Adjoint Methods in Plasma Physics and Charged Particle Dynamics

or

When the solution to your problem
is not your problem.

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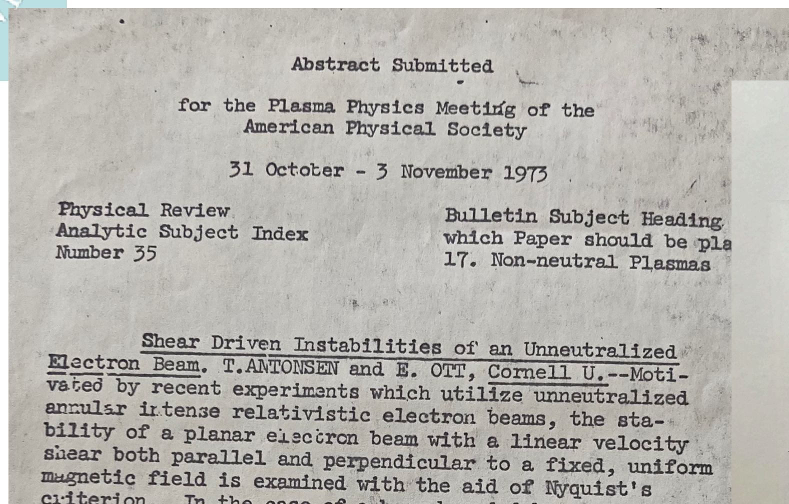
First, Some Nostalgia – TMA-DPP 50th


Abstract Submitted

for the Plasma Physics Meeting of the
American Physical Society

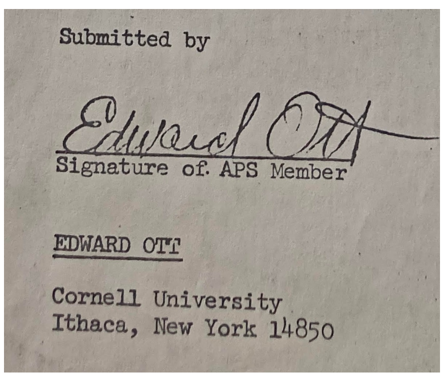
31 October - 3 November 1973

Physical Review
Analytic Subject Index
Number 35

Bulletin Subject Heading
which Paper should be pla
17. Non-neutral Plasmas

Shear Driven Instabilities of an Unneutralized Electron Beam. T.ANTONSEN and E. OTT, Cornell U.--Motivated by recent experiments which utilize unneutralized annular intense relativistic electron beams, the stability of a planar electron beam with a linear velocity shear both parallel and perpendicular to a fixed, uniform magnetic field is examined with the aid of Nyquist's criterion. In the case of a beam bounded by cond

Submitted by
my advisor

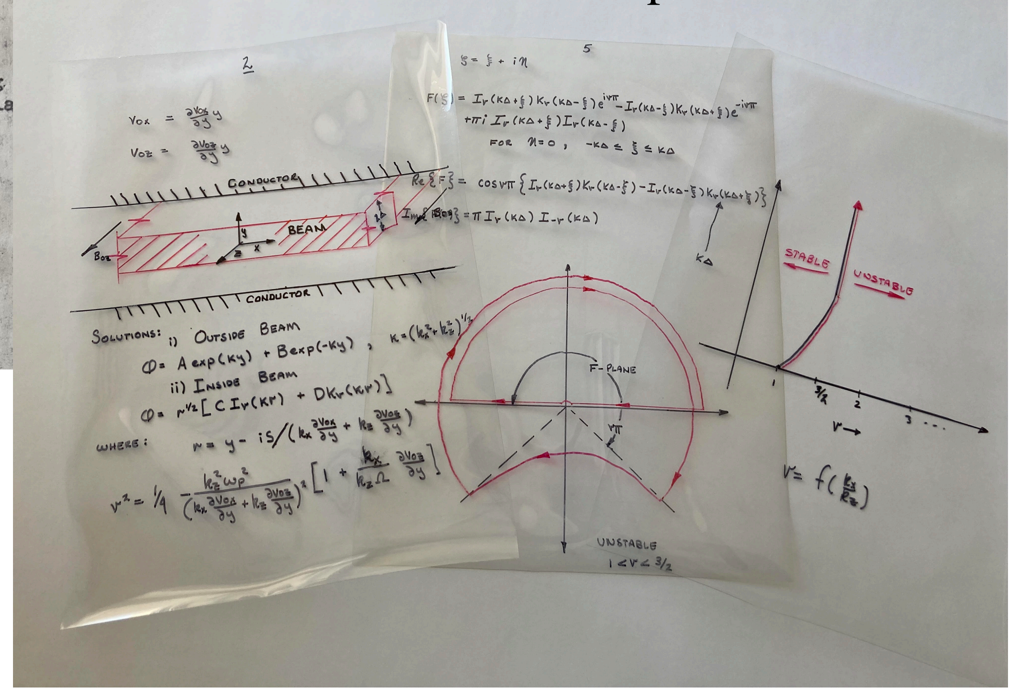

Submitted by

Edward Ott
Signature of APS Member

EDWARD OTT

Cornell University
Ithaca, New York 14850

These are called transparencies



Adjoint Method: What it does

- Efficiently finds the dependence of system performance on parameters by solving a system problem different from the one proposed. - **Adjoint Problem**

- Requires identification of a **Metric** or **Figure of Merit (FoM)**

$$\boldsymbol{F}(\mathbf{a}) \qquad \mathbf{a} \text{ list of parameters}$$

or

$$\mathbf{d}\,\boldsymbol{F}(\mathbf{a})/\mathbf{d}\,\mathbf{a}$$

- Solution of adjoint problem gives ***F*** or d***F***/**d** ***a***

Useful for:

Optimization

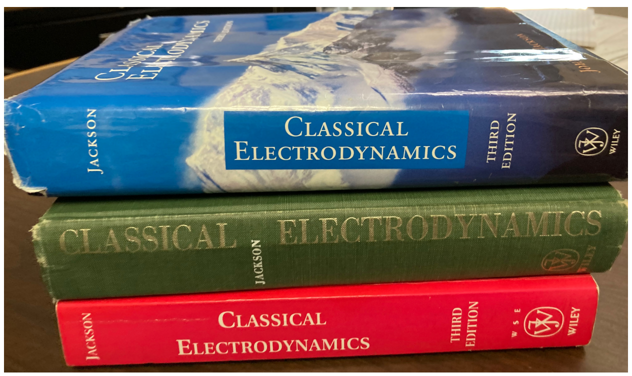
Sensitivity Studies

Wikipedia
The Free Encyclopedia

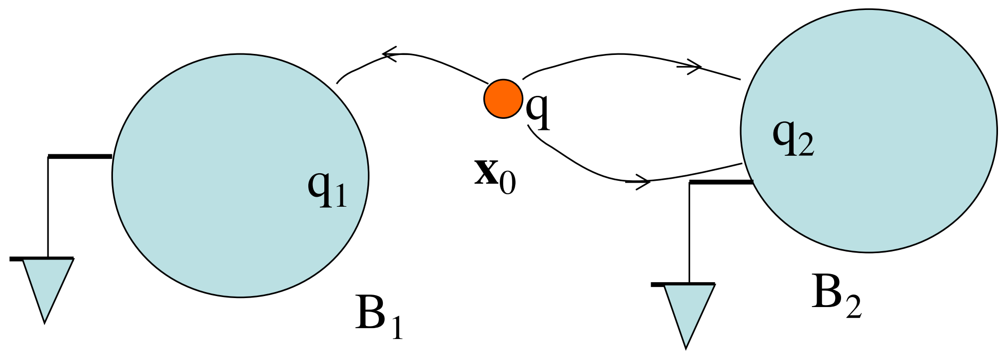
Basic Adjoint Example (More Nostalgia)

Jackson, Classical Electrodynamics Problems 1.12 and 1.13

The book is notorious for the difficulty of its problems, and its tendency to treat non-obvious conclusions as self-evident.[4][6] A 2006 survey by the American Physical Society (APS) revealed that 76 out of the 80 U.S. physics departments surveyed require all first-year graduate students to complete a course using the third edition of this book.[6][7]



A charge q is placed at an arbitrary point, $\mathbf{x}_o$, relative to two grounded, conducting electrodes.



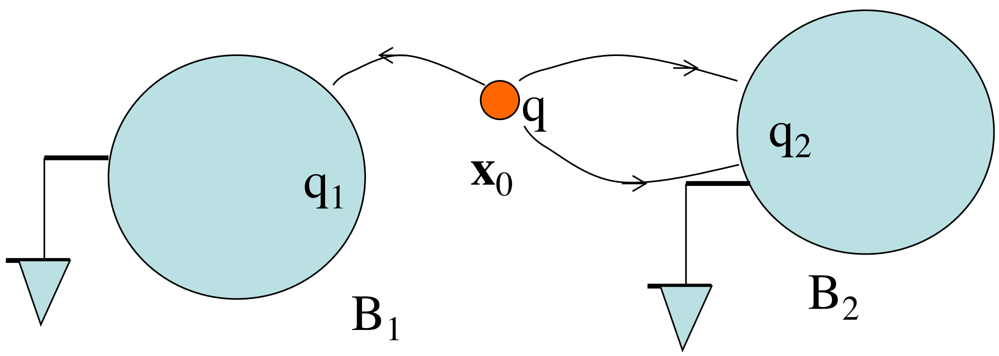
What is the charge q_1 on the surface of electrode 1?

q_1 **Figure of Merit**
$\mathbf{x}_0$ **Parameters**

<u>Now Repeat for different $\mathbf{x}_0$</u>

Direct Solution

Prob #1	Solve	BCs:	answer	
Your Problem	$\nabla^2\phi = -q\delta(\mathbf{x}-\mathbf{x}_0)$	$\phi\vert_{B1} = \phi\vert_{B2} = 0$ $\phi(x\to\infty)=0$	$q_1 = \int_{B1} d^2x\, \mathbf{n}\cdot\nabla\phi$	Repeated for each $\mathbf{x}_0$



* Must first find potential throughout space.
* Then evaluate E-field on surface of B_1.
* If $\mathbf{x}_0$ changes everything must be redone.

Adjoint Solution – Green's Reciprocation Theorem

<u>Prob #2</u>
Adjoint Problem
(Not your problem)

Solve

$$\nabla^2 \psi = 0$$

BCs:

$$\psi\big|_{B1} = 1,$$
$$\psi\big|_{B2} = \psi(x \to \infty) = 0$$

<u>Done once</u> !

