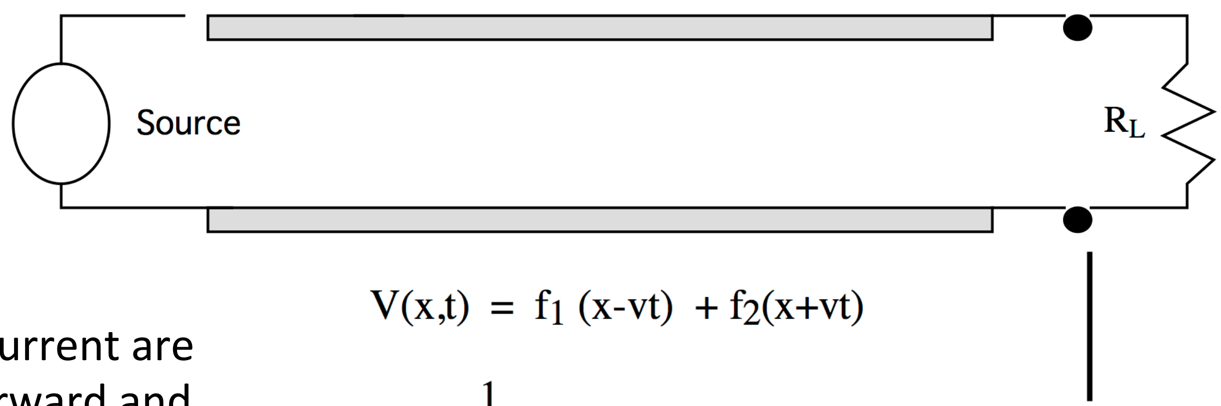
ENEE381

Signals on Transmission Lines

Transmission Line



$$V(x,t) = f_1(x-vt) + f_2(x+vt)$$

Voltage and current are the sum of forward and backward pulses

$$I(x,t) = \frac{1}{Z_0}\,[\,f_1(x-vt) - f_2(x+vt)\,],$$

$$I(0,t) = \frac{1}{Z_0}\,[\,f_1(-vt) - f_2(+vt)\,] = \frac{1}{R_L}\,[\,f_1(-vt) + f_2(+vt)\,] = \frac{V(0,t)}{R_L}$$

$$f_2(+vt) = \rho\, f_1(-vt),$$

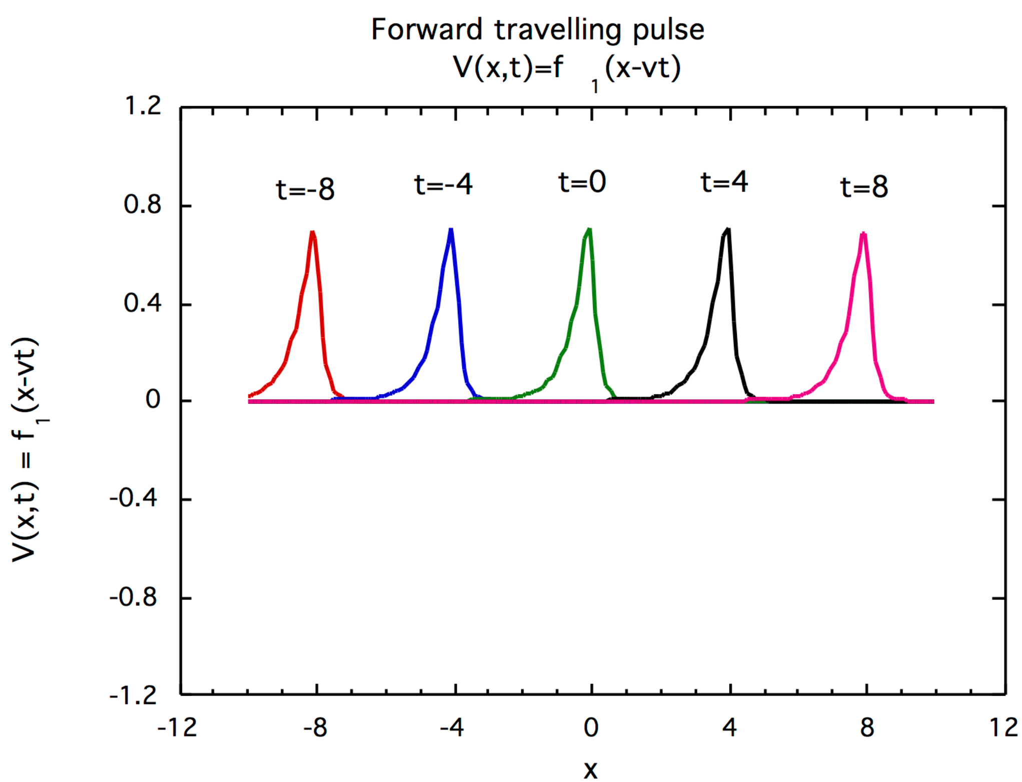
where the reflection coefficient ρ is given by,

