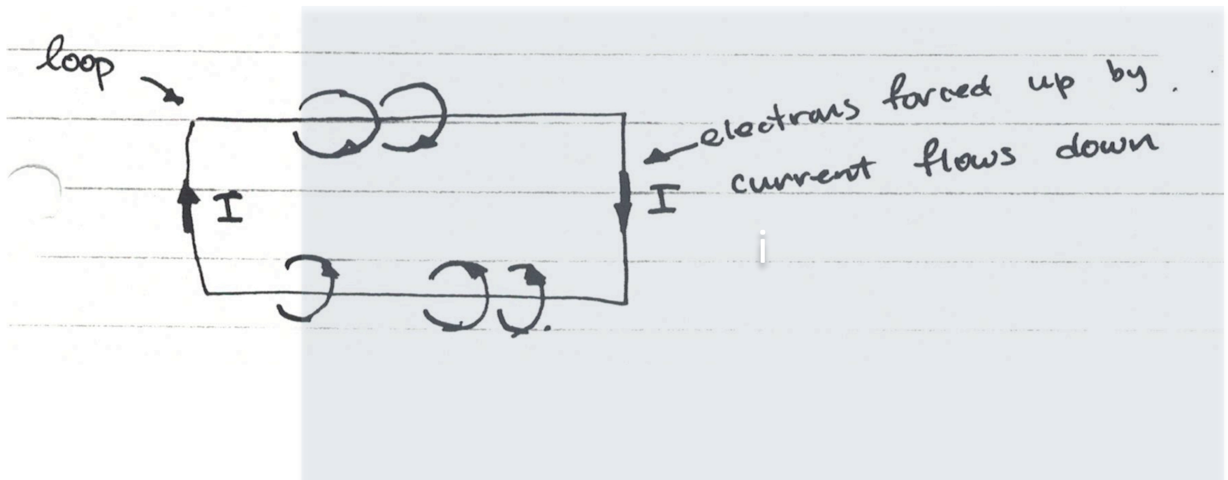
$$\left(\vec{\mathbf{E}}+\vec{\mathbf{v}}\times\vec{\mathbf{B}}\right)=0$$

$$\left(\vec{\mathbf{E}}+\vec{\mathbf{v}}\times\vec{\mathbf{B}}\right)_y=E_y-v_xB_0=0$$

Electrostatic field $\vec{\mathbf{E}}=-\nabla\phi,\quad \oint_{loop} d\vec{\mathbf{l}}\cdot\vec{\mathbf{E}}=0$

$$EMF=\oint_{loop} d\vec{\mathbf{l}}\cdot\left(\vec{\mathbf{v}}\times\vec{\mathbf{B}}\right)=-hvB_0$$

Case #3: Conducting Loop



$$EMF = \oint_{loop} d\vec{\mathbf{l}} \cdot \left(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}} \right) = 0$$

Induced currents keep flux constant