Apply Green's Theorem

$$\int_V d^3x \left(\psi \nabla^2 \phi - \phi \nabla^2 \psi \right) = \int_S d^2x\, n \cdot \left(\psi \nabla \phi - \phi \nabla \psi \right)$$

$\nabla^2 \psi \to 0$, $\quad \phi \nabla \psi \to 0$

$$\nabla^2 \phi = -q\delta(\mathbf{x} - \mathbf{x}_0)$$

When the dust settles:

$$-q\psi(\mathbf{x}_0) = q_1$$

Answer

George Green 1793-1841

The Green of Green Functions

In 1828, an English miller from Nottingham published a mathematical essay that generated little response. George Green's analysis, however, has since found applications in areas ranging from classical electrostatics to modern quantum field theory.

Lawrie Challis and Fred Sheard Physics Today Dec. 2003

his family built a house next to the mill, Green spent most of his days and many of his nights working and indeed living in the mill. When he was 31, Jane Smith bore him a daughter. They had seven children in all but never married. It was said that Green's father felt that Jane was not a suitable wife for the son of a prosperous tradesman and landowner and threatened to disinherit him.

- Born in Nottingham (Home of Robin Hood)
- Father was a baker
- At age 8 enrolled in Robert Goodacre's Academy
- Left after 18 months (extent of formal education pre age 40)
- Worked in bakery for 5 years
- Sent by father to town mill to learn to be a miller

- Fell in love with Jane, the miller's daughter.
- Green's father forbade marriage.
- Green had 7 children with Jane.
- Took up mathematics.
- Self published work in 1828
- With help, entered Cambridge 1833, graduated 1837.
- Theory of elasticity, refraction, evanescence
- "Discovered" by Lord Kelvin in 1840.
- Died of influenza, 1841

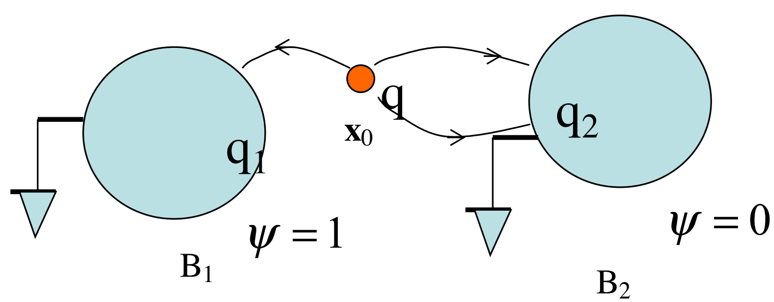
Green's Mill: still functions

Features of Problems Suited to an Adjoint Approach

1. Many computations need to be repeated.
   (many different locations of charge, q)

2. Only a limited amount of information about the solution is required.
   (only want to know charge on electrode #1)

Trick is to find the right combination of conditions on problem 2 to solve problem 1.



Adjoint Methods in Science and Engineering

E3232 *Journal of The Electrochemical Society*, **164** (11) E3232-E3242 (2017)

JES Focus Issue on Mathematical Modeling of Electrochemical Systems at Multiple Scales in Honor of John Newman

Adjoint Method for the Optimization of the Catalyst Distribution in Proton Exchange Membrane Fuel Cells

James Lamb,[a,b] Grayson Mixon,[a,b] and Petru Andrei[a,b,*,z]

[a]*Department of Electrical and Computer Engineering, Florida A&M University-Florida State University College of Engineering, Tallahassee, Florida 32310, USA*
[b]*Aero-Propulsion, Mechatronics and Energy Center, Florida State University, Tallahassee, Florida 32310, USA*

Adjoint method for the optimization of insulated gate bipolar transistors

Cite as: AIP Advances 9, 095301 (2019); doi: 10.1063/1.5113764
Submitted: 7 June 2019 • Accepted: 23 August 2019 •
Published Online: 3 September 2019

Adjoint shape optimization applied to electromagnetic design

Christopher M. Lalau-Keraly,[1,*] Samarth Bhargava,[1] Owen D. Miller,[2] and Eli Yablonovitch[1]

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RESEARCH | OPEN ACCESS

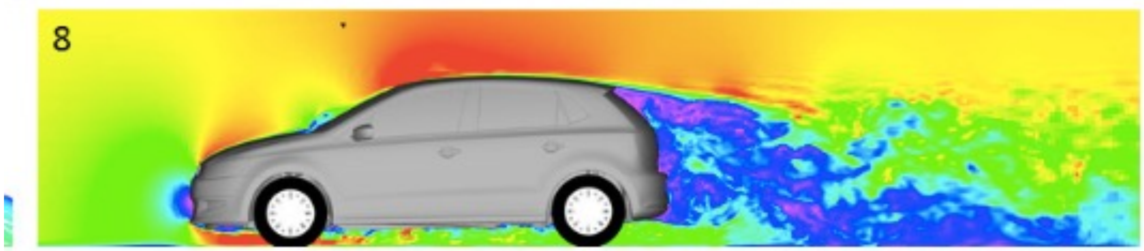
Courtesy, Elizabeth Paul

Adjoint methods for car aerodynamics

Carsten Othmer

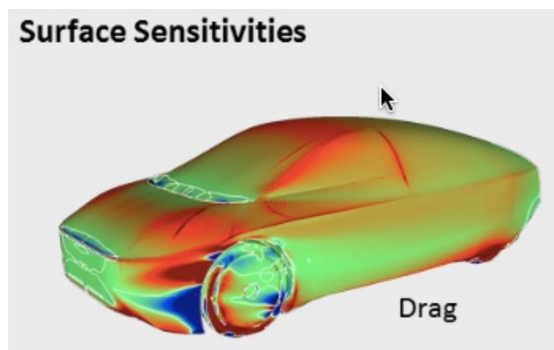
Journal of Mathematics in Industry 2014 4:6 | **DOI:** 10.1186/2190-5983-4-6 |

Received: 30 March 2013 | **Accepted:** 5 March 2014 | **Published:** 3 June 2014



Optimize shape via steepest descent to minimize drag.

Super Computer



Result is also aesthetically appealing.

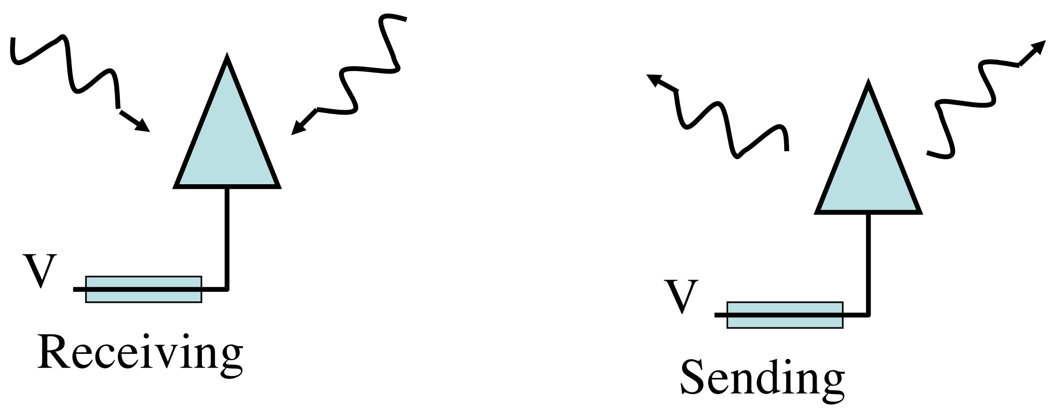
1985 Volvo 240 DL

Oops, coding error ? Possible local minimum?

Relation to Reciprocity

Example:

- Antenna sending and receiving radiation patterns are equal due to time reversal symmetry of Maxwell's Equations.
- Direct calculation of receiving pattern requires many simulations
- Instead, calculate sending pattern and invoke reciprocity



Other Examples of Reciprocity

Collisional Transport: Onsager Symmetry of off-diagonal elements of transport matrices.

Temperature gradient	➔	Electric current
Electric field	➔	Heat flux

<u>Neoclassical Tokamak Transport</u>

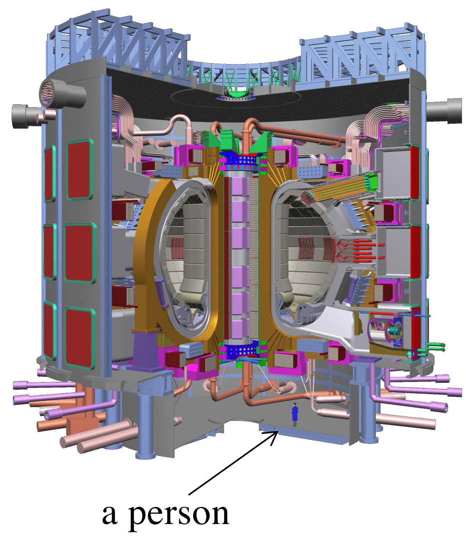
Pressure gradient	➔	Bootstrap Current
Toroidal E-field	➔	Ware particle flux

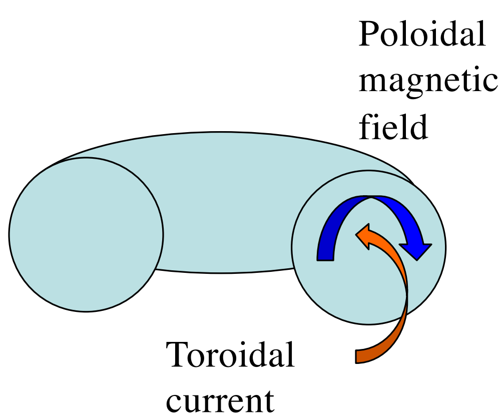
F. Hinton, and R. Hazeltine, Rev. Mod Phys, 48 (2) , 1976

RF Current Drive in Fusion Plasmas

Magnetic Confinement: ITER

US-EU-Russia-Japan-India Collaboration
Will be built in Cardarache France
Completion 2016??
http://www.iter.org/





Injecting RF waves can drive a toroidal current. N. Fisch PRL (1978)

RF Current Drive Efficiency

Original Langevin Treatment: Nat Fisch, PRL (1978)

RF pushes particles in velocity space.

$\vec{\Gamma}$ = RF induced velocity space particle flux

Collisions relax distribution back to equilibrium.