$$\rho = \frac{R_L - Z_0}{R_L + Z_0}.$$

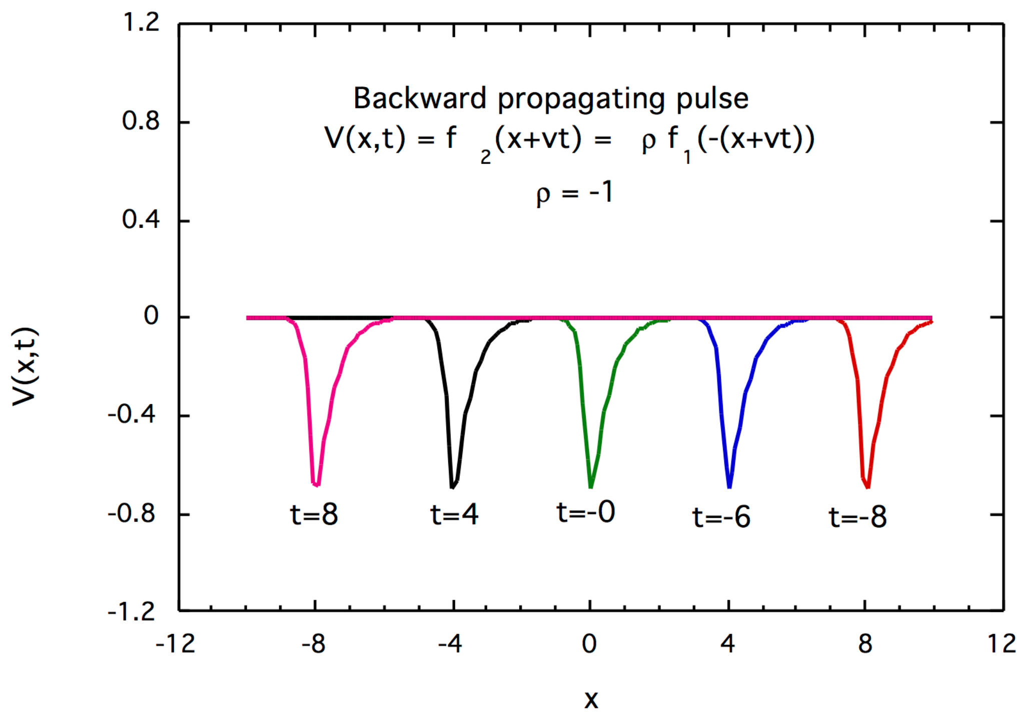
Forward Pulse

The incident pulse is chosen to be

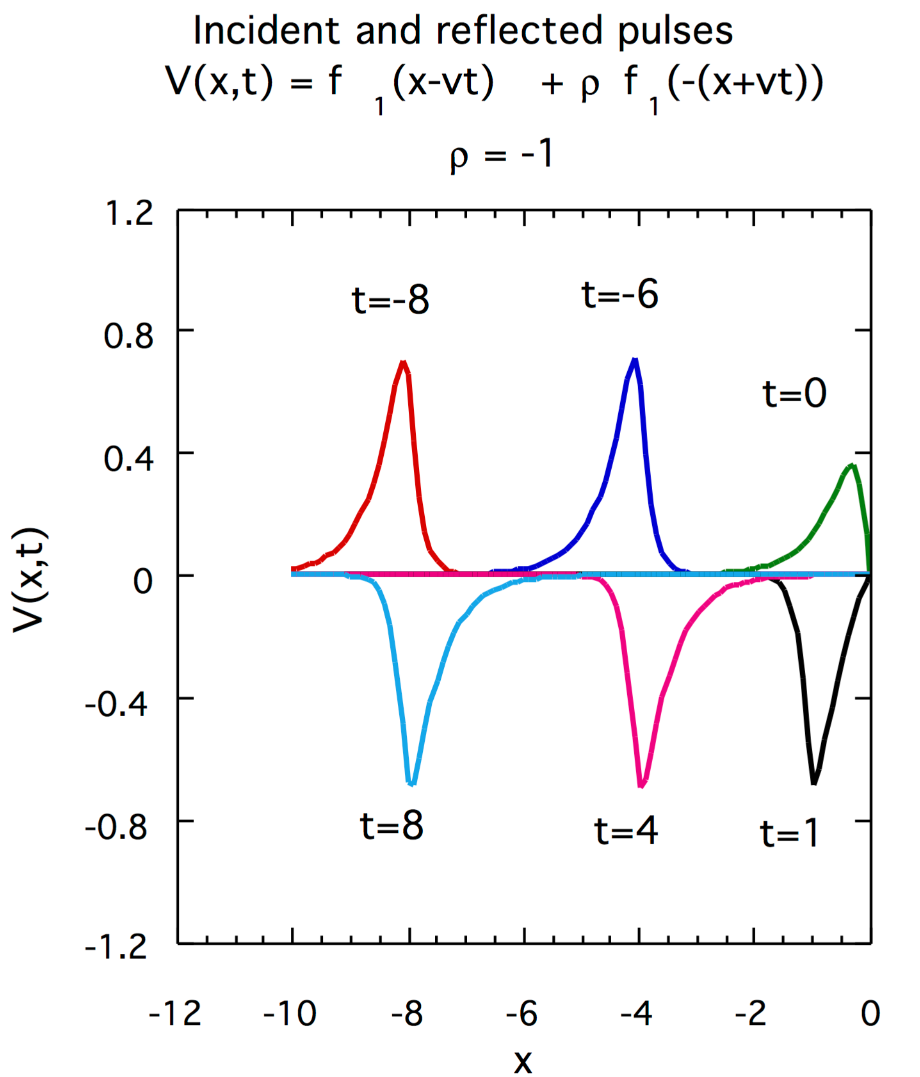
$$f_1(x) = \exp[\,-\,2x\;-\,4\sqrt{x^2 + .01}\;\;]$$