What is the current generated per unit power dissipated? J/P_D

$$J = \int d^3v\, \vec{\Gamma} \cdot \frac{\partial}{\partial v}\Psi \qquad P_D = \int d^3v\, \vec{\Gamma} \cdot \frac{\partial}{\partial v}\varepsilon$$ - energy

ψ sensitivity function, inversely proportional to collision rate

RF Current Drive Efficiency

RF pushes particles in velocity space. $\vec{\Gamma}$

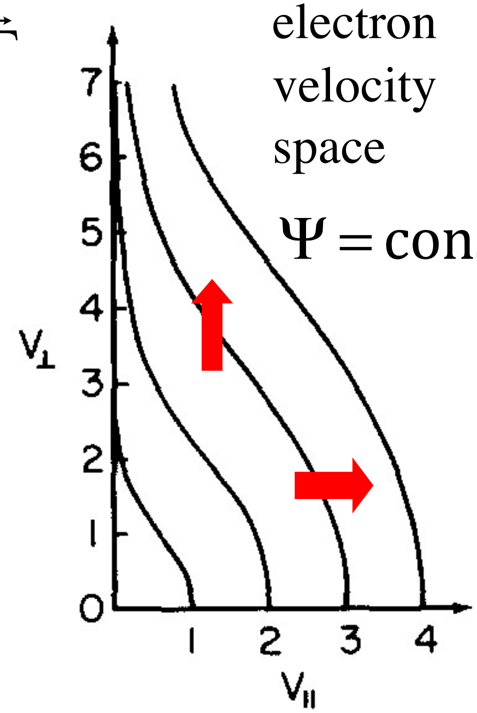
Collisions relax distribution back to equilibrium.

$$J = \int d^3v \, \vec{\Gamma} \cdot \frac{\partial}{\partial v} \Psi$$

N. Fisch: Current generated in parallel direction even if push is in perpendicular direction

electron velocity space

$\Psi = \text{const.}$



antisymmetric in $v_{\parallel}$

Adjoint Approach:

S. Hirshman, PoF, 23, 1238 (1980),
TMA and KR Chu, PoF 25, (1982),
M. Taguchi, J. Phys. Soc. Jpn (1982)

For a Homogeneous Plasma, we want to solve steady state kinetic equation

$$\frac{\partial f}{\partial t} = 0 = C(f) - \frac{\partial}{\partial \mathbf{v}} \cdot \Gamma$$

Problem #1

Linearized collision operator — RF induced velocity space flux - **parameters**

Then find parallel current **FoM** $J_{\|} = -e \int d^3v \, v_{\|} f$

Adjoint problem: Spitzer-Harm $g(v_{\perp}, v_{\|})$
Distribution function driven by a DC electric field.

$$eE_{\|} \frac{\partial f}{\partial v_{\|}} \sim -e v_{\|} f_M = C(g)$$

Problem #2

Parallel current

$$J_{\|} = \int d^3v \, \Gamma \cdot \frac{\partial}{\partial \mathbf{v}} \left(\frac{g}{f_M} \right)$$

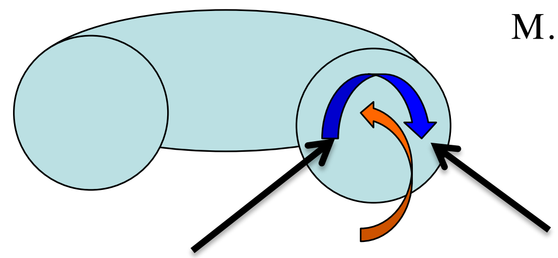
Toroidal Geometry Makes a Difference,

Streaming

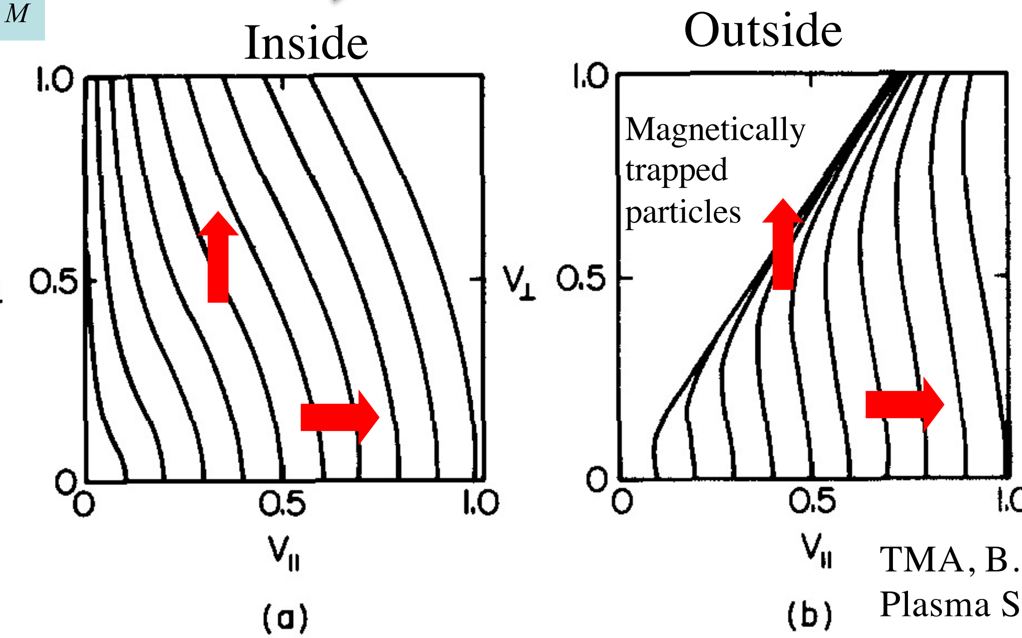
$$v_{\parallel}\mathbf{b}\cdot\nabla g - e v_{\parallel} f_M = C(g)$$

$$J = \int \frac{dl}{B} d^3v\, \Gamma \cdot \frac{\partial}{\partial v} \, g\Big/ f_M$$

TMA and KR Chu, PoF 25, (1982),
M. Taguchi, J. Phys. Soc. Jpn (1982)



$\Psi = \text{const.}$



TMA, B. Hui, IEEE Trans Plasma Sci., 1986

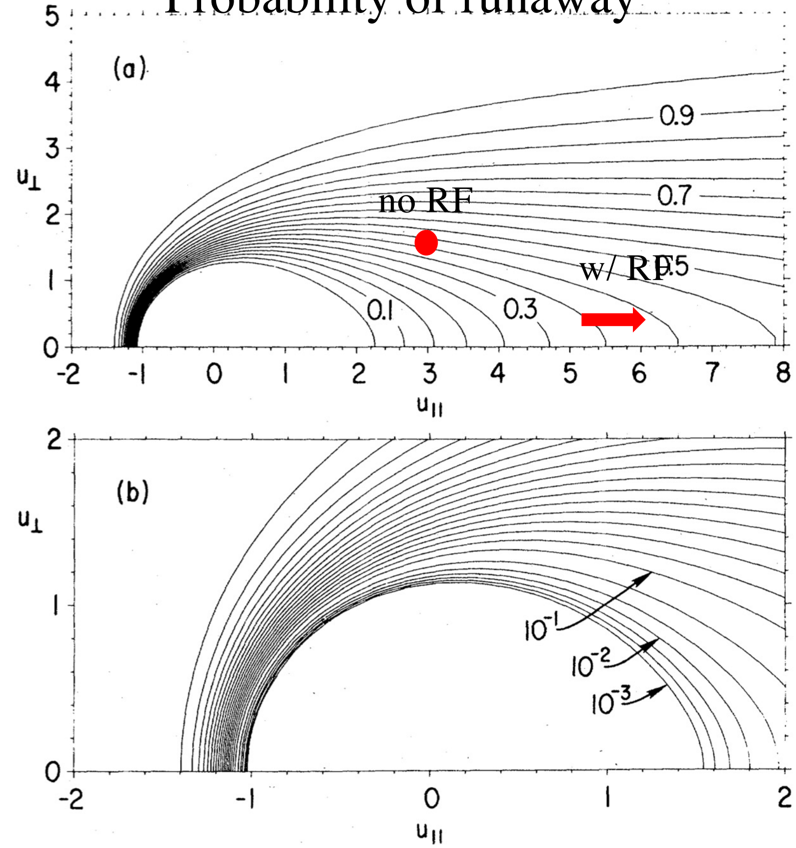
Toroidal effects
Ohkawa, T., 1976, General Atomic Company Report No. A1384
Parks, P. B., and F. B. Marcus, 1981, Nucl. Fusion 21, 1207

Extensions to Energetic Particles– Fisch and Karney

Energetic Particles lead to dynamic distribution functions

1. Probability of runaway (no RF)
2. Runaway rate with RF
3. Energy flow to stored poloidal field

Probability of runaway



u - normalized to Dreicer velocity

Fisch, N. J., 1985a, Phys. Fluids 28, 245.
Fisch, N. J., and C. F. F. Karney, 1985b, Phys. Fluids 28, 3107.
Karney, C. F. F., and N. J. Fisch, 1986, Phys. Fluids 29, 180.
Fisch, Reviews of Modern Physics, Vol. 59, No. 1, January 1987

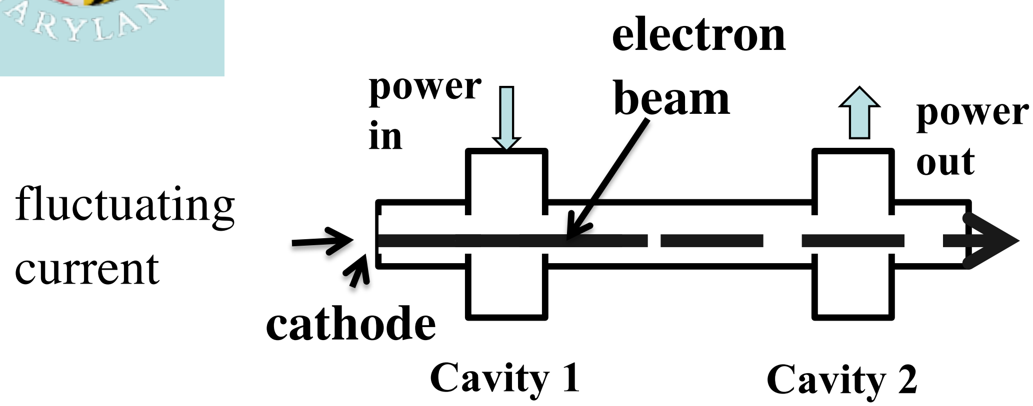
Recent Adjoint Approaches

- Shot noise on gyrotron beams, TMA, W. Manheimer and A. Fliflet, PoP (2001).
- Beam optics sensitivity function, TMA, D. Chernin, J. Petillo, Phys. Plasmas 26, 013109 (2019);
- Stellarator Optimization and Sensitivity, E. Paul, M. Landreman, TMA, *J. Plasma Phys.* (2019), v*ol*. 85, 905850207, *J. Plasma Phys.* (2021), v*ol*. 87, 905870214
- Optimization of Flat to Round Transformers in Particle Accelerators, L. Dovlatyan, B. Beaudoin, S. Bernal, I. Haber, D. Sutter and TMA, Phys Rev Accel and Beams V25, 044002 (2022).
- Adjoint Equations for Beam-Wave Interaction and Optimization of TWT Design, A. Vlasov, TMA, D. Chernin and I. Chernyavskiy, IEEE Trans. Plasma Sci. V. March (2022).

Give a child a hammer and everything becomes a nail.

Signal to Noise Ratio in Klystrons & Gyro-Klystrons

TMA, W. Manheimer and A. Fliflet, PoP (2001).



Signal to noise ratio determined by ratio of injected signal power in Cavity 1 to fluctuating beam power due to discrete electronic charge.