Backward Pulse



Forward + Backward

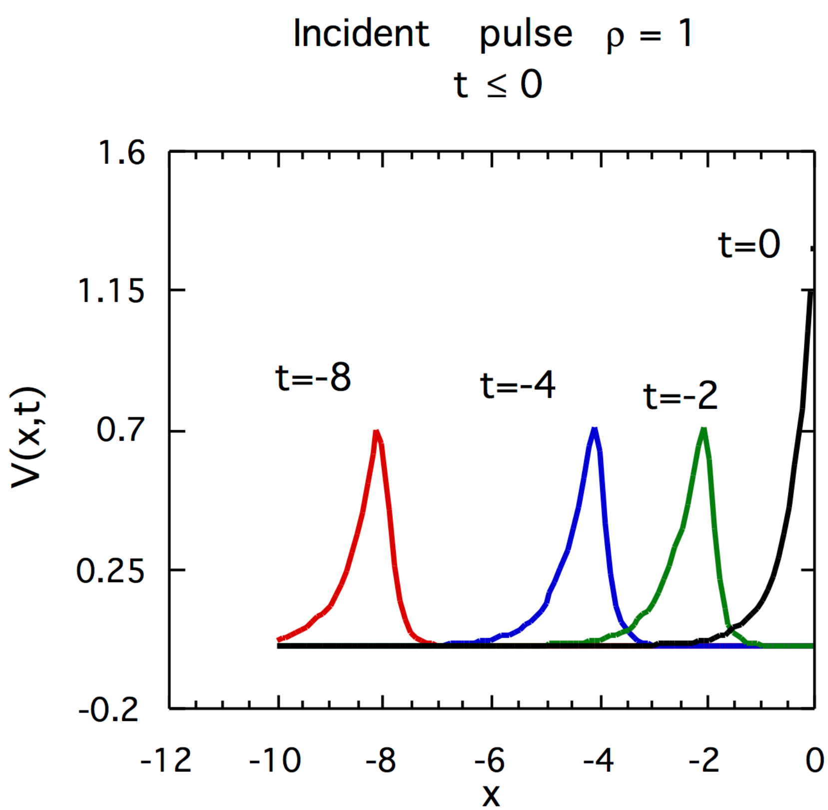


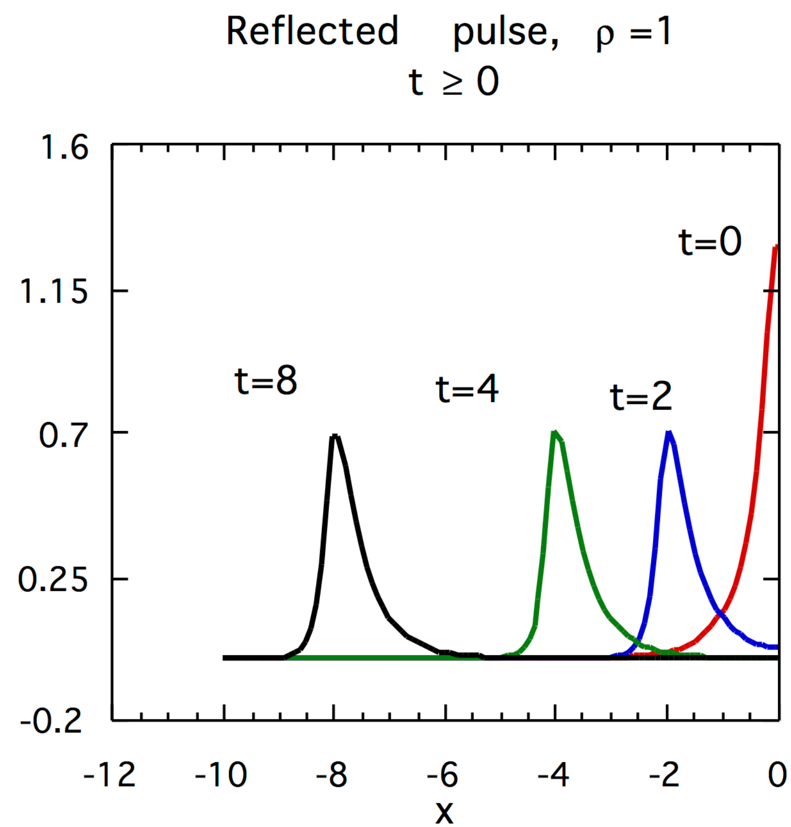
$$\rho = \frac{R_L - Z_0}{R_L + Z_0},$$

$$R_L = 0$$

$$\rho = -1$$

Forward + Backward



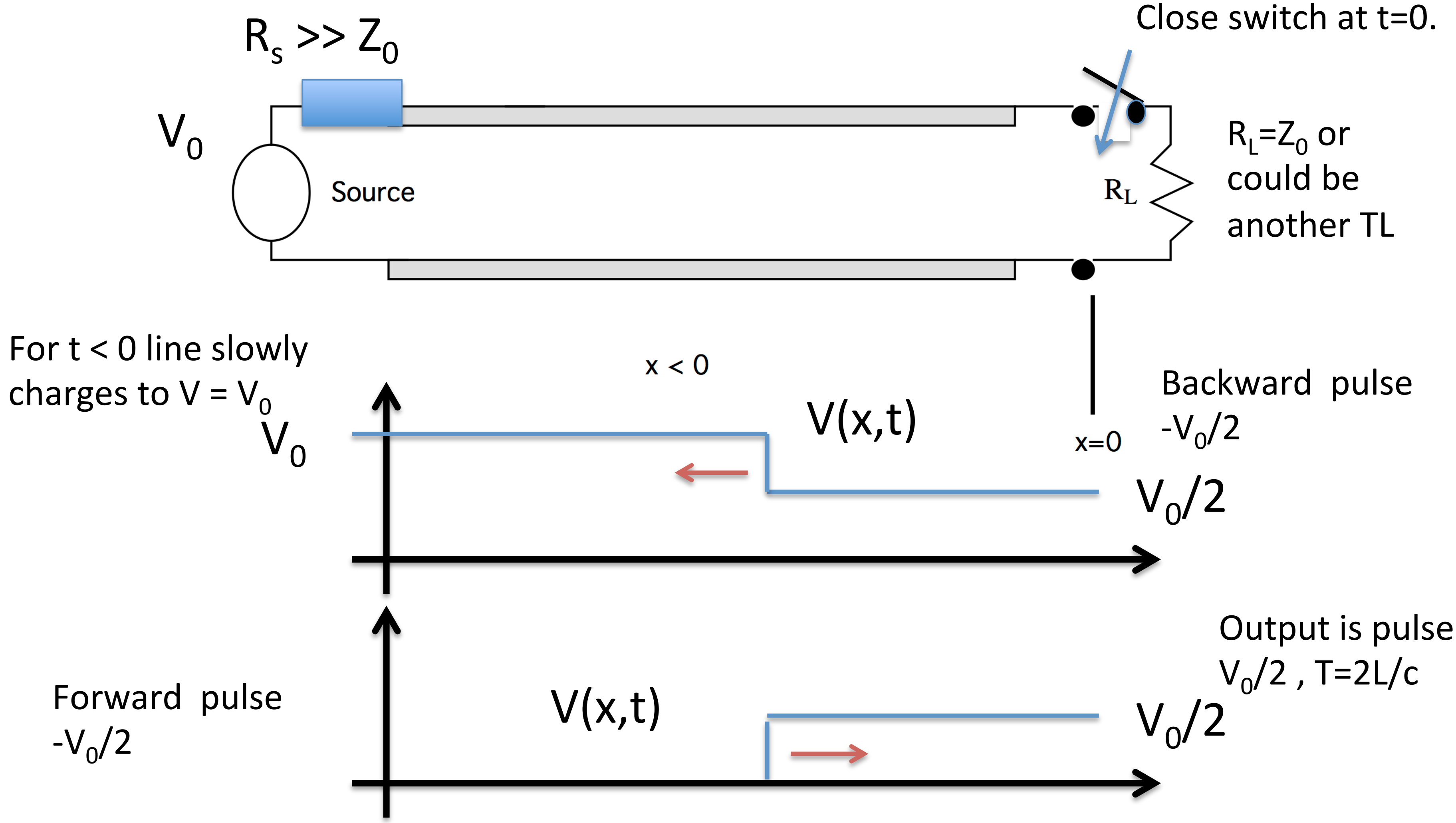


$$\rho = \frac{R_L - Z_0}{R_L + Z_0},$$

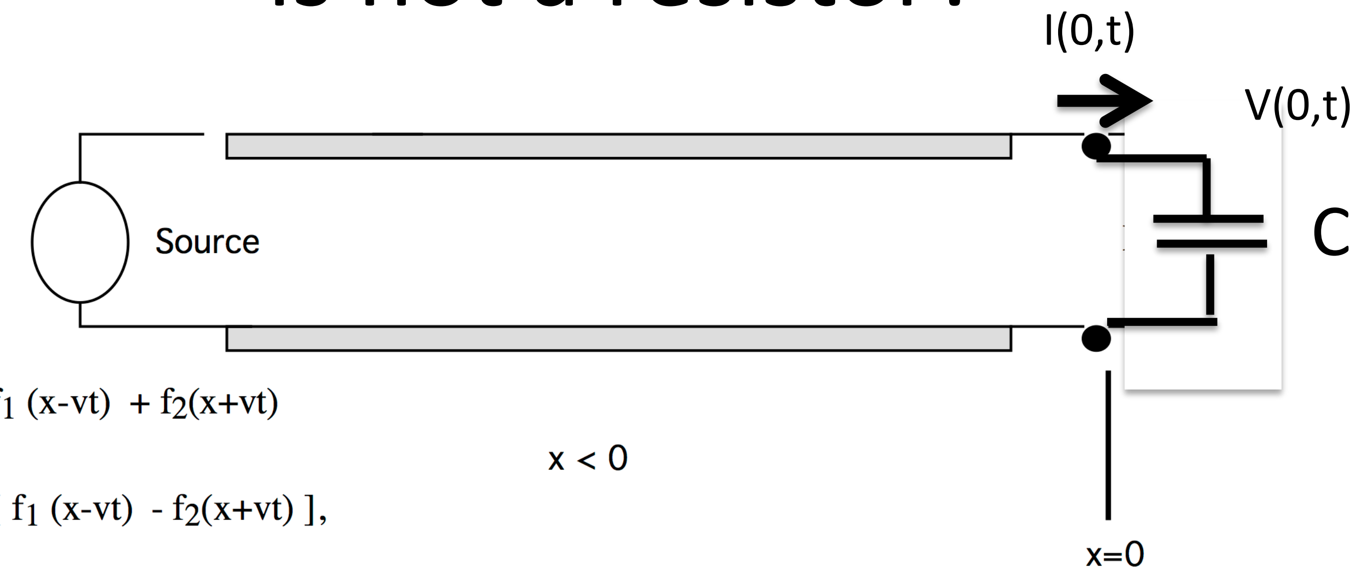
$$R_L = \infty$$

$$\rho = 1$$

Pulse Forming Network



What happens when the termination is not a resistor?



$$V(x,t) = f_1(x-vt) + f_2(x+vt)$$

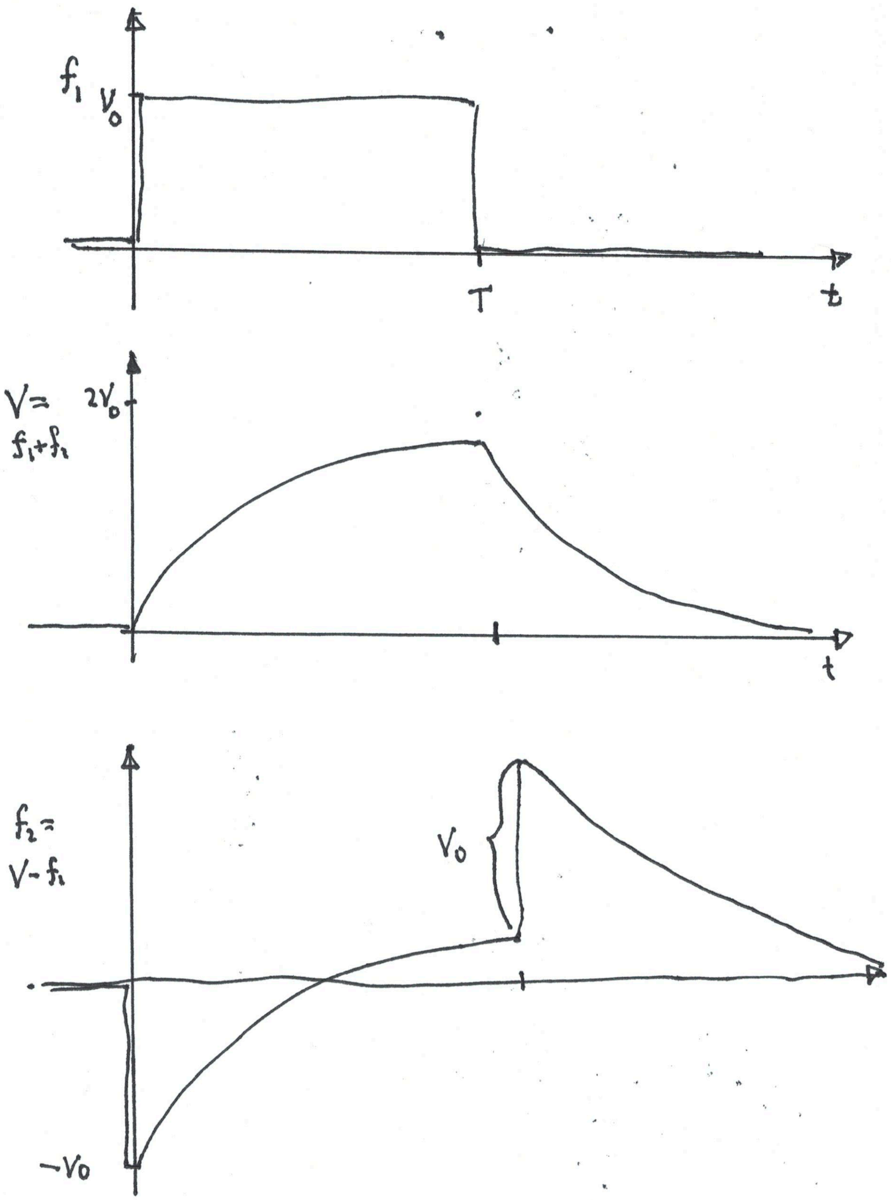
$$I(x,t) = \frac{1}{Z_0}[f_1(x-vt) - f_2(x+vt)],$$