Shot noise: If arrival times in cavity 1 are independent and identically distributed, fluctuations are a white noise process.

$$\langle I^2(t)\rangle = \int \frac{d\omega}{2\pi} e\langle I\rangle$$

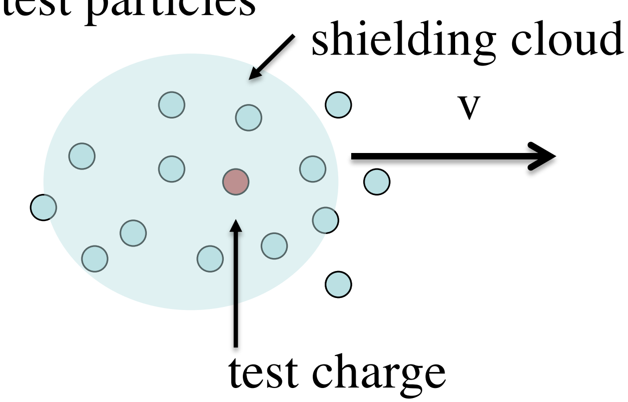
But, this is wrong: electrons become correlated on transit from cathode to Cavity 1. Significantly reduces noise level.

Shielding Cloud

Direct calculation: Problem 1

For an ensemble (N>>1) of initial conditions at the cathode of test electrons, calculate the shielding cloud and total current fluctuation that excites the relevant mode in the cavity. Must be done for each test charge

Method of dressed test particles



Adjoint approach: Problem 2

For a given cavity mode profile, integrate the kinetic equation (once) backward in time to find the sensitivity function at the cathode, average over initial ensemble of test electrons.

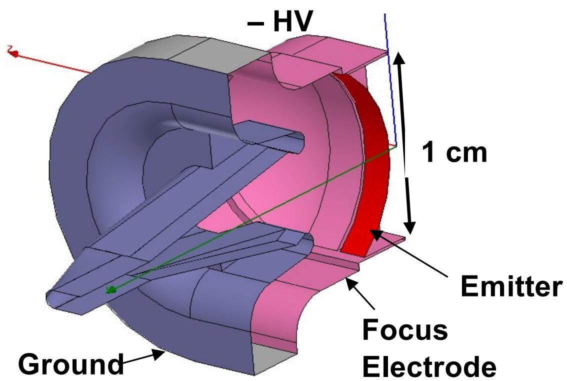
Shielding cloud is potentially **unstable** for **Gyro-Klystrons** **(must taper guiding magnetic field)**

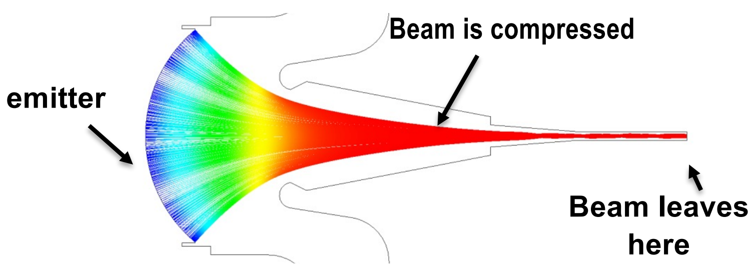
Global Beam Sensitivity Function for Electron Guns

TMA, D. Chernin, J. Petillo, Phys. Plasmas 26, 013109 (2019).

Thermionic Cathode Electron Gun

Solid Model of Electrodes





Goal

Evaluate a function that gives the variation of specific beam **FoMs** to variations in electrode potential/position

Can be used to

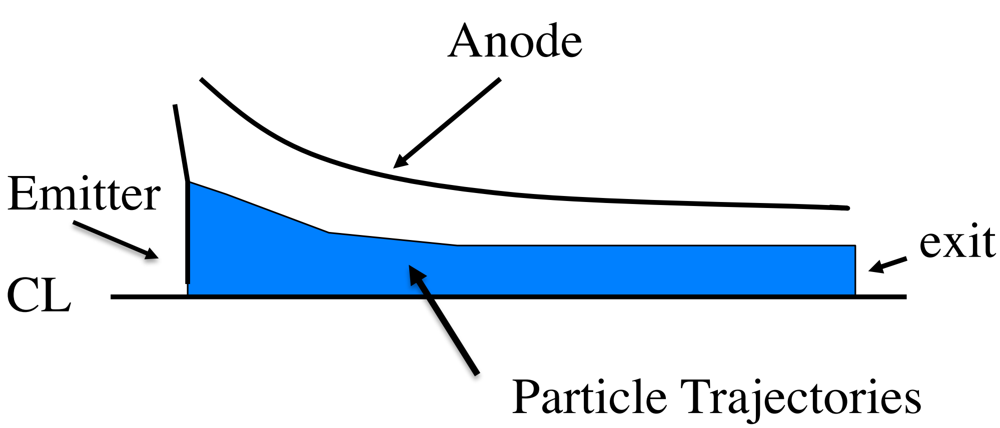
- establish manufacturing tolerances
- optimize gun designs

Should be embedded in gun code (e.g. Michelle)

What shape to make electrodes?

Particle in Cell (PIC) Code Michelle

Code solves the Steady state equations of motion for N particles j=1, N in self consistent fields



Start with vacuum fields
Solve for trajectories

$$\frac{d}{dt}\mathbf{x}_j = \mathbf{v}_j, \quad \frac{d}{dt}\mathbf{p}_j = q\left(\mathbf{E}(\mathbf{x}_j) + \mathbf{v}_j \times \mathbf{B}(\mathbf{x}_j))\right)$$

Accumulate a charge density on grid

$$\rho(\boldsymbol{x}) = \sum_j I_j \int_0^{T_j} dt\, \delta(\boldsymbol{x} - \boldsymbol{x}_j(t))$$

Solve Poisson Equation

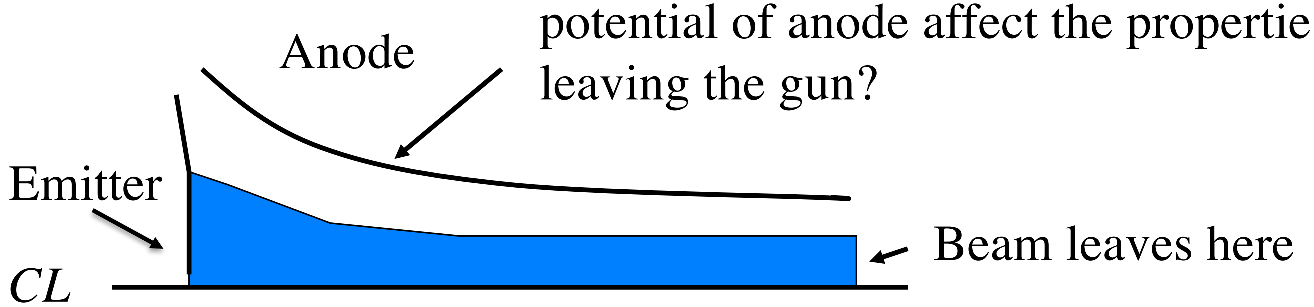
$$-\nabla^2 \Phi = \rho / \varepsilon_0$$

Iterates until converged

Petillo, J; Eppley, K; Panagos, D; et al., IEEE TPS 30, 1238-1264 (2002).

Beam Sensitivity Function

Basic question: How do small changes in position or potential of anode affect the properties of the beam leaving the gun?



Beam characterized by FoM, function of particle coordinates

$$F = F(\mathbf{p}_j, \mathbf{x}_j)_{z=L}$$

Example RMS size

$$F = \sum_j \left| \mathbf{x}_{\perp j} \right|^2$$

Conventional Approach: Solve directly (Problem 1)

Do many simulations with different anode potentials, positions

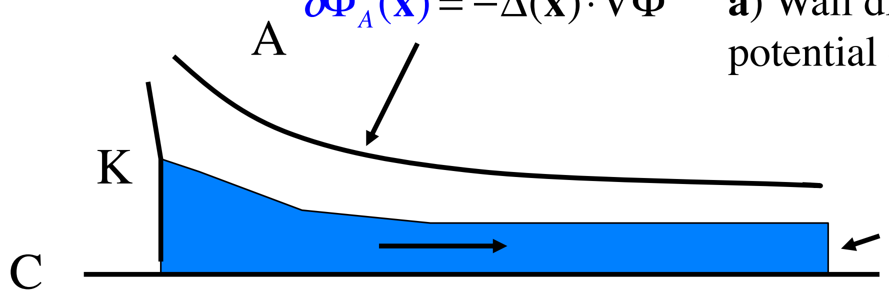
Select the best based on Figure of Merit (FoM) measured at the exit.

We need an adjoint problem

$\delta\Phi_A(\mathbf{x}) = -\Delta(\mathbf{x}) \cdot \nabla\Phi$

a) Wall displacement changes potential on unperturbed surface.

Problem #1
Direct



b) Leads to change in Figure of Merit F, function of electron coordinates.

$$F = F(\mathbf{p}_j, \mathbf{x}_j)_{z=L}$$

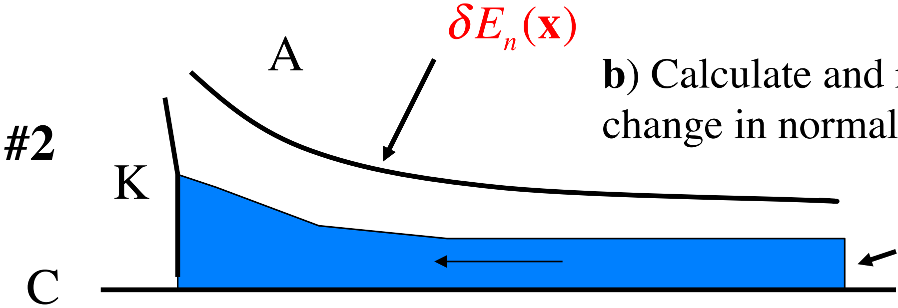
$$\delta F = \sum_j \left(\delta \mathbf{p}_j \frac{\partial F}{\partial \mathbf{p}_j} + \delta \mathbf{x}_j \frac{\partial F}{\partial \mathbf{x}_j} \right)_{z=L}$$

x

$\delta E_n(\mathbf{x})$

b) Calculate and record change in normal E.