At x = 0

$$I(0,t) = C\frac{dV(0,t)}{dt}$$

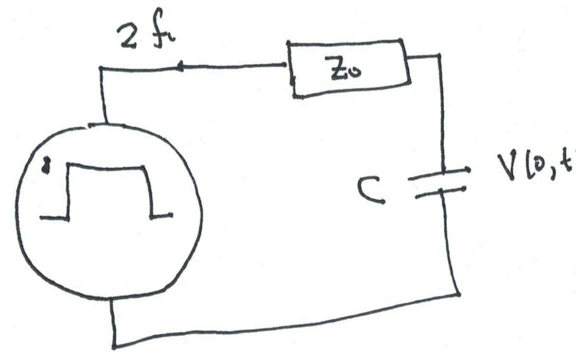
$$I(0,t) = \frac{1}{Z_0}(f_1 - f_2) = \frac{1}{Z_0}(2f_1 - (f_1 + f_2)) = \frac{1}{Z_0}(2f_1 - V(0,t))$$

$$2f_1 = V(0,t) + \tau\frac{dV(0,t)}{dt}, \qquad \tau = Z_0 C$$



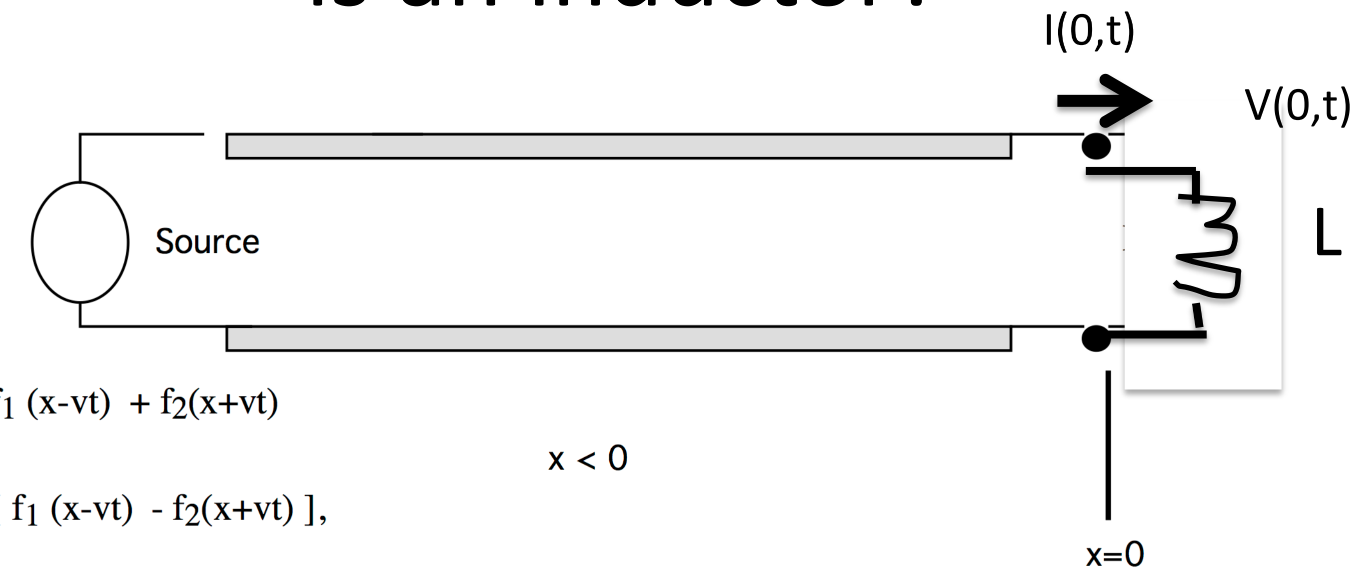
$$2f_1 = V(0,t) + \tau \frac{dV(0,t)}{dt}, \qquad \tau = Z_0 C$$

$$f_2 = V(0,t) - f_1$$



Capacitor initially acts like a short circuit, then transitions to an open circuit

What happens when the termination is an inductor?



$V(x,t) = f_1(x-vt) + f_2(x+vt)$

$I(x,t) = \frac{1}{Z_0}[f_1(x-vt) - f_2(x+vt)],$

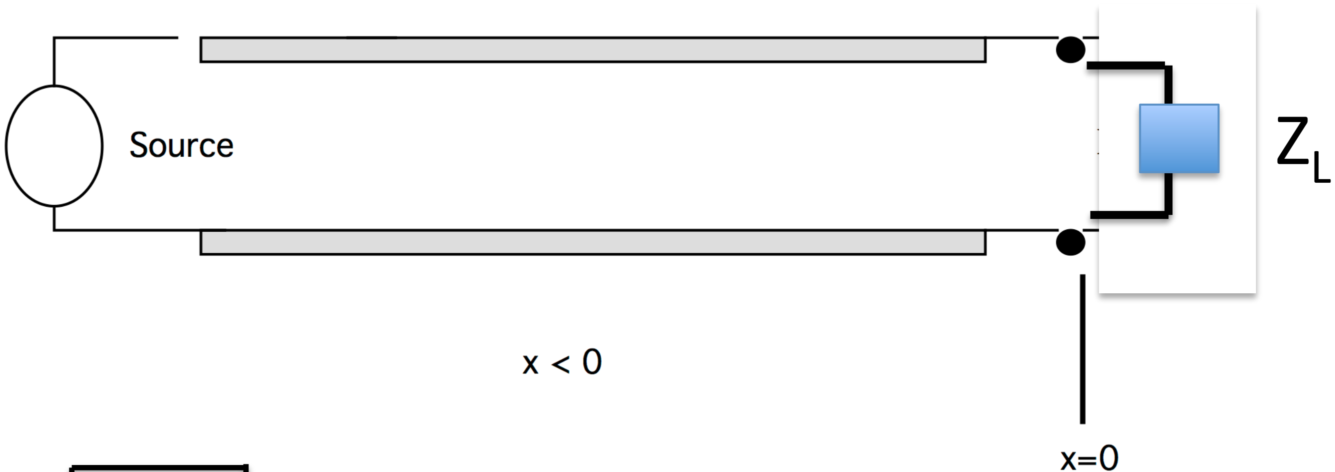
At x = 0

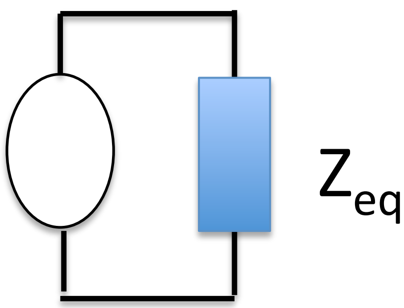
$$V(0,t) = L\frac{dI(0,t)}{dt}$$

$$V = f_1 + f_2 = 2f_1 - (f_1 - f_2) = 2f_1 - Z_0 I(0,t) = L\frac{dI(0,t)}{dt}$$

$$2f_1 = Z_0\left(I(0,t) + \tau\frac{dI(0,t)}{dt}\right), \qquad \tau = L/Z_0$$

What about AC signals?





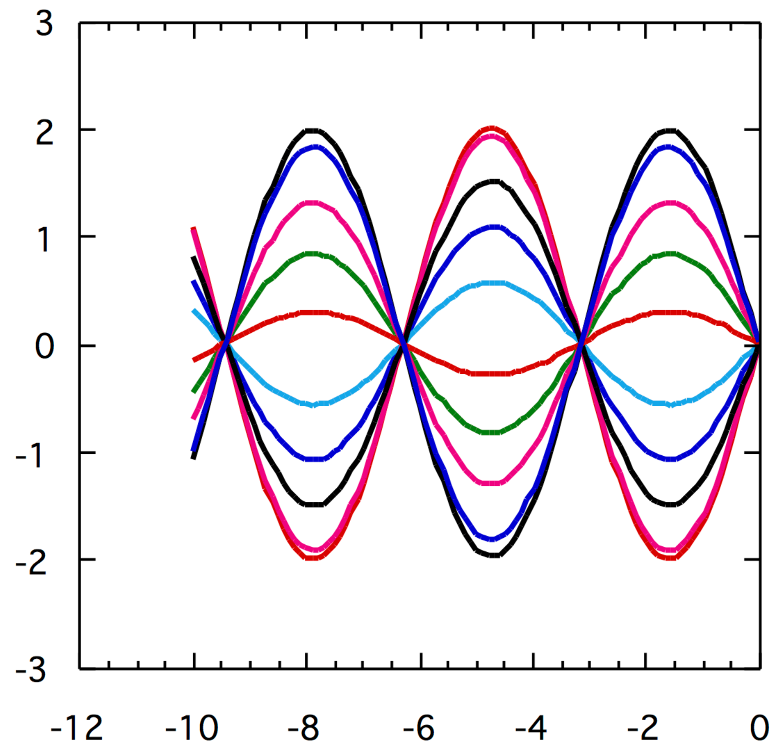
The impedance presented to the source is modified to Z_{eq}, Depends on the load, the length and the characteristic impedance of the line.

Standing Waves

$$V = \mathrm{Re}\left\{ \hat{V}_{inc} \left(e^{-jkz} + \rho e^{jkz} \right) e^{j\omega t} \right\}$$

$$I = \mathrm{Re}\left\{ \frac{\hat{V}_{inc}}{Z_0} \left(e^{-jkz} - \rho e^{jkz} \right) e^{j\omega t} \right\}$$

Plots of Ex(z,t) at different times



$$\rho = -1$$

Nodes are short circuits

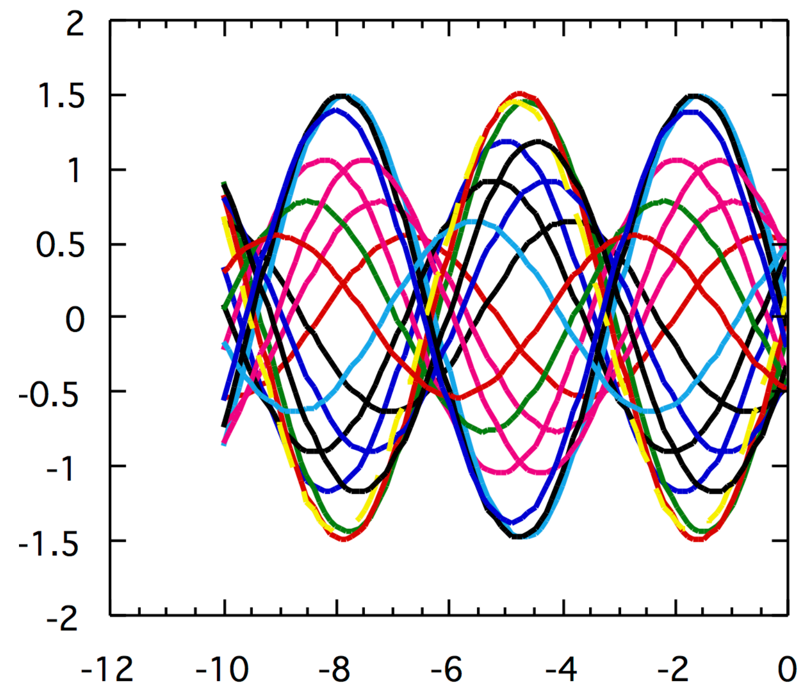
Anti nodes are open circuits

Standing Waves

$$V = \text{Re}\left\{\hat{V}_{inc}\left(e^{-jkz} + \rho e^{jkz}\right)e^{j\omega t}\right\}$$

$$I = \text{Re}\left\{\frac{\hat{V}_{inc}}{Z_0}\left(e^{-jkz} - \rho e^{jkz}\right)e^{j\omega t}\right\}$$

Plots of Ex(z,t) at different times

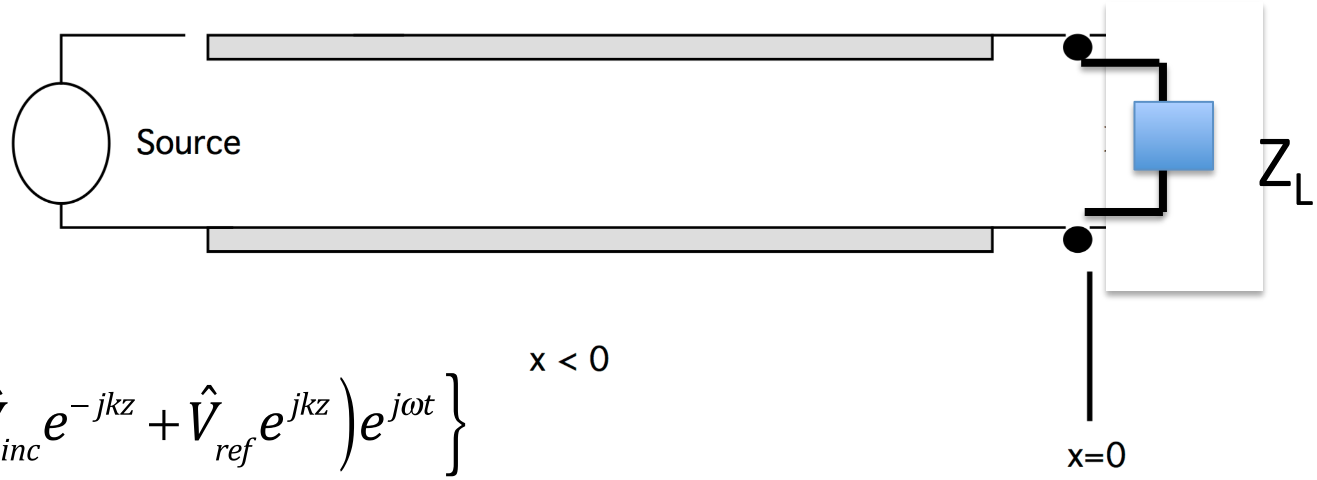


V(z,t)