Problem #2
Adjoint



a) Perturb electron coordinates in a prescribed way based on FoM, then reverse momenta and send back

Electrons run backwards

δE_n Is the sensitivity function

Sensitivity function

$$\delta F \propto \int_S da\, \delta\Phi_A(\mathbf{x}) \delta E_n(\mathbf{x})$$

Why does it work? Hamilton's Equations Conserve Symplectic Area

<u>2023 John Dawson Award for Excellence in Plasma Physics Research</u>

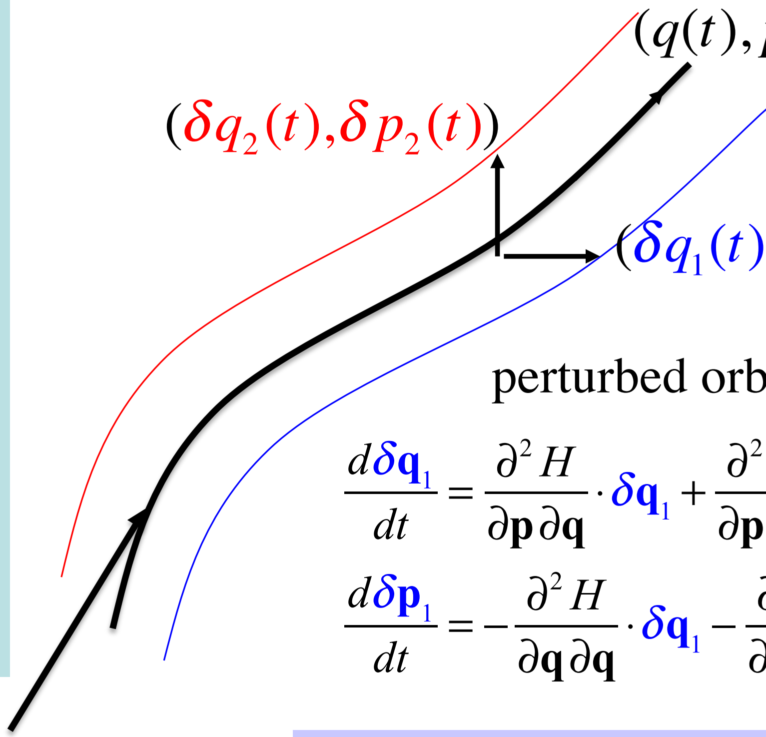
Philip Morrison, Hong Qin, and Eric Sonnendrücker

"For establishing and shaping the field of structure-preserving geometric algorithms for plasma physics."

$H(\boldsymbol{p},\boldsymbol{q},t)$

$$\frac{d\mathbf{q}}{dt}=\frac{\partial H}{\partial \mathbf{p}}$$

$$\frac{d\mathbf{p}}{dt}=-\frac{\partial H}{\partial \mathbf{q}}$$



perturbed orbit #1

$$\frac{d\delta\mathbf{q}_1}{dt}=\frac{\partial^2 H}{\partial\mathbf{p}\,\partial\mathbf{q}}\cdot\delta\mathbf{q}_1+\frac{\partial^2 H}{\partial\mathbf{p}\,\partial\mathbf{p}}\cdot\delta\mathbf{p}_1$$

$$\frac{d\delta\mathbf{p}_1}{dt}=-\frac{\partial^2 H}{\partial\mathbf{q}\,\partial\mathbf{q}}\cdot\delta\mathbf{q}_1-\frac{\partial^2 H}{\partial\mathbf{q}\,\partial\mathbf{p}}\cdot\delta\mathbf{p}_1$$

perturbed orbit #2

$$\frac{d\delta\mathbf{q}_2}{dt}=\ldots$$

$$\frac{d\delta\mathbf{p}_2}{dt}=-\ldots$$

Reference trajectory in 6N dimensional phase space

$$\frac{d}{dt}\left(\delta\mathbf{p}_1\cdot\delta\mathbf{q}_2-\delta\mathbf{p}_2\cdot\delta\mathbf{q}_1\right)=0$$

Area conserved for any choice of 1 and 2

Reference Solution + Two Linearized Solutions

$$(\mathbf{x}_j,\mathbf{p}_j) \rightarrow (\mathbf{x}_j,\mathbf{p}_j) + (\delta\mathbf{x}_j,\delta\mathbf{p}_j)$$

$$\rho(\mathbf{x}) \rightarrow \rho(\mathbf{x}) + \delta\rho(\mathbf{x})$$

$$\Phi(\mathbf{x}) \rightarrow \Phi(\mathbf{x}) + \delta\Phi(\mathbf{x})$$

Reference Solution Perturbation

Two Linearized Solutions

$[\delta x_j(t),\delta p_j(t)]$ true – changes in anode

$[\delta \hat{x}_j(t),\delta \hat{p}_j(t)]$ adjoint – we pick

Subject to different BC's

Can show difference in symplectic area entering and leaving is given by surface integral of perturbed fields

$$\sum_j I_j\left(\delta\hat{\mathbf{p}}_j\cdot\delta\mathbf{x}_j - \delta\mathbf{p}_j\cdot\delta\hat{\mathbf{x}}_j\right)\Big|_0^L = -q\varepsilon_0\int_S da\left[\delta\Phi\,\mathbf{n}\cdot\nabla\delta\hat{\Phi} - \delta\hat{\Phi}\,\mathbf{n}\cdot\nabla\delta\Phi\right]$$

Conservation of Symplectic Area meets Green's Theorem !

What is the change in a generic figure of merit?

$$F = F(\mathbf{p}_j, \mathbf{x}_j)_{z=L}$$

$$\delta F = \sum_j \left(\delta \mathbf{x}_j \frac{\partial F}{\partial \mathbf{x}_j} + \delta \mathbf{p}_j \frac{\partial F}{\partial \mathbf{p}_j} \right)_{z=L}$$

$$\delta\Phi\big|_B = -\,\Delta(\mathbf{x}) \cdot \nabla\,\Phi\big|_B$$

Generalized Green's Theorem

$$\sum_j I_j \left(\delta\mathbf{x}_j \cdot \delta\hat{\mathbf{p}}_j - \delta\mathbf{p}_j \cdot \delta\hat{\mathbf{x}}_j \right)\Big|_0^L = -q\varepsilon_0 \int_S da \left[\delta\Phi\, \mathbf{n} \cdot \nabla \delta\hat{\Phi} - \delta\hat{\Phi}\, \mathbf{n} \cdot \nabla \delta\Phi \right]$$

Problem #2 Perturbed trajectories at exit proportional to grad-F

$$I_j \delta\hat{\mathbf{p}}_j\big|_T = \frac{\partial F}{\partial \mathbf{x}_j}, \quad I_j \delta\hat{\mathbf{x}}_j\big|_T = -\frac{\partial F}{\partial \mathbf{p}_j},$$

$$\delta\hat{\Phi}(\mathbf{x}) = 0$$

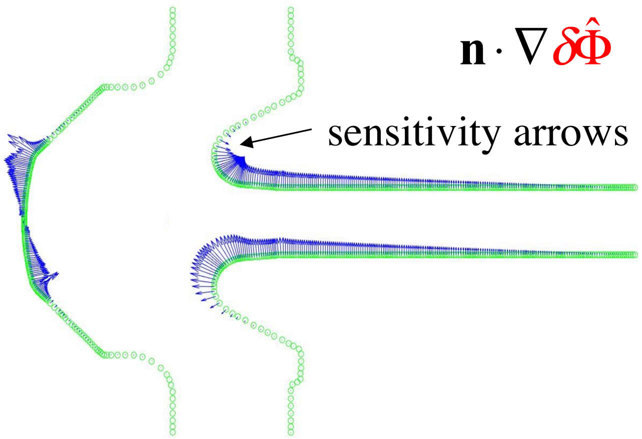
Sensitivity Function

$$\delta F = -q\varepsilon_0 \int_S da\, \delta\Phi \left(\mathbf{n} \cdot \nabla \delta\hat{\Phi} \right)$$

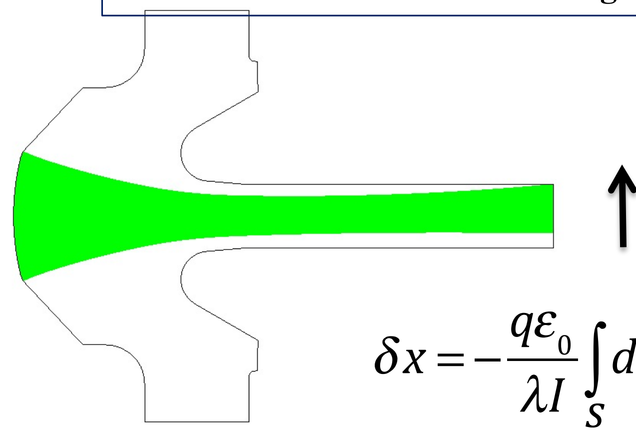
Vertical Displacement of the Beam

Vector plot of the 'sensitivity' or Green's function

$$\mathbf{n} \cdot \nabla \delta\hat{\Phi}$$



'Direct' MICHELLE Simulation with Perturbed Anode Voltages



$$\delta x = \sum_j \frac{1}{N} \delta x_j$$

$$\delta x = -\frac{q\varepsilon_0}{\lambda I} \int_S da\,\mathbf{n} \cdot \delta\Phi \nabla \delta\hat{\Phi}$$

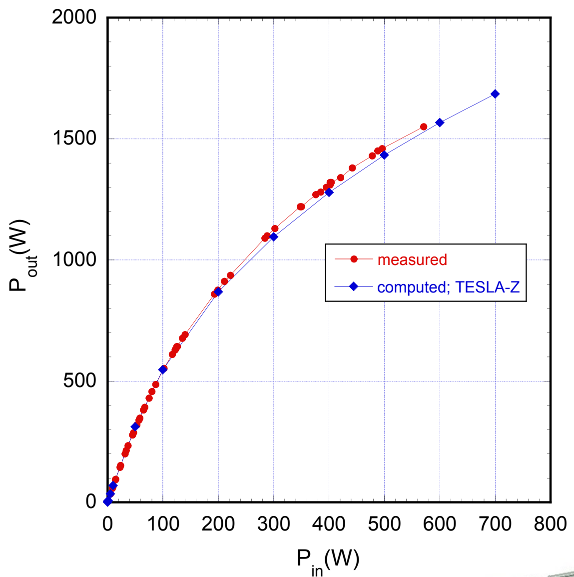
Predicted displacement / Calculated displacement = 0.9969

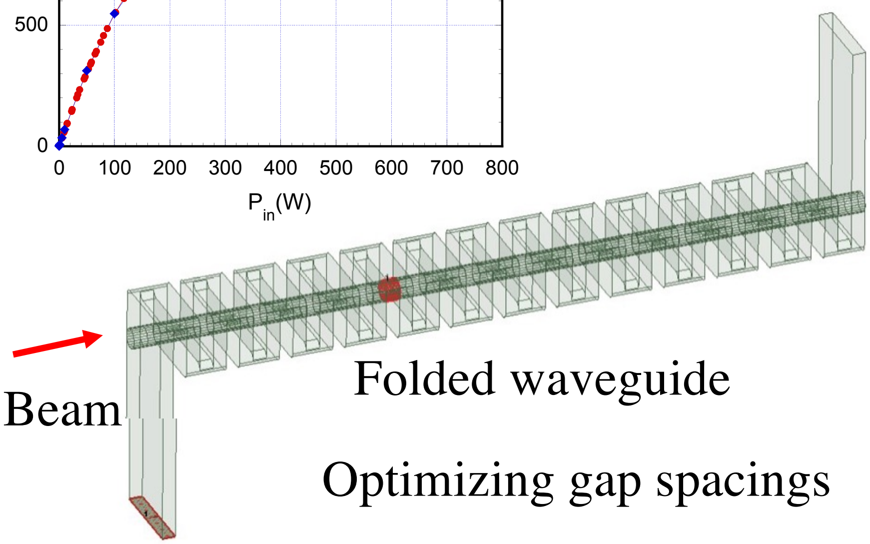
Currently being updated to include B-field:

John Petillo, Serguei Ovtchinnikov, Aaron Jensen (Leidos), Philipp Borchard (Dymenso)
Kyle Kuhn, Heather Shannon, Brain Beaudoin TMA. (U. Maryland) ONR STTR

Optimization of TWT Design Using Adjoint Approach

A. Vlasov, TMA, D. Chernin, I. Chernyavskiy, IEEE Trans Plasma Science 2023.





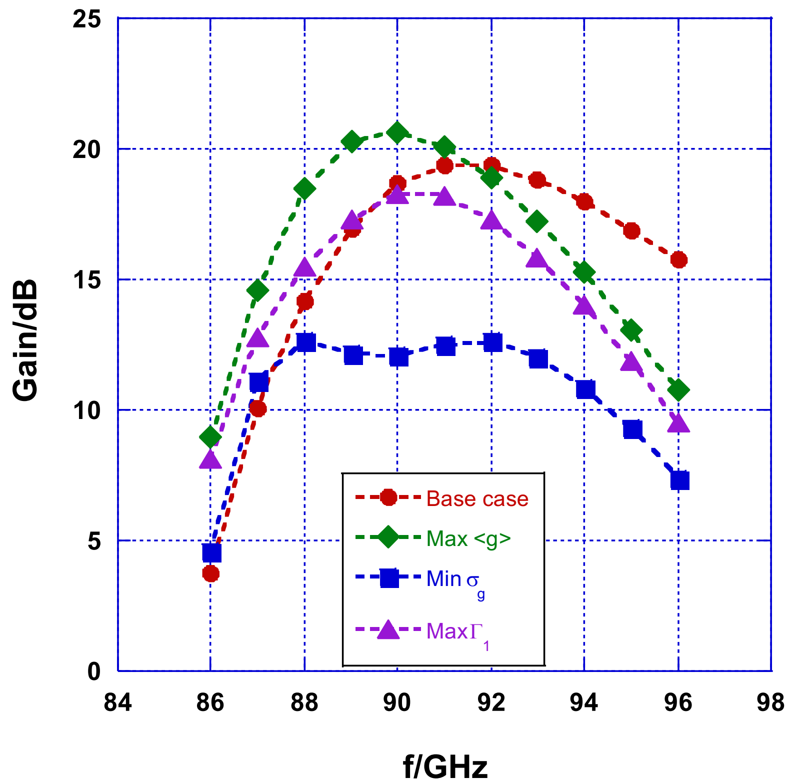
Output

Beam

Folded waveguide

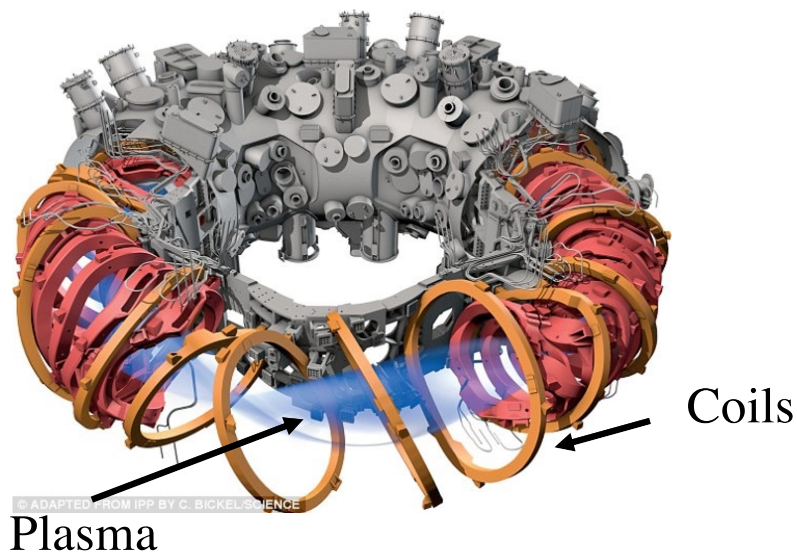
Optimizing gap spacings

Input



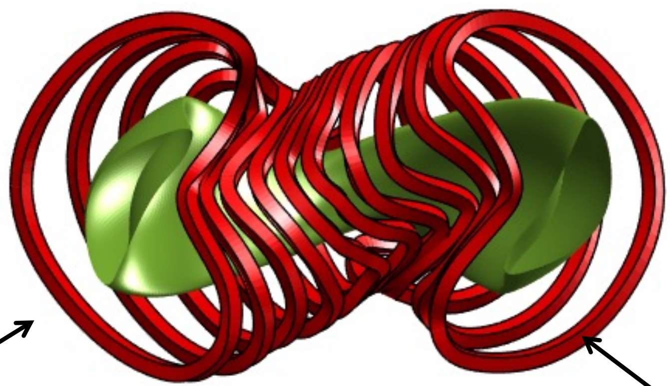
Optimization of Small Signal Gain

3D MHD Equilibria



Wendelstein 7-X
Max Planck Institute for Plasma Physics (IPP)
Greifswald, Germany   Completed 2015

3D MHD Toroidal Equilibrium



Coils

In plasma

$$-\nabla p + \frac{\mathbf{J} \times \mathbf{B}}{c} = 0$$

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J}$$

Basic Question: How do changes in coil currents or shapes affect equilibrium?

Alternatively, how do changes in the shape of the outermost flux surface affect equilibrium?

Adjoint Equations for Stellarator Equilibria

Shape Gradient Sensitivity Functions

Elizabeth Paul

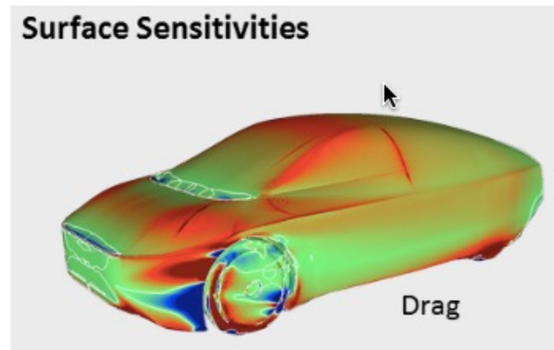
2021 APS Marshall N. Rosenbluth Outstanding Doctoral Thesis Award

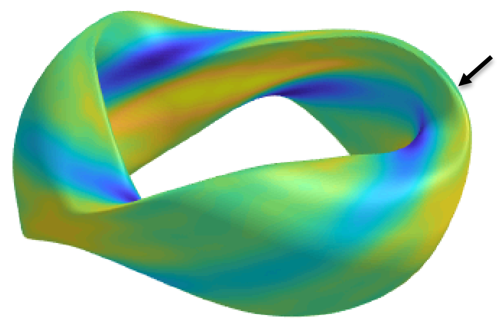
"Adjoint methods for stellarator shape optimization and sensitivity analysis" UMD 2020

Currently Assistant Prof. Columbia

Landreman and Paul, (2018) Nucl. Fusion 58 076023,
TMA , E. Paul, M. Landreman, J. Plasma Phys. (2019),
E. Paul, M. Landreman, TMA, J. Plasma Phys. (2021),
R. Nies, E. Paul, S. Hudson, and A. Bhattacharjee, J. Plasma Phys. (2022), vol. 88, 905880106

DRAG





Colors show sensitivity of rotational transform to displacement of outer flux surface

Linear Perturbations to Equilibrium
Similar to MHD stability $-\Delta W(\xi)$

ξ - field line displacement

$$\mathbf{B} \Rightarrow \mathbf{B} + \nabla \times \left(\xi \times \mathbf{B} - \delta\Phi_P \nabla \zeta \right)$$

Changes in magnetic field

$$\nabla p \Rightarrow \nabla \left(p + \xi \cdot \nabla p \right) + \nabla \cdot \delta \underline{\underline{\mathbf{P}}}$$

Added pressure tensor

$$\mathbf{J}_C \Rightarrow \mathbf{J}_C + \delta \mathbf{J}_C$$

Changes in current/shape of coils
or Shape of outer flux surface

$$\xi \cdot \mathbf{n} \big|_{\text{Boundary}}$$

Generalized Forces and Responses

Responses

Vacuum fields

Plasma displacement

Toroidal current profile

$$\begin{pmatrix} \delta \mathbf{A}_V \\ \xi \\ d\delta I_T / d\psi \end{pmatrix} = \underline{\underline{O}} \begin{pmatrix} \delta \mathbf{J}_C \\ \nabla \cdot \delta \underline{\underline{P}} \\ \delta \Phi_P \end{pmatrix}$$

Forces:

Coil currents

Pressure tensor

Rotational transform

More generically, for two different perturbations

$$\delta x_i^{(1)} = \sum_j O_{ij}\, \delta F_j^{(1)} \qquad \delta x_i^{(2)} = \sum_j O_{ij}\, \delta F_j^{(2)}$$

Self-Adjoint Symmetry Gives

$$\sum_j \left\{ \delta x_i^{(1)} \delta F_i^{(2)} - \delta x_i^{(2)} \delta F_i^{(1)} \right\} = 0$$

Example Adjoint Relation

Make this appear to be change in FoM

Adjoint added pressure tensor

Adjoint sensitivity function

$$\int_{VP} d^3x \left(\xi \cdot \nabla \cdot \delta \underline{\underline{\mathbf{P}}}_L \right) = -\frac{1}{4\pi} \int_S d^2x \, \mathbf{n} \cdot \xi \left(\delta \mathbf{B} \cdot \mathbf{B} \right) = 0$$

True interior displacement

True outer flux surface displacement

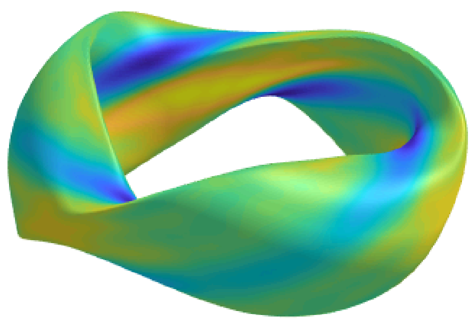
Surface and Coil Sensitivity

Adjoint Approach to gradient calculation

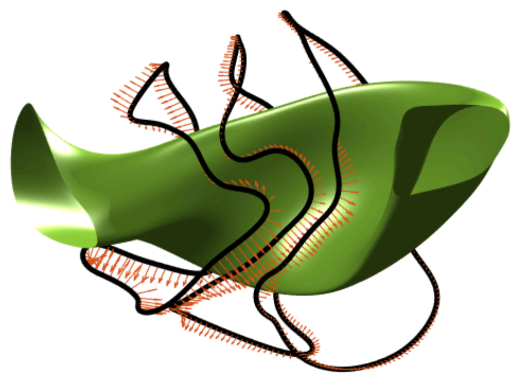
> 500 X Speed – Up over direct calculation