Z [m]

$$\rho = -0.5$$

Impedance is complex and depends on where V/I is measured



$$V = \mathrm{Re}\left\{\left(\hat{V}_{inc}e^{-jkz} + \hat{V}_{ref}e^{jkz}\right)e^{j\omega t}\right\}$$

$$V = \mathrm{Re}\left\{\hat{V}_{inc}\left(e^{-jkz} + \rho e^{jkz}\right)e^{j\omega t}\right\} = \mathrm{Re}\left\{\hat{V}(z)e^{j\omega t}\right\}$$

$$I = \mathrm{Re}\left\{\frac{\hat{V}_{inc}}{Z_0}\left(e^{-jkz} - \rho e^{jkz}\right)e^{j\omega t}\right\} = \mathrm{Re}\left\{\hat{I}(z)e^{j\omega t}\right\}$$

The reflection coefficient at the load

$$\rho = \frac{Z_L - Z_0}{Z_L + Z_0}$$

At z = -L

$$Z_{eq} = \frac{\hat{V}(-L)}{\hat{I}(-L)} = Z_0 \left.\frac{\left(e^{-jkz} + \rho e^{jkz}\right)}{\left(e^{-jkz} - \rho e^{jkz}\right)}\right|_{z=-L} = Z_0 \frac{1 + \rho e^{-2jkL}}{1 - \rho e^{-2jkL}}$$

The reflection coefficient atz=-L

$$\rho(L) = \frac{Z_{eq} - Z_0}{Z_{eq} + Z_0} = \rho e^{-2jkL}$$

Equivalent Impedance

$$Z_{eq} = Z_0 \frac{1+\rho e^{-2jkL}}{1-\rho e^{-2jkL}}$$

$$Z_{eq} = Z_0 \frac{\left(Z_L + Z_0\right)e^{jkL} + \left(Z_L - Z_0\right)e^{-jkL}}{\left(Z_L + Z_0\right)e^{jkL} - \left(Z_L - Z_0\right)e^{-jkL}}$$

$$Z_{eq} = Z_0 \frac{Z_L \cos\left(kL\right) + jZ_0 \sin\left(kL\right)}{jZ_L \sin\left(kL\right) + Z_0 \cos\left(kL\right)}$$

If line has losses effect of reflections is diminished

$$k = k' - jk'', \quad e^{-2jkL} = e^{-2k''L} e^{-2jk'L}$$

$$Z_{eq} = Z_0 \frac{1+\left(\rho e^{-2k''L}\right)e^{-2jk'L}}{1-\left(\rho e^{-2k''L}\right)e^{-2jk'L}}$$

$$\text{If L} \rightarrow 0, \ \sin(\text{kL}) \rightarrow 0, \ \cos(\text{kL}) \rightarrow 1, \qquad Z_{eq} \rightarrow Z_L$$

$$\text{If kL} \rightarrow \frac{\pi}{2}, \ \sin(\text{kL}) \rightarrow 1, \ \cos(\text{kL}) \rightarrow 0 \qquad Z_{eq} \rightarrow Z_0^2 / Z_L$$

Special Cases

$$Z_{eq} = Z_0 \frac{1+\rho e^{-2jk_1 L}}{1-\rho e^{-2jk_1 L}}$$

$$L = n\lambda/2, \quad 2k_1 L = 2\pi n \qquad Z_{eq} = Z_0 \frac{1+\rho}{1-\rho} = Z_L$$

Remember half-wave window

$$L = \left(n + \frac{1}{2}\right)\lambda/2, \quad 2k_1 L = 2\pi\left(n + \frac{1}{2}\right) \qquad Z_{eq} = Z_0 \frac{1-\rho}{1+\rho} = Z_0^2 / Z_L$$

Quarter-wave transformer

Admittance vs Impedance

$$I = \mathrm{Re}\left\{\left(\hat{I}_{inc} e^{-jkz} + \hat{I}_{ref} e^{jkz}\right) e^{j\omega t}\right\}$$

$$V = \mathrm{Re}\left\{Z_0 \left(\hat{I}_{inc} e^{-jkz} - \hat{I}_{inc} e^{jkz}\right) e^{j\omega t}\right\}$$

Same formulas apply to current amplitudes.