Uses VMEC & DIAGNO
Hirshman and Whitman, 1983 Phys. Fluids 25 3553
H.J. Gardner 1990 *Nucl. Fusion* 30 1417

Different Figures of Merit Possible
Plasma pressure – beta
Rotational transform
Toroidal current
Neoclassical radial transport -1/ν regime
Energetic particle drifts
Quasi-symmetry



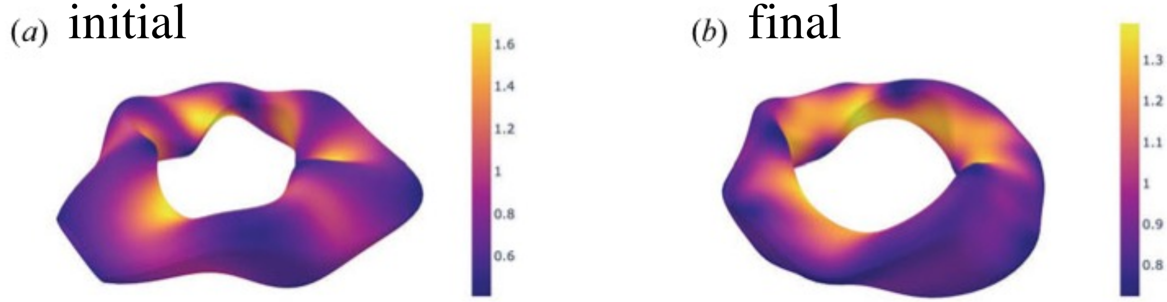
Surface shape sensitivity



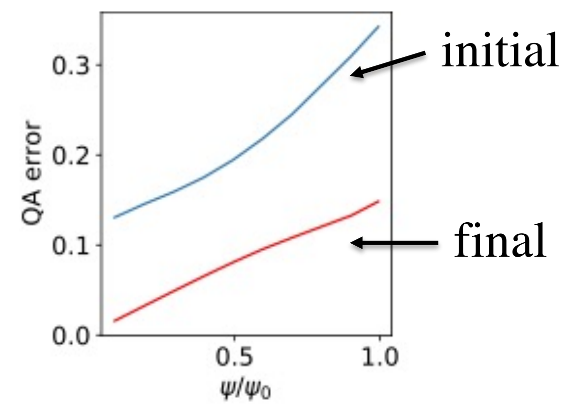
Coil location sensitivity

J. Plasma Phys. (2021), *vol.* 87, 905870214
Published by Cambridge University Press
doi:10.1017/S0022377821000283

Gradient-based optimization of 3D MHD equilibria



Optimized for “Quasi-symmetry” on axis
(good particle confinement)



Challenges

Sensitivity function

$$\int_{VP} d^3x \left(\xi \cdot \nabla \cdot \delta \underline{\underline{\mathbf{P}}}_L\right) = -\frac{1}{4\pi}\int_S d^2x \, \mathbf{n}\cdot \xi \left(\delta \mathbf{B} \cdot \mathbf{B}\right) = 0$$

Make this appear to be change in FoM

Surface displacement

Limited number of FoMs can be put in this form.
Formulation must be compatible with 3D - Equilibrium codes

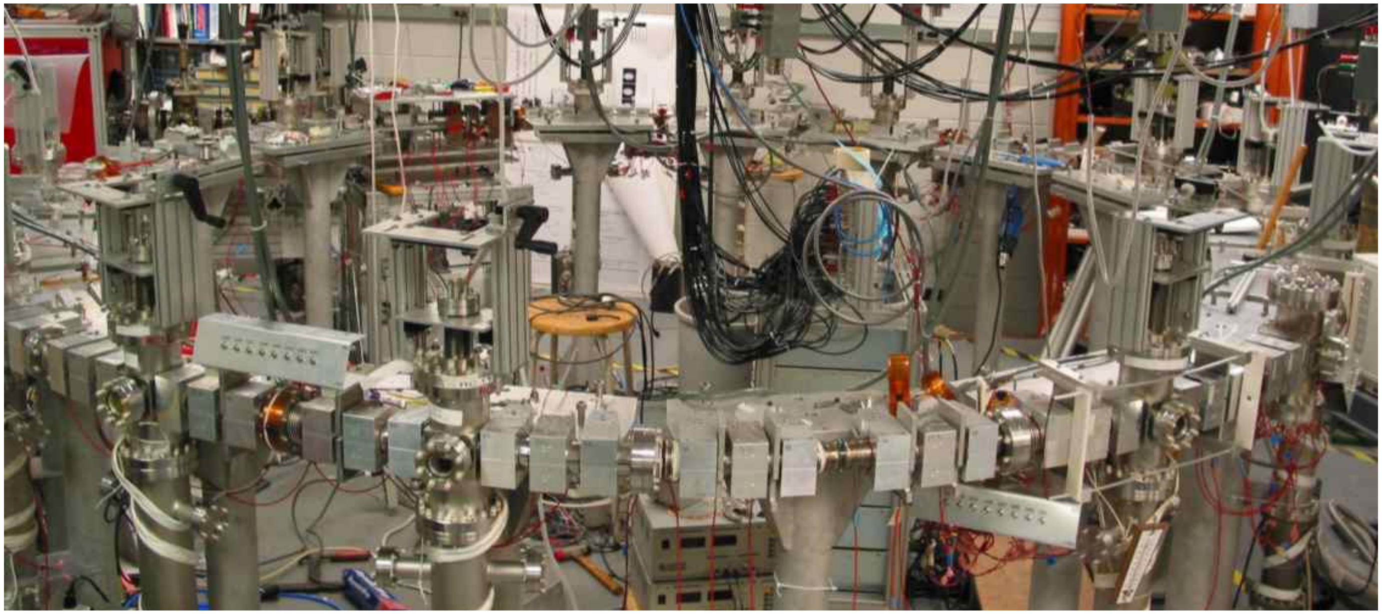
Minimize Energy
VMEC - S. P. Hirshman and J. C. Whitson, (1983).
SPEC - S. R. Hudson, R. L. Dewar, G. Dennis, M. J. Hole, M. McGann, G. von Nessi, and S. Lazerson, (2012).

Solve Force Balance
DESC – D. W. Dudt, E. Kolemen, (2020), D.W. Dudt , R. Conlin1, D. Panici and E. Kolemen, (2023)

Includes automatic differentiation to compute FoM gradient Program takes code, breaks into primitive operations and computes derivatives

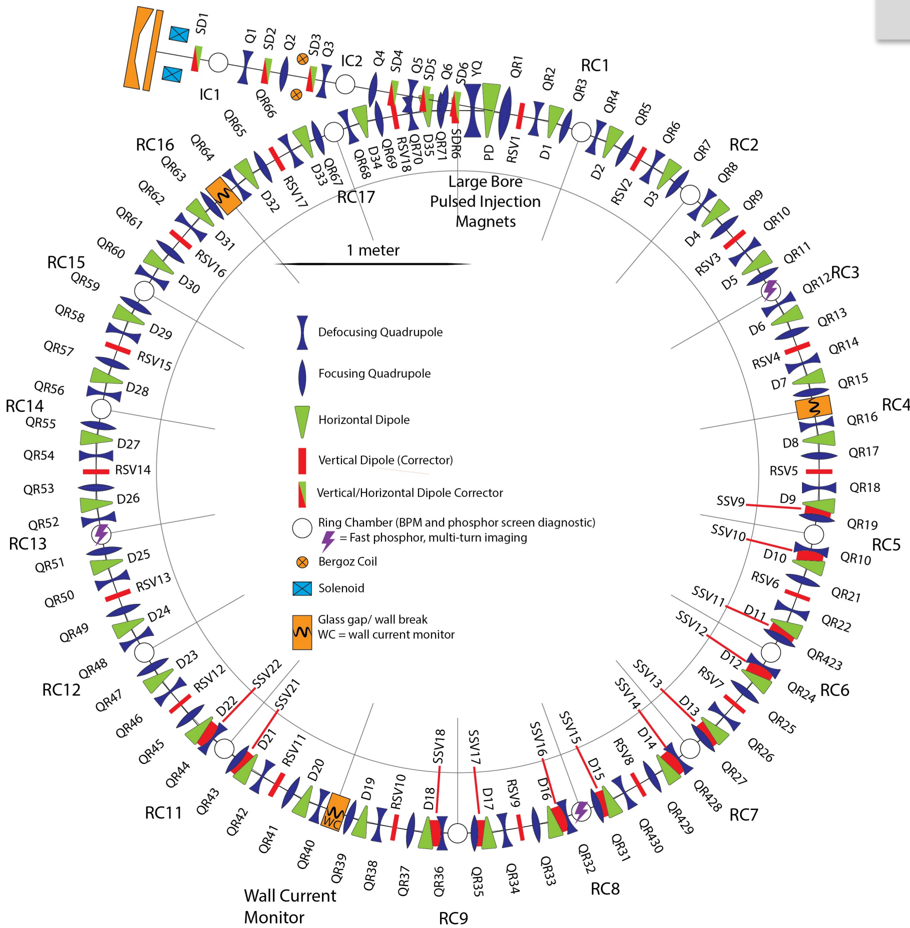
Optimization of Focusing Magnets in Accelerator Lattices

The University of Maryland Electron Ring



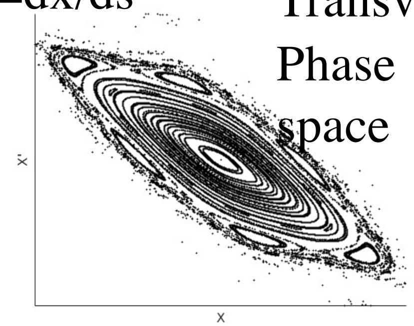
UMER is a fully functional electron storage ring

UMER Systems and Layout



- 167 Magnets, power supplies & controls.

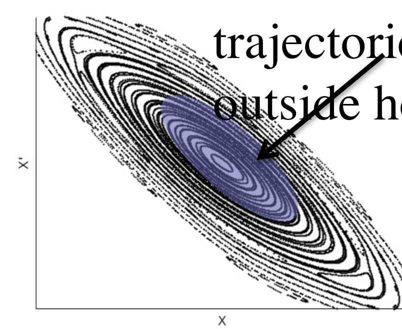
x'=dx/ds



Transverse Phase space

x

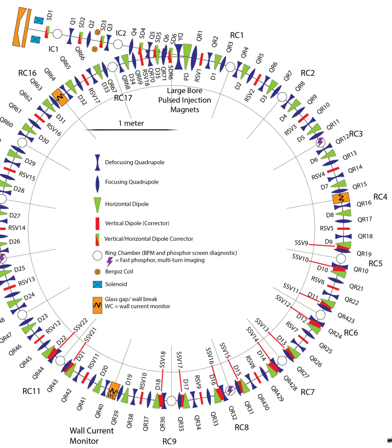
x'=dx/ds



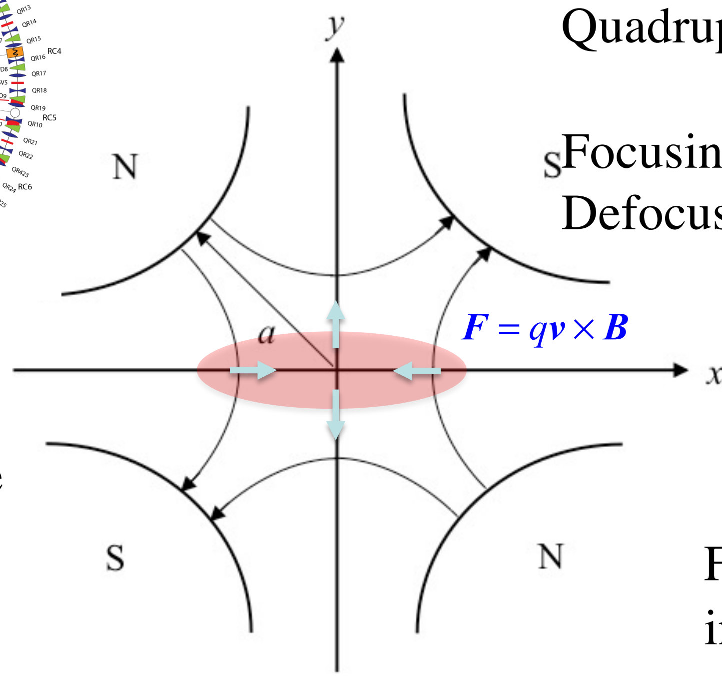
Want to avoid trajectories outside here

x

Focusing Basics



32 Quadrupole magnets



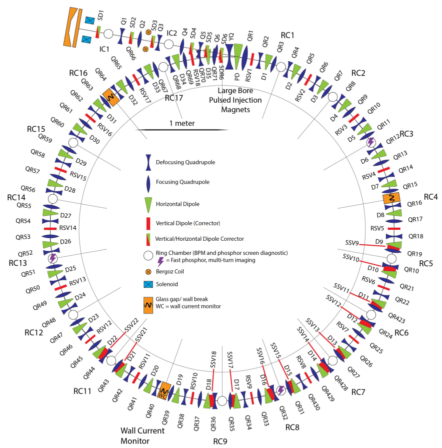
Quadrupole Magnetic Field

Focusing in x-direction
Defocusing in y-direction

qv – out of page

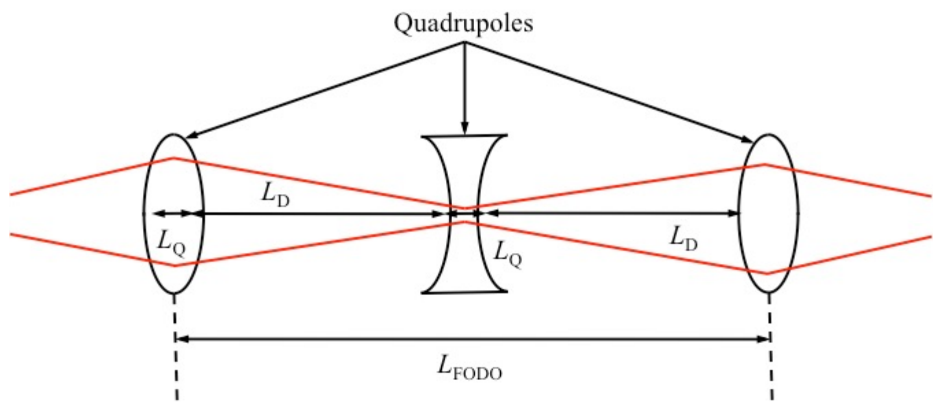
Field strength increases linearly with distance from the axis

FODO Lattice



32 Quadrupole magnets

Alternate focusing and defocusing orientations



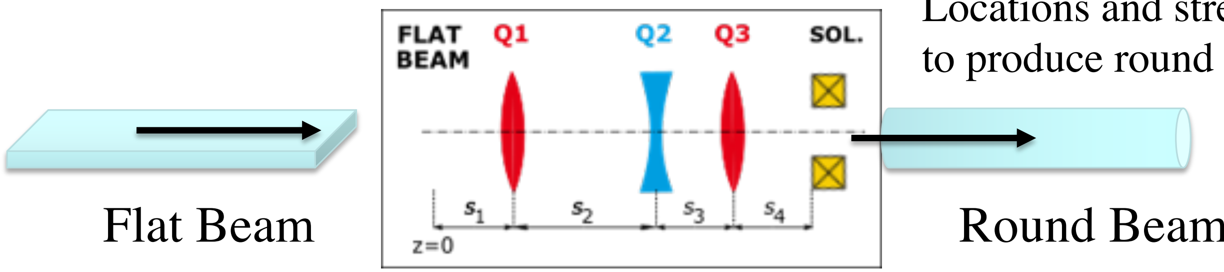
Qualitative explanation: beam passes through focusing quad when it is big (strong fields). Passes through defocusing quad when it is small (weak fields). Net effect is focusing.

Beam distribution depends on many parameters How to optimize?
Can we become Lords of the Ring?

Optimization of Flat to Round Transformers Using Adjoint Techniques

L. Dovlatyan, B. Beaudoin, S. Bernal, I. Haber, D. Sutter and T. M. A. Phys. Rev. Acc. Beams (2022).

Locations and strengths of magnets optimized to produce round beam in solenoid



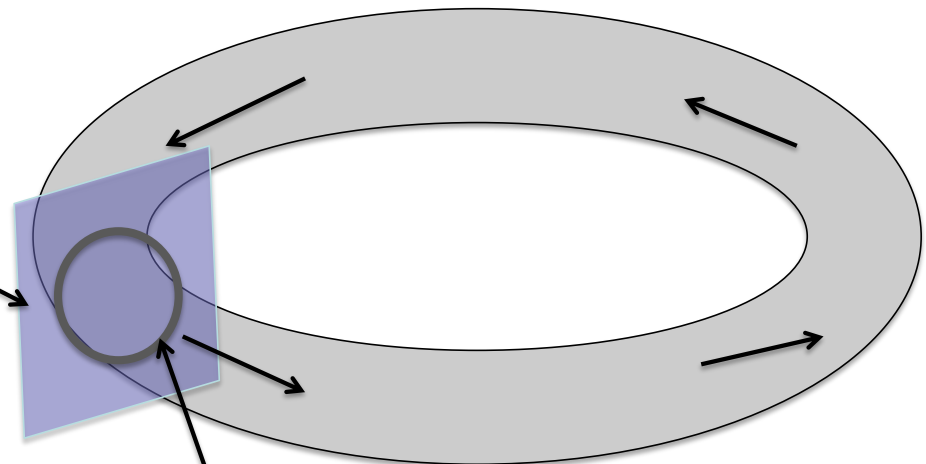
Flat to Round and Round to Flat transformers are proposed for cooling of hadron beams. Ya. Derbenev, Adapting optics for high-energy electron cooling, Tech. Rep. UM-HE-98-04-A (1998).

Cylindrical hadron beam cools via collisions when co-propagated with electron beam. Optimized when beams overlap and transverse energy is minimum.

Circular Accelerators-Periodicity

Solve Eqs. of motion and self fields

Particles return to initial plane.
Need to maintain periodicity of distribution, not individual orbits



Start here with 4D particle coordinates

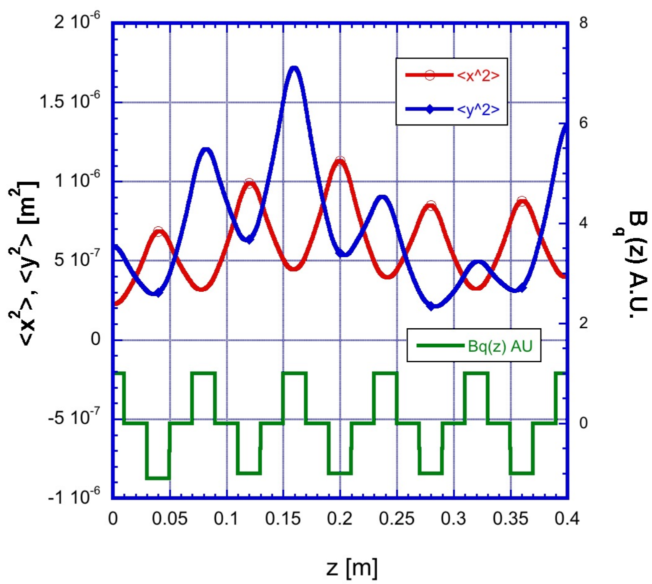
Big problems:
Do periodic distributions exist? Most likely no.
How to relaunch particles to optimize?

Constrained Optimization

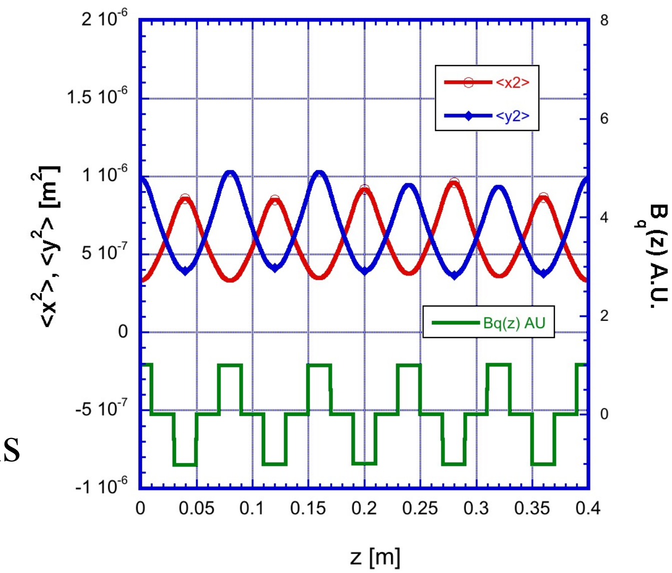
"Adjoint with a Chaser"

Test Problem – 10 Quadrupole lattice

Initial



Final



Transverse moments

Quadrupole strengths

Beam not in equilibrium
Large beam waist excursion

Beam moments become periodic
Excursions minimized (FoM)

Conclusion

Adjoint methods are a powerful way to evaluate parameter dependences in many systems involving charged particle dynamics.

Issue: coding complexity
Adjoint vs Automatic differentiation?

Thank you.

Acknowledge: ONR, DoE, AFOSR, Simons Foundation

Fuel Efficiency 1985 Volvo 240 DL