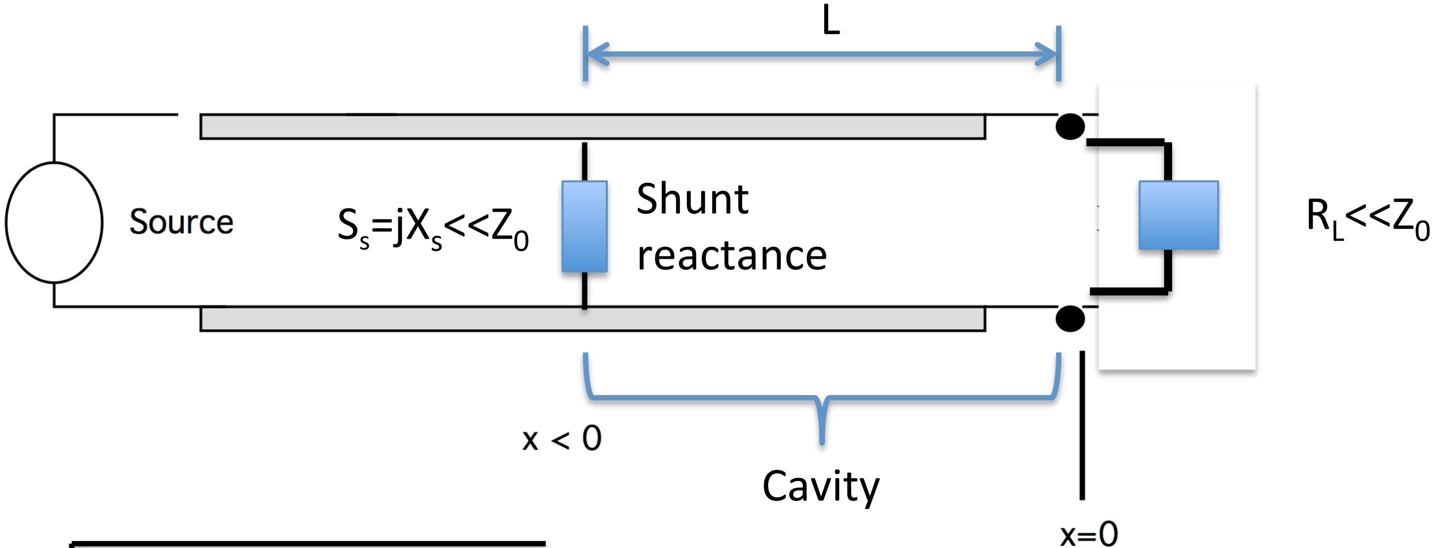
$$I = \mathrm{Re}\left\{\hat{I}_{inc}\left(e^{-jkz} + \rho_I e^{jkz}\right) e^{j\omega t}\right\} = \mathrm{Re}\left\{\hat{I}(z) e^{j\omega t}\right\}$$

$$V = \mathrm{Re}\left\{Z_0 \hat{I}_{inc}\left(e^{-jkz} - \rho_I e^{jkz}\right) e^{j\omega t}\right\} = \mathrm{Re}\left\{\hat{V}(z) e^{j\omega t}\right\}$$

Current reflection coefficient = - voltage reflection coefficient

$$\rho_I = -\rho \quad \rho_I = -\frac{Z_L - Z_0}{Z_L + Z_0} = \frac{Y_L - Y_0}{Y_L + Y_0}$$

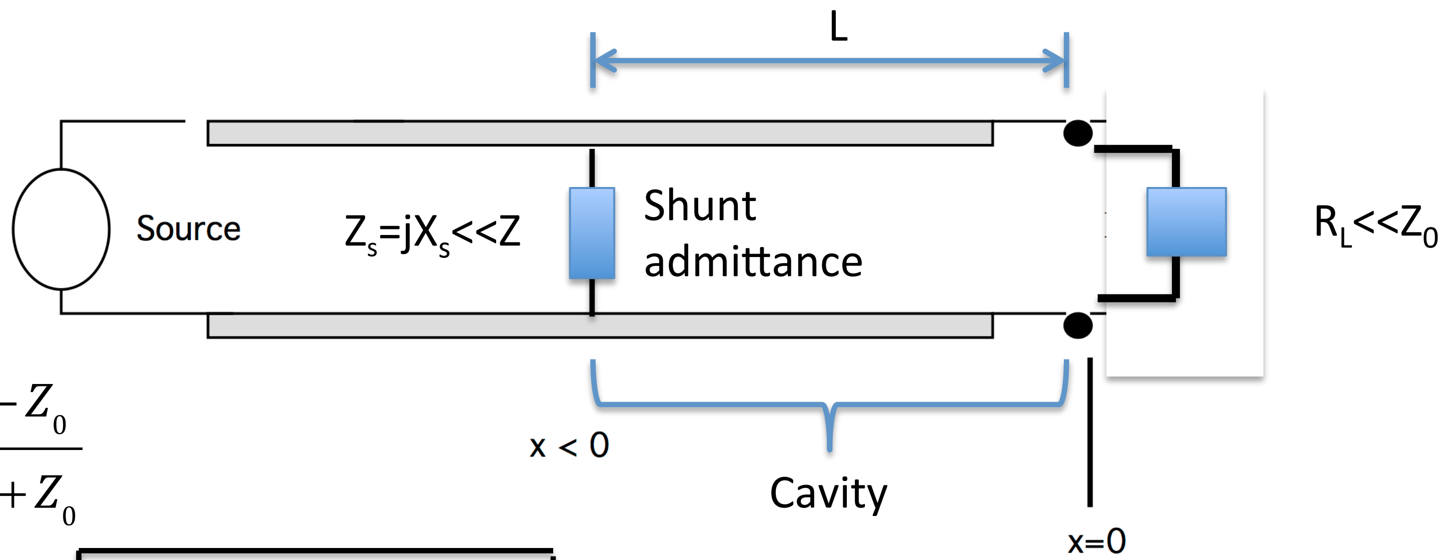
High Q cavity model



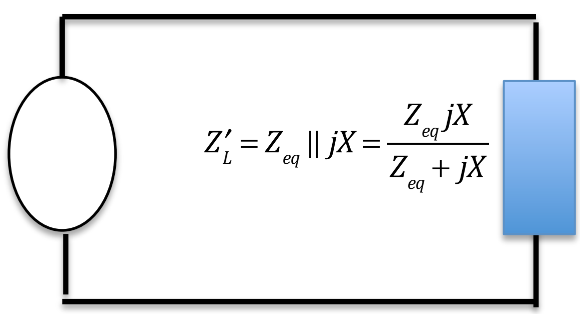
$$Z'_L = Z_{eq} \parallel jX = \frac{Z_{eq}\, jX}{Z_{eq} + jX}$$

$$\rho == \frac{Z'_{eq} - Z_0}{Z'_{eq} + Z_0}$$

High Q Cavity Model



$$\rho_{cav} = \frac{\left(Z_{eq} \parallel jX_s\right) - Z_0}{\left(Z_{eq} \parallel jX_s\right)_s + Z_0}$$



$$Z_{eq} = Z_0 \frac{R_L \cos(kL) + jZ_0 \sin(kL)}{jR_L \sin(kL) + Z_0 \cos(kL)} \simeq jZ_0 \tan(kL) + R_L$$

$$\rho_{cav} \simeq -\frac{j\left(Z_0 \tan(kL) + X_s\right) + R_L - X_s^2 / Z_0}{j\left(Z_0 \tan(kL) + X_s\right) + R_L + X_s^2 / Z_0} = -\frac{j2\left(\omega - \omega_c\right)/\omega_c + Q_{\text{int}}^{-1} - Q_{ext}^{-1}}{j2\left(\omega - \omega_c\right)/\omega_c + Q_{\text{int}}^{-1} + Q_{ext}^{-1}}